Bremsstrahlung and Electron Scattering Cross Sections in Au for 247-Mev Electrons and Positrons*

PHILIP C. FISHER†
University of Illinois, Urbana, Illinois
(Received June 29, 1953)

The absolute cross sections for gold for the top 40 percent of the bremsstrahlung spectrum have been measured for electrons and positrons of 247-Mev mean energy. A magnetic cloud chamber containing two Au foils was used. The foils were positioned to intercept an electron or alternately a positron beam provided by a magnetic field analysis of the electron-positron pair members produced in a Pb foil situated in the x-ray beam of the 300-Mev betatron. With the possible exception of the top 2 percent of the spectrum, electron and positron bremsstrahlung were the same. For quantum energies in the interval 0.604 to 0.927 of maximum, the Bethe-Heitler spectrum is 8.7±2.3 percent above the experimental spectrum. In the same interval the Maximon-Bethe-Davies theory agrees with experiment. Scattering by electrons was also observed as a function of energy of the struck or lowest energy electron. For these particles having energies between 20 and 130 Mev, the scattering cross section ratios of experiment to theory are 0.96±0.06 and 1.06±0.08 for incident electrons and positrons, respectively. The cross section ratio of electrons to positrons in the same energy interval is 1.31±0.13 for experiment and 1.46 for theory. All errors are probable errors.

INTRODUCTION

THE Bethe-Heitler[1] (hereafter referred to as BH) and Wheeler-Lamb[2] theories of bremsstrahlung, applicable for nuclear and electronic fields, respectively, have generally been used to describe the radiative energy losses experienced by electrons and positrons in passing through matter for particle energies much larger than mc^2. A first-order Born approximation was used for the BH derivation. For relativistic electrons or positrons on Au the possibly overly stringent conditions for the validity of this approximation are not fulfilled. Maximon, Bethe, and Davies[3,4] (hereafter referred to as MBD) have derived a nuclear bremsstrahlung cross section without the use of the Born approximation. The MBD theory yields a cross section of the order of 10 percent smaller than that predicted by the BH theory.

Curtiss[5] has investigated the shape and magnitude of the bremsstrahlung spectrum produced by 60-Mev electrons in traversing Pb foils. There have been no experiments with particles of energy greater than 60 Mev of sufficient accuracy to precisely determine either the shape or the absolute magnitude of the bremsstrahlung cross section. This experiment is primarily concerned with supplying these two pieces of information for both electrons and positrons striking Au, for particles radiating more than two-thirds of their energy.

When a particle experiences radiation on passage through a foil located in a magnetic cloud chamber, the straggling is detected from the different track curvatures exhibited on opposite sides of the foil. By determining the number and energy of such radiative events, the shape of the high-energy part of the bremsstrahlung spectrum is found. From the total number of foil traversals made by the incident particles and the energy spectrum of these particles the absolute cross section for radiation is found.

A by-product of the experiment is the information obtained on the elastic scattering of positrons and electrons by electrons in the foil if the struck or lowest energy electron receives more than 10 Mev. Comparisons with the Bhabha[6] and Moller[7] theories are made.

THE EXPERIMENT

Two Au foils of effective thickness 0.3799±0.0009 g/cm² were placed parallel to one another in a 33 cm diameter cloud chamber. The foils were each 7.6 cm high and 20.3 cm long and were located 6.5 cm and 20 cm from the center of an Al-covered particle entrance window in the glass cloud-chamber ring. This thickness and arrangement was based on compromising four factors: first, thick enough foils to give a reasonable number of pictures; second, sufficient chord lengths of almost all straggled tracks to allow accurate energy measurement; third, thin enough foils to hold the occurrence of successive events to a minimum; fourth, sufficient distance in front of the first foil to distinguish electron produced events from stray x-ray produced events, and to allow rejection of obviously degenerate energy and/or poorly directed (collimator scattered) tracks.

The apparatus alignment is shown in Fig. 1. The source of particles was the pairs produced by allowing x-rays from the 300-Mev betatron to impinge on a $\frac{1}{16}$-in. thick, 1-cm by 2-cm Pb sheet placed in an 11 500-gauss

* This work was supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
† Now at Los Alamos Scientific Laboratory, Los Alamos, New Mexico.
[1] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **146**, 83 (1934).
[2] J. A. Wheeler and W. E. Lamb, Phys. Rev. **55**, 858 (1939).
[3] L. C. Maximon and H. A. Bethe, Phys. Rev. **87**, 156 (1952).
[4] H. Davies and H. A. Bethe, Phys. Rev. **87**, 156 (1952).
[5] C. D. Curtiss, Phys. Rev. **89**, 123 (1953).
[6] H. J. Bhabha, Proc. Roy. Soc. (London) **154**, 195 (1936).
[7] C. Moller, Ann. Physik **14**, 531 (1932).

analyzing field about 3.7 m from the betatron target. Electron pair members produced in the x-ray converter passed through two small primary collimators before entering the cloud chamber. The first collimator was formed by the 1.3-cm separated pole faces of the analyzing magnet and a Pb brick supposedly placed to intercept most electrons of energy less than 265 Mev. The second collimator was located at the cloud chamber and was a 5-cm thick Pb brick with a 0.3-cm by 12.6-cm aperture. Immediately behind and centered with respect to this was the 0.7-cm by 14-cm 5-mil Al foil covered slot in the glass cloud-chamber ring.

By simultaneously reversing polarity of the analyzer and cloud-chamber fields, oppositely charged pair members having the same energy distribution were brought into the chamber.

The cloud-chamber tracks were photographed stereoscopically using a pair of cameras and a single mirror. Pulsed lights provided a horizontal sheet of light in the chamber which was at least 5.8 cm high and a chamber diameter in width. A coating of Aquadag of about 0.00014 g/cm^2 was applied to each side of each Au foil to cut down reflection of light to the roof and floor of the chamber. Light scattered at approximately 90° from the track droplets sensitized the film. One hundred foot ro'ls of Kodak linograph-ortho films were employed because of this film's small shrinkage and good track to background contrast. Each roll of film recorded incident particles of one sign of charge, alternate rolls recording incident particles of the opposite sign.

A magnetic pulse time delay circuit[8] was used for timing and sequencing the cloud-chamber functions, the betatron, and the diesel driven generator which supplied the cloud chamber's pulsed magnetic field. The chamber was expanded when the magnetic field reached the maximum value. Particles entered the chamber 0.6 second after expansion and were allowed to grow 0.12 second before the pulse lights were flashed. The chamber was fast compressed 0.01 second later. A 15-second cycle time was used.

The maximum value of the magnetic field averaged over the whole experiment was $10\,020\pm180$ gauss. The true maximum fields were 0.5 percent higher for electrons and the same amount lower for positrons than the average field; the fields increased uniformly with time from -0.4 percent to $+0.4$ percent during the experiment. This drift produced a probable uncertainty of ±0.2 percent for which the data to be presented cannot be corrected. A calibrated fluxmeter was used to standardize a meter indicating coil current against the peak magnitude of the field pulse. The point on the current peak field curve corresponding to 1800 gauss was checked using a proton magnetic moment detector.

Analysis of the pictures involved counting all foil traversals and measuring the radii and angles of deflection, α and φ in planes parallel and perpendicular

[8] C. R. Emigh, Rev. Sci. Instr. **21**, 142 (1950).

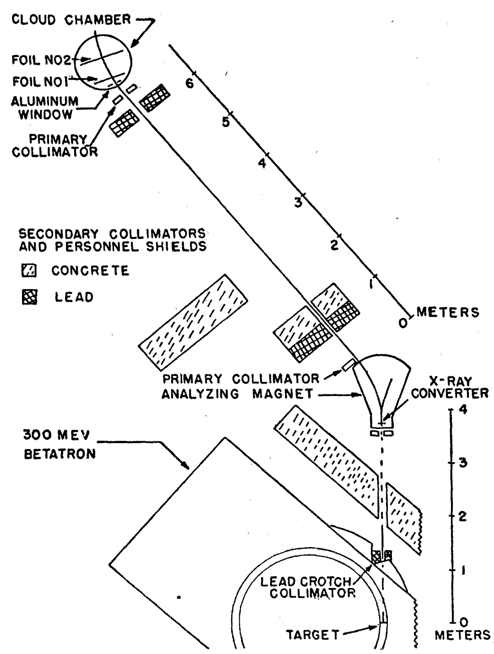

FIG. 1. Schematic for obtaining a particle beam in the chamber.

to the magnetic field, of any particles which had experienced events. Events were labeled as to type in accordance with the following conventions: bremsstrahlung if one particle came out of the foil with an energy ≤ 108 Mev; elastic electron scattering if two electrons came out each having an energy ≥ 10 Mev with their combined energy being ≥ 130 Mev; elastic positron scattering if an electron of energy ≥ 10 Mev came out, the combined energy of the electron and positron out being ≥ 130 Mev; and pair production if three or two particles came out with the particle with electric charge of opposite sign than that of the incident particle having an energy ≥ 10 Mev and the combined energy of the emergent particles being ≥ 130 Mev. The reasons for limitation of the accepted scattered electrons to energies ≥ 10 Mev were ease of observation and exclusion of events produced by a combined bremsstrahlung and Compton effect.

To avoid the difficulties of identification and measurement encountered when events occurred near the boundaries of the lighted region strict track acceptance criteria were established. These restricted to set values, α, φ, the horizontal and vertical positions in the chamber, and the minimum energy of particles entering either foil. In addition any aggregate of two or more tracks crossing the first foil which could not readily be resolved into its components before crossing the foil resulted in discarding the entire picture.

Angles were measured with a device similar to that described by Brueckner *et al.*[9] Energies up to 130 Mev were found using an instrument constructed from the

[9] Brueckner, Hartsough, Hayward, and Powell, Phys. Rev. **75**, 555 (1949).

design of Emigh.[10] Above 110 Mev an optical track straightening device employing a prism was used. This device will be described in a paper to be published later. The latter energy region is associated with the incident particles. To obtain this energy spectrum for the incoming electrons, all tracks of every tenth accepted picture were measured between the foils. Only one-thirtieth of the accepted pictures were measured to obtain the incident positron energy spectrum.

Energy lost by ionization in the foil was accounted for by adding to each measured energy $\Delta E \sec\alpha \sec\varphi$, the sum being called the total energy. ΔE is calculated from Bloch's theory[11] and the assumption that the radiative and scattering events occurred in the center of the foil.

During the initial stages of analysis, pictures were occasionally deliberately discarded because of elastic scattering and pair production events. About one-third of the pictures were analyzed before this discrimination was corrected. It was necessary to examine the remaining two-thirds of the pictures twice to be certain that all pair and scattering events were properly recorded.

THE THEORY

A. Bremsstrahlung

When high-energy electrons, or positrons, pass through a foil in a cloud chamber those bremsstrahlung events which are observed are caused by interaction of the incident particle with the two distinct entities of which the foil is composed; nuclei and electrons. In the process arising from the atomic electron interaction, the recoil momentum of the orbital electron is in general so small that it will not be seen in the chamber. Consequently this process cannot be distinguished from bremsstrahlung caused by a nucleus, and a correction must be applied to the number of events seen to find that component attributable to the nuclei alone. If the foil is sufficiently thick, corrections must also be made for multiple events. Thus consideration must be given those particles which undergo several successive radiations in passage through the foil, as the total emergent energy spectrum will be distorted from that due to a single radiation process. Another correction arises from the conversion of bremsstrahlung into electron-positron pairs and the resultant impossibility of identifying that particle which experienced the radiation straggling. Finally the possibility of a particle radiating more than one quantum while passing through the field of a single nucleus must be considered.

In 1934, BH published their theory of bremsstrahlung which took into account the electronic screening of the nucleus by using a Fermi-Thomas model atom. The perturbation which causes an incident particle of energy E_0 to transform to an energy E and a quantum of energy $k=E_0-E$ is the interaction of the particle with the atomic field and the interaction of the particle with the radiation field giving rise to the emission of the quantum E_0-E. As the Born approximation was used, the result is accurate provided $Zc/137v_1 \ll 1$ and $Zc/137v_2 \ll 1$, where v_1 and v_2 are the velocities of the electron in the initial and final state. For relativistic electrons in Au, these reduce to $0.58 \ll 1$ and indicate that some error may exist. The MBD theory accounts for the same process without the use of the Born approximation. To be valid this theory requires E_0, $E \gg mc^2$ and scattering angles of the order of mc^2/E_0, which are the only angles contributing significantly to the total cross section. With the exception of a constant the cross sections of the two theories are identical.

No difference between electron and positron bremsstrahlung is predicted by either theory. The reason being that the electric charge of the particle appears in the matrix element which when squared leads to the cross section. However some difference in behavior is expected for positive and negative particles. Emission of radiation is caused by the influence of the nuclear electric field, modified by the orbital electrons. A positive charge is repelled and should not approach the nucleus as closely as a negative charge, and so should interact less strongly than a negative particle.

Expressions for the probability of bremsstrahlung in an atomic field are

no screening, $2 \leq \gamma \leq 15$,

$$\varphi_\nu(E_0, E) = \{4Z^2/(137E_0^2k)\}\{e^2/mc^2\}^2 \times\{E_0^2+E^2-2EE_0/3\} \times\{\ln 2E_0E/(mc^2k) - \tfrac{1}{2} - c(\gamma) - f(Z)\}; \quad (1)$$

partial screening, $0 \leq \gamma \leq 2$,

$$\varphi_\nu(E_0, E) = \{Z^2/(137E_0^2k)\}\{e^2/mc^2\}^2 \times\{[E_0^2+E^2][\varphi_1(\gamma) - (4/3)\ln Z - 4f(Z)] - (2EE_0/3)[\varphi_2(\gamma) - (4/3)\ln Z - 4f(Z)]\}, \quad (2)$$

where $\gamma = 100mc^2k/E_0EZ^{\frac{1}{3}}$. Placing $f(Z)=0$ yields the BH theory while $f(Z)=0.287\pm0.004$ for Au gives the MBD theory.[12] The functions $\varphi_1(\gamma)$, $\varphi_2(\gamma)$, and $c(\gamma)$ are given in tabular form by BH.

Wheeler and Lamb have deduced the theory for bremsstrahlung when an incident particle interacts with an orbital electron. Their formula is sufficiently similar to the nuclear bremsstrahlung formula that for this experiment sufficient accuracy is obtained by using

$$\varphi_{\text{total}} = \varphi_{\text{nuclear}}\left[1+\frac{\varphi_{\text{electronic}}}{\varphi_{\text{nuclear}}}\right] = \varphi_{\text{nuclear}}[1.015]. \quad (3)$$

The cross section which includes the effect of two successive radiations has been formulated by Curtiss.[13]

[10] C. R. Emigh, Ph.D. thesis, University of Illinois, 1951 (unpublished).

[11] See, e.g., W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, London, 1944), 2nd edition, p. 218.

[12] H. A. Bethe (private communication).

[13] C. Curtiss, reference 5, Eq. (4).

Accurate numerical integration of his equation is difficult because the small number answer is obtained from a difference of large numbers. To avoid this, Eq. (2) was rearranged to give

$$\varphi(E_0, E) = B(E_0, E; \varphi_1) + C(E_0, E; \varphi_1 - \varphi_2).$$

In this formulation B is roughly about ten times larger than C in the region of interest here. Using the approximation, arrived at empirically, $\varphi_1 = (68.66 + 6.928\gamma)/(3.306+\gamma)$, Curtiss' formula can be integrated analytically for all terms of order B^2. The integrations are rather long but straightforward. The only analytic approximation for $\varphi_1 - \varphi_2$ that was judged to be sufficiently accurate, lead to such tedious and involved numerical manipulation after integration that the small contribution of the BC and C^2 terms is found by numerical integration.

A cross section for the creation of pairs from bremsstrahlung may be derived by using the theories of BH. $\varphi(E_0, E_0-k)$ and $\varphi(k)$ are defined to be, respectively, the probabilities that an electron of energy E_0 will radiate a quantum of energy k, and that a quantum k will create an electron-positron pair. On the average the quantum will be produced halfway through the foil so that only the remaining half of the foil is available for the creation of a pair. Therefore the probability that in traversing the foil an electron will create a pair of energy between k and $k+dk$ by this two stage process is

$$d\sigma = nt\varphi(E_0, E_0-k)\cdot nt\varphi(k)dk/2. \tag{4}$$

As screening is partially effective in each stage of the event, this expression must be evaluated numerically. Since both φ's were derived using the Born approximation, Eq. (4) is valid regardless of whether an electron or a positron initiates the event.

B. Elastic Scattering by Electrons

For the incident energies used here scattering of electrons and positrons by electrons is of interest because different interactions of the incident and struck particle are predicted according to whether the charges carried by the particles are of the same or different sense. Since the interaction of the two particles is treated as a first-order perturbation by both Moller and Bhabha, their theories are accurate provided $c/137v \ll 1$. This condition is obviously fulfilled for particles of relativistic velocity. The actual scattering observed is distorted from that due to the processes under investigation because of the multiple events arising from the use of thick foils. After the scattering process has occurred the struck, or lowest energy electron, may radiate part of its energy in traversing the remainder of the foil.

The average radiative energy loss of the struck electron was computed with the aid of an approximate relation given by Rasetti,[14] corrected by a multiplying factor of 0.91 to correspond with the bremsstrahlung results found in the next section. In conjunction with this relation the Bloch ionization loss theory was used, this providing a sufficiently accurate result. The difference between the theoretically expected number of events for the true scattered energy and the theoretically expected number of events for the measured scattered energy which neglected radiation loss, may be applied as a correction to the experiment. The second and more important correction is due to those bremsstrahlung created pairs in which the energy sharing is so unequal that it allows only observation of the electron pair member. This correction was evaluated experimentally.

[14] F. Rasetti, *Elements of Nuclear Physics* (Prentice-Hall, Inc., New York, 1936), p. 72.

THE RESULTS

A. Bremsstrahlung

The data compiled from the analysis of 163 110-electron and 161 585-positron foil traversals for radiation events are presented in Table I.

TABLE I. The observed number of radiation events, the BH corrected experimental cross sections for the sum of the positron and electron events, and the theoretical cross sections found from integration of the BH theory over the incident spectrum of Fig. 2. The cross sections are in units of barns/Mev.

Straggled particle energy in Mev	Number of positron events	Number of electron events	BH corrected experimental cross sections	BH theoretical cross sections
0.511–6	167	155	0.0967±0.0064[a]	0.1025±0.0012
6–12	161	173	0.1326±0.0061	0.1668±0.0020
12–18	175	181	0.1436±0.0067	0.1891±0.0023
18–24	210	218	0.1764±0.0078	0.2027±0.0024
24–30	219	239	0.1909±0.0084	0.2128±0.0026
30–36	266	240	0.2128±0.0091	0.2215±0.0027
36–42	256	284	0.2284±0.0097	0.2297±0.0028
42–48	237	274	0.2166±0.0096	0.2377±0.0029
48–54	253	272	0.2234±0.0099	0.2459±0.0030
54–60	269	254	0.2233±0.0100	0.2545±0.0031
60–66	244	317	0.2405±0.0104	0.2641±0.0032
66–72	286	291	0.2479±0.0106	0.2747±0.0033
72–78	293	308	0.2589±0.0108	0.2868±0.0035
78–84	335	335	0.2896±0.0115	0.2996±0.0037
84–90	321	344	0.2881±0.0114	0.3137±0.0039
90–96	347	360	0.3070±0.0118	0.3304±0.0041
96–102	465	460	0.4024±0.0135	0.3484±0.0043
102–108	345	361	0.3077±0.0118	0.3675±0.0046

[a] Annihilation accounted for.

Numbers of straggled particles were determined for different energy bins, each of which was 6 Mev wide with the exception of that bin corresponding to the highest energy radiation which was (6–0.511) Mev wide. Analysis of the angular distribution of emerging particles showed symmetry and equality within statistics in both the horizontal and vertical cloud-chamber planes for any given bin. Within statistics the horizontal and vertical position distributions of the events in each bin were equal to the corresponding distributions of the incident particles. From these comparisons the asymmetry in the ease of observation and measurement of events introduced by the narrow vertical, as com-

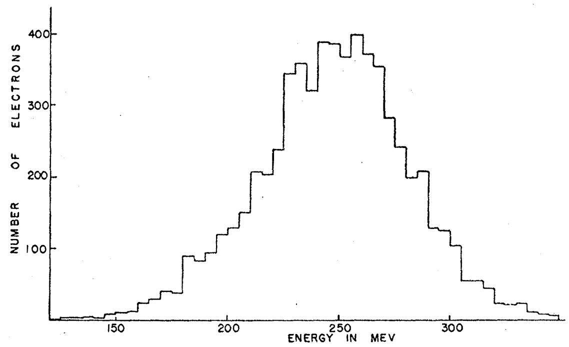

FIG. 2. Incident electron spectrum with energies computed from average magnetic field. Mean energy is 247.3 Mev.

pared to broad horizontal, lighted region does not appear to have affected the efficiency of the data collecting process which was 100 percent efficient for each bin.

For any bin the experimental cross section, φ_e, in units of cm^2/Mev is found from $\varphi_e = AN/TLdW$, where N is the number of straggled particles in the bin, T the number of foil traversals, A the atomic weight of the target, d the foil density, W the bin width, and L the Avogadro number. The errors assigned to φ_e are probable errors and are calculated from the statistical error in the number of events and from the experimentally found variation with energy of the number of events per bin, and the error in any one energy measurement. Because of this error in energy there is a displacement of events both in to and out of a bin occuring across both edges of a bin. As the bremsstrahlung process is statistical by nature the combined error is found by taking the square root of the sum of the squares of the errors in each one of the four component fluxes and in the number of events actually placed in a bin. The resulting probable error for the number in any one bin after including errors in foil thickness is found to vary approximately from $0.7\sqrt{N}$ to $1\sqrt{N}$ as the energy increases, instead of remaining a constant $0.675\sqrt{N}$ as would be the case if only the number of events actually found in the bin were considered.

The theoretical cross sections, φ_t, for a bin are found by integrating the theories over the incident electron energy distribution and then integrating these results over the straggled particle energies included in the bin. Figure 2 shows the incident electron spectrum used in the calculations. Because only 6300 elements are contained in the distribution, a statistical error amounting to about 1.2 percent is given φ_t. The spectrum is the weighted average of the measured second and calculated first foil distributions, the weights being assigned according to the number of particle traversals experienced by the respective foils. The spectrum incident on the first foil was found by assuming the BH bremsstrahlung theory to be 9 percent high and then calculating that spectrum which after traversal of the first foil would give the measured emergent spectrum. In terms of φ_e the resultant distribution is not critically dependent on the percentage chosen. Use of the 9 percent figure will, however, eventually receive some justification.

Before theory and experiment can be compared the corrections already discussed must be applied. These corrections are independent of the sign of the charge of the incident particle. Consequently it is feasible to compare electron and positron bremsstrahlung in all but the lowest straggled energy bin directly on the basis of the experimentally observed events, provided allowance is made in the errors assigned for the purely statistical error resulting from integration of the theoretical cross sections over the incident spectrum. The lowest straggled energy bin is excluded from this comparison because of the necessity of correcting the positron results for the annihilation processes still to be discussed.

The experimental ratio of positron to electron events must be increased by 3.0 ± 0.4 percent to yield the true

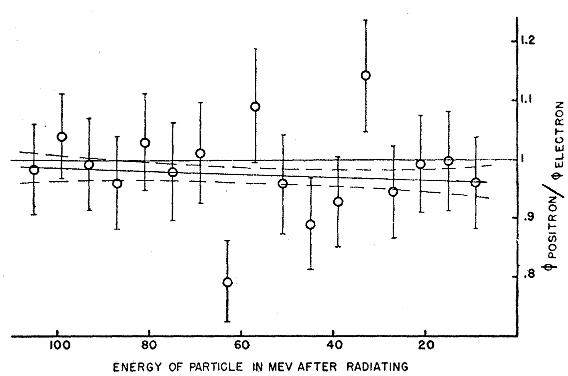

FIG. 3. Relative bremsstrahlung cross sections for positrons to electrons. The broken lines give the probable error in the straight line fitted to the data by the weighted least-squares method.

ratio of the cross sections. Approximately one-third of this increase is due to the difference in the number of foil transits for the two types of particles. A correction in energy resulting from use for the energy measurements of a magnetic field averaged over the entire experiment accounts for the remaining 2 percent and the ± 0.4 percent uncertainty. One-half of the 2 percent increase arises from the number of events being roughly proportional to the true bin width. The remaining 1 percent is due to the different average incident energies for the two types of particles, as these energies appear to be identical when the average magnetic field is used.

Figure 3 is a plot of the corrected ratio of positron to electron events. Following the method of Birge[15] a straight line was fitted to the data by a weighted least squares approximation. Taken over the whole range of 6 to 108 Mev the weighted average ratio is 0.973 $\pm 0.013 \pm$ the error of 0.004 discussed above. It is

[15] R. T. Birge, Phys. Rev. 40, 207 (1932).

apparent that for the interval of straggled energies considered, positrons and electrons yield very closely the same shape and magnitude for the bremsstrahlung cross section. On the basis of this conclusion positron and electron data in this interval are added to give experimental values to compare to the theoretical cross sections.

Table II gives the numerical values of the corrections made to the experimental cross section. The electronic bremsstrahlung is taken as $0.015\varphi_e$. Integration of Eq. (4) over the incident spectrum yields the correction for the double process involving bremsstrahlung and pair production. Here the assumption is made that the BH theories involved were too large by 9 percent. This conclusion has been established for pair production by the work of Lawson,[16] and DeWire *et al.*,[17] and will be

TABLE II. Corrections to bremsstrahlung cross sections, φ_e, of combined positron and electron events. All corrections in units of barns/Mev.

Straggled particle energy in Mev	$k/(E_0-mc^2)$ for center of bin	Electron electron brems-strahlung	Bremsstrah-lung pair production	Calculated BH double brems-strahlung	Total corrected correction using BH theory
0.511–6	0.988	−0.0016	0.0015	−0.0131	−0.0588[a]
6–12	0.965	−0.0022	0.0029	−0.0186	−0.0150
12–18	0.940	−0.0024	0.0033	−0.0173	−0.0137
18–24	0 915	−0.0030	0.0036	−0.0157	−0.0127
24–30	0.890	−0.0030	0.0037	−0.0143	−0.0114
30–36	0.866	−0.0035	0.0039	−0.0133	−0.0108
36–42	0.841	−0.0037	0.0040	−0.0124	−0.0102
42–48	0.816	−0.0035	0.0041	−0.0116	−0.0092
48–54	0.791	−0.0036	0.0043	−0.0109	−0.0085
54–60	0.766	−0.0036	0.0044	−0.0102	−0.0078
60–66	0.742	−0 0038	0.0046	−0.0096	−0.0073
66–72	0.717	−0.0040	0.0047	−0.0091	−0.0070
72–78	0.692	−0.0041	0.0048	−0.0086	−0 0066
78–84	0.667	−0.0046	0.0050	−0.0081	−0.0064
84–90	0.642	−0.0045	0.0052	−0.0076	−0.0057
90–96	0.617	−0.0048	0.0054	−0.0071	−0.0054
96–102		−0.0063	0.0056	−0.0066	−0.0063
102–108		−0.0048	0.0058	−0.0061	−0.0042

[a] Includes positron annihilation correction of −0.0476.

seen to be approximately valid for bremsstrahlung on the basis of the present experiment. To the accuracy required here the theory of MBD yields the same result as the adjusted BH theory, and, as in the case of the input spectrum, the actual correction to φ_e is not critically dependent on the precise value by which the BH theory must be altered. The double bremsstrahlung correction was found by integration over the incident spectrum for both the BH and MBD theories. As this process contributes appreciably to φ_e it is necessary to first make the correction and then adjust the correction on the basis of the deviation found between theory and experiment. The appropriate numerical factor of $(0.92)^2$ for the adjusted BH theory was found by using the integrated cross section for straggled particle energies between 18 and 96 Mev. When this is done the MBD and BH double bremsstrahlung corrections are equal within 2 percent in all but the (6–0.511)-Mev bin. No correction is made for three successive radiations although on the basis of the double radiation process the triple radiation process could conceivably give a contribution of the order of 1 percent to the cross sections in the lowest straggled energy bins measured.

In addition to the above factors, the phenomenon of positron annihilation requires consideration. The complete loss of energy of a positron by radiation is indistinguishable in the cloud chamber from a one or two quantum annihilation process. The theory for two quantum annihilation has been developed by Dirac[18] and has been confirmed within an experimental error of ±25 percent by Gilbert and Colgate,[19] who apparently assume that the positron and electron bremsstrahlung cross sections are equal. Heitler[20] has given the cross section for the one quantum process but this has not been tested experimentally. After integration over the incident spectrum the annihilation theories predict that of the 167 events found in the lowest positron energy bin, 46 should be ascribed to the two quantum and 3 to one quantum process. Making this correction the ratio of positron to electron bremsstrahlung in this bin is then 0.78±0.08. There is not sufficient accuracy to definitely determine whether a difference does or does not exist in the bremsstrahlung of the two types of particles. Consequently in comparing the experimental results with the theories, a triangular symbol is used in the figures for the annihilation corrected lowest energy bin instead of the circular symbol used for all other bins.

Figures 4 and 5 present the completely corrected data. It appears that the BH theory is incorrect in magnitude. It also appears that there is some discrepancy between theory and experiment near the tip of the spectrum.

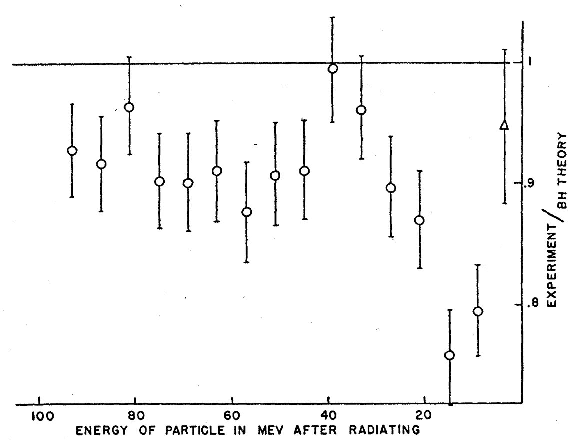

FIG. 4. Bremsstrahlung cross section relative to BH theory.

[16] T. L. Lawson, Phys. Rev. **75**, 433 (1949).
[17] DeWire, Ashkin, and Beach, Phys. Rev. **82**, 447 (1951).
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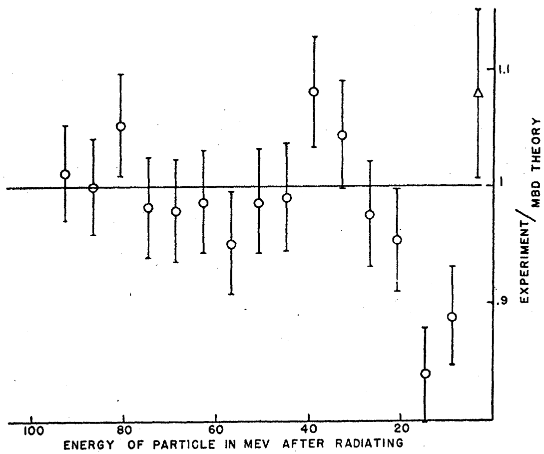

FIG. 5. Bremsstrahlung cross section relative to MBD theory.

The experimental result would be quite perplexing if data had not been taken for straggled particle energies greater than 42 Mev. Although the theories seem invalid for low straggled energies, the double bremsstrahlung corrections are fairly accurate because of their relative independence of observed straggled energies in this region.[21]

The corrected experimental cross sections in the 96 to 108 Mev interval given in Table I appear to contain systematic errors and so are not shown in Figs. 4, 5, and 6. In this interval the scale of the instrument used to determine straggled particle energies was greatly compressed. Inspection of the energy distributions of events in the various bins showed approximate equality with the theoretically expected distributions for bins containing energies less than 96 Mev. The energy distributions indicate that the observer definitely practiced a discrimination of putting almost all events near 102 Mev in the 96- to 102-Mev bin. Although more care was exercised near 108 Mev the tendency was to place possibly doubtful events in the 102- to 108-Mev bin. For fear of influencing the observer, the experimental results were deliberately not evaluated until the analyzing had been completed. Consequently the errors went unnoticed and no correction can be made. More care was taken in the measurement of the few scattered electrons having energies in this region and no systematic displacement was found.

The bremsstrahlung cross section found from adding the electron and positron results in the interval 18 to 96 Mev is 18.62±0.39 barns experimentally as compared to 20.24 barns for the BH and 18.60 barns for the MBD theories. Agreement with the latter theory is good while the BH theory is found to be 8.7±2.3 percent higher than experiment. In the 0.511 to 18 Mev interval the experimental cross sections are 91.7±4.3 percent of the MBD and 82.1±3.9 percent of the BH predictions for positrons, and 100.4±4.3 percent and 89.8±3.9 percent, respectively, for electrons. The errors given are the sums of the absolute values of the probable error, found from φ_e and φ_t, and 0.4 percent arising from the uncertainty in magnetic field. All results neglect the three successive radiation and two quanta from one nuclear encounter types of event, and assume the Wheeler-Lamb electronic bremsstrahlung theory. The result for the 0.511- to 18-Mev positron energy interval is additionally dependent on the validity of the annihilation theories.

TABLE III. The observed number of electron scattering events, and the observed number of obvious bremstrahlung plus pair production events.

Scattered electron energy in Mev	Number of positron-electron events	Number of positron-positron events	Number of electron-electron events	Number of electron-positron events
10–20	85	11	139	25
20–30	38	7	47	9
30–40	16	4	25	3
40–50	13	2	22	4
50–60	8	0	8	3
60–70	4	0	6	0
70–80	4	0	7	1
80–90	3	0	5	1
90–100	3	0	6	0
100–110	3	1	4	1
110–120	1	0	3	1
120–130	1	1	2	0
130	11	0	0	0

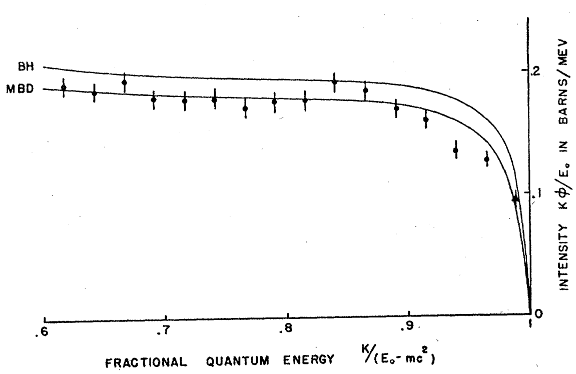

FIG. 6. Quanta spectrum resulting from superposition of spectra from all bins of incident particle spectrum.

Figure 6 summarizes the positron electron bremsstrahlung results in terms of intensity, $k\varphi/E_0$, as a function of the average fractional energy loss $k/(E_0-mc^2)$, where k equals the incident energy, E_0, minus the energy of the electron after radiating. The average values of k/E_0 and $k/(E_0-mc^2)$ were found by integration over the incident spectrum of Fig. 2. The corrected experimental values used are those found with the aid of the adjusted BH theory corrections.

[21] *Note added in proof.*—Bethe, Maximon, and Low, in Phys. Rev. **91**, 417 (1953) point out that an error was made by MBD in their calculation of the bremsstrahlung cross section. The comparison made here is with the uncorrected MBD calculation since the corrected calculation is not yet available.

B. Elastic Scattering by Electrons

Table III and Figs. 7 and 8 give the data compiled from the analysis of 97 833 positron and 113 125 electron foil traversal for elastic scattering by electrons. To draw attention to the small number of events observed the results are expressed in terms of events. The theoretical values are those found from integration over the incident spectrum of Fig. 2.

The total corrections to the experimental data are given in Figs. 7 and 8. The method of determining the correction for the radiation loss of the struck electron has already been described. Correction for those two stage bremsstrahlung plus pair production processes which were counted as scatterings was achieved using information found experimentally. If an electron enters

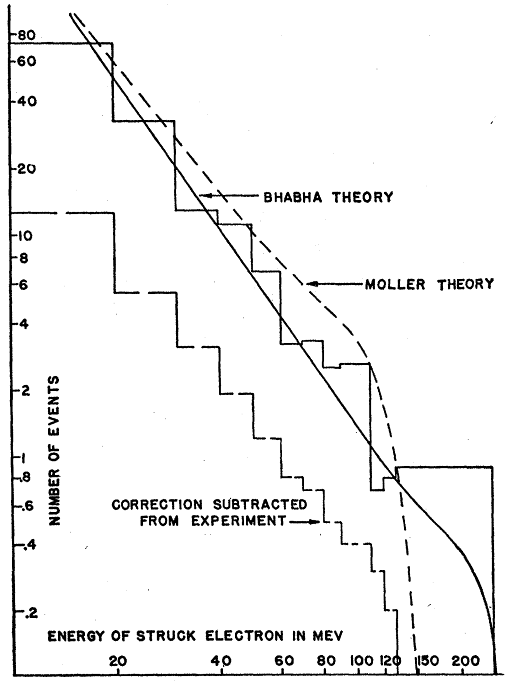

FIG. 7. Comparison of the observed positron-electron scattering events with the theoretically predicted values.

the foil and two electrons emerge the occurrence can be attributed to either elastic scattering or the double process. However it is also possible for an electron and positron to emerge. The number and positron energy distribution of the latter type of event was measured. On the assumption that the double process electron positron event occurs as often as the double process electron electron event the number of electron electron events due to the double process can be found. A similar behavior is exhibited by incident positrons. In the preceding section the bremsstrahlung of positrons and electrons was found to be approximately equal. Consequently it is permissible for the accuracy required here to combine the obvious double process events (i.e., incident electron emerging as positron and electron) of incident positrons and of incident electrons to achieve a slightly greater accuracy in the correction. This was the procedure followed.

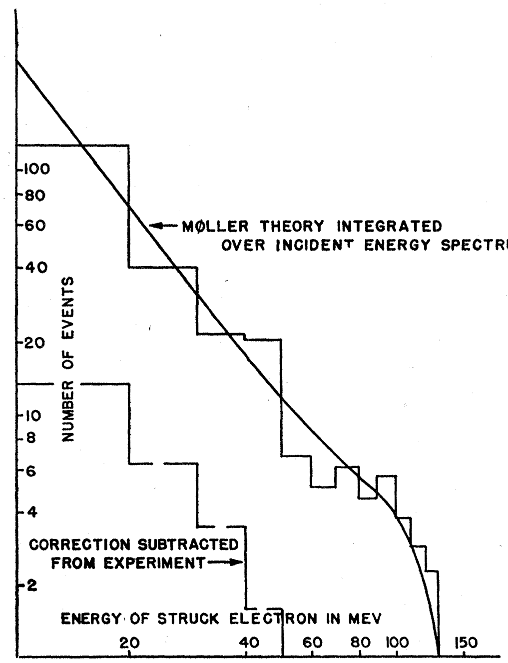

FIG. 8. Comparison of the observed electron-electron scattering events with the theoretically predicted values.

Agreement with the Møller electron electron scattering theory is rather better than the statistics warrant. The same is true of the Bhabha theory of positron electron scattering, with the exception of the 10 to 20 Mev and 130- to 250-Mev bins. For energies of the struck electron between 20 and 130 Mev, where the best experimental fits are found, the ratio of experimental to theoretical cross section is 0.956 ± 0.057 for electrons and 1.065 ± 0.075 for positrons. The cross-section ratio of electrons to positrons in the same energy interval is 1.31 ± 0.13 for experiment and 1.46 for theory. Errors are probable errors based only on the corrected number of events.

It has been stated that the pictures were analyzed twice for scattering processes, the first examination having yielded results differing from the theories by as much as a factor of two for the lowest struck electron energy bins. In view of this and the large errors attendant on the final number of events found, the information presented should be regarded only as a rough substantiation of the theories involved.

The author wishes to express his gratitude for the guidance given him by Professor D. W. Kerst, under whose direction this experiment was performed. He also wishes to acknowledge the helpful discussions of the work held with Professor A. T. Nordsieck, Professor A. O. Hanson, and Dr. C. R. Emigh. Much of the cloud-chamber equipment was designed and built by the latter.