Angular Distributions from (n,p) Nuclear Reactions*

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It is shown that angular distributions of high-energy proton groups from (n,p) reactions should evidence sharp maxima near the forward direction, which arise from neutrons with large impact parameters interacting with protons out in the "tails" of the initial nucleus. The positions of these peaks are characterized by the allowed values of orbital angular momenta l_n with which the neutron can be captured to form the final state. A study of such distributions therefore may well lead to information concerning spins and parities of nuclear energy levels, as in the case of stripping reactions. For most nuclei, compound nucleus formation should contribute little to such proton groups. For light nuclei, in order that the sharp maxima stand out above the compound nucleus background, it is probably necessary that the incident neutron energy not be near a resonance of the compound nucleus.

I. INTRODUCTION

ANALYSIS of the total cross sections for processes of the type $X(n,p)Y$ for neutron energies of 14 Mev and for fairly heavy nuclei[1] has indicated that an important contribution comes from interaction with surface protons. This is because there is an appreciable probability that a surface proton can receive by direct collision almost all the energy of the incident neutron, thus overcoming the Coulomb barrier, and leaving the final nucleus in a state of low excitation. Compound nucleus formation, on the other hand, due to the rapid increase in density of states with excitation for the final nucleus, favors the emission of lower-energy protons, which in these cases are subject to a large Coulomb barrier and can contribute little to the reaction.

The energy range in which the surface effect is important is most likely to be about 10–30 Mev. Nuclear "transparency" establishes the high-energy limit. At energies below 10 Mev the immediate formation of a compound nucleus may be suppressed,[2] so the interaction of the incident neutron with just one other nucleon might be possible throughout the entire target nucleus, rather than only at the surface.

The conclusion of reference 1 raises the question as to the appearance of the angular distributions resulting from such surface interactions, i.e., the angular distributions resulting from neutrons with large impact parameters interacting with protons which have penetrated outside the main core of the initial nucleus and whose wave functions in this region are the familiar "exponential" tails. In the present paper it is shown that these angular distributions are similar to those obtained in stripping reactions[3] and possess sharp maxima near the forward direction, maxima which depend on the allowed values of the orbital angular momenta with which the neutron can be captured. Moreover, since these sharp peaks involve many partial waves of the incident neutrons, it seems quite possible that they will stand out against the background due to compound nucleus formation even for considerably lighter nuclei than those considered in reference 1. The possibility exists, therefore, that (n,p) angular distribution experiments can yield information about the properties of the energy levels of a wide range of nuclei.

Such experiments should be most satisfactory for the higher-energy discrete proton groups. Energy discrimination for these groups can distinguish them from the background of protons from compound nucleus formation since, even for these nuclei whose Coulomb barriers cannot suppress it, this background mostly includes protons having energies considerably less than the incident neutron energy, and having a practically continuous energy distribution. For quite light nuclei, where there is no great increase in density of states with excitation for the final nucleus, it would probably be necessary that the incident neutron energy not be near a resonance of the compound nucleus. The anticipated total cross section for one high-energy group might be estimated as several millibarns for each initial participating proton. This is computed, for 14 Mev, by correcting the 0.6-barn total $(n\text{-}p)$ scattering cross section both for the fraction of time the initial and final particles spend outside their respective nuclei, and for certain angular effects.

If the process should be dominated by the least bound single-particle proton state of the initial nucleus, i.e., that state whose tail extends furthest from the nuclear core, the angular distributions become especially simple. They are then characterized by the set of values for l where,

$$l_n+l_p \geq l \geq |l_n-l_p|;$$

l_p and l_n are, respectively, the orbital angular momentum of a proton in the state of least binding of the

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[1] H. McManus and W. T. Sharp, Phys. Rev. **87**, 188 (1952), and (to be published).

[2] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953).

[3] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

initial nucleus, and the orbital angular momentum of the captured neutron (which in this event goes into a neutron state of least binding of the final nucleus).

Of course, the considerations of this paper not only apply to (n,p) reactions, but also to the types (n,n'), (p,n), and (p,p').

Our calculations are done in the impulse approximation. Section II presents the analysis of the simple case where only the least-bound states of the initial and final nuclei are important, while Sec. III considers what can be learned if such a strong selection cannot be imposed.

II. TRANSITIONS BETWEEN SINGLE STATES

In this section we consider those cases where the incident neutron interacts only with protons of the initial nucleus which we assume to be both outside the main body of the nucleus and also in a single state, *viz.*, the proton state of least binding. These considerations will apply therefore only to cases where protons in other states are substantially more tightly bound. Then contributions from the tightly bound protons are suppressed because the neutrons with the large impact parameters which are needed to get a rapidly varying angular distribution see mainly the proton tails of greatest extent, and also because the neutrons with sufficiently small impact parameter to see proton tails with small extension are more likely to be captured into a compound nucleus. The considerations of this section are later applied in Sec. III, where many tails participate. Although the simple rules of Sec. II are not likely to apply for many nuclei, their possible application in any one experiment should not be overlooked.

Experiments suggest that protons of least binding frequently have a definite orbital angular momentum, even in those cases where spin coupling considerations allow an admixture of two or more orbital momenta. On such a picture the proton is ejected from the state of least binding and known orbital momentum; the neutron is captured into a similar state, also characterized by a definite orbital momentum, leaving the final nucleus in one of its low-lying states.

We assume then that a proton, initially in a bound state of orbital angular momentum l_p and projection m_p, is knocked out by the incoming neutron, which is captured into a state of orbital angular momentum l_n and projection m_n. We also suppose that the interaction between the two particles takes place only outside the nucleus. These assumptions suggest using the "impulse approximation" method of Chew,[4] for the assumption that the reaction takes place outside the nucleus means just that neither particle interacts with the nucleus during the time they are interacting with each other. Indeed, our case is almost exactly the simple example Chew treats in Sec. II of his paper.

Suppose $G_t(\mathbf{r}_p)$ and $G_{t'}'(\mathbf{r}_n)$ are the wave functions of the bound neutron and proton in states t and t', respectively, and $e^{i(\mathbf{k}\cdot\mathbf{r}_n)}$ and $e^{i(\mathbf{k}'\cdot\mathbf{r}_p)}$ are the incident and outgoing plane waves. Also, suppose $V_{np}(\mathbf{r}_p-\mathbf{r}_n)$ is the neutron-proton interaction, r_0 is the nuclear radius, and $\mathbf{R}=\frac{1}{2}(\mathbf{r}_p+\mathbf{r}_n)$, $\mathbf{r}=(\mathbf{r}_p-\mathbf{r}_n)$. Then the matrix element for the transition may be written as

$$\int_{r_n,\,r_p\geq r_0} d^3r_n d^3r_p G_{t'}'^*(\mathbf{r}_n)e^{-i(\mathbf{k}'\cdot\mathbf{r}_p)}V_{np}\Psi(\mathbf{r}_n,\,\mathbf{r}_p). \quad (1)$$

Here Ψ is the complete wave function for the process. Following Chew, we introduce the impulse approximation by replacing Ψ by the approximate function Ψ_a, where

$$\Psi_a=\int d^3k_p g_t(\mathbf{k}_p)\varphi_{\mathbf{k}_p,\,\mathbf{k}}(\mathbf{r}_n,\,\mathbf{r}_p) \quad (2)$$

$$=\int d^3k_p g_t(\mathbf{k}_p)e^{i(\mathbf{R}\cdot\mathbf{k}_p)+i(\mathbf{R}\cdot\mathbf{k})}\varphi_{\frac{1}{2}(\mathbf{k}_p-\mathbf{k})}(\mathbf{r}). \quad (3)$$

The functions φ are two-body scattering wave functions, $\varphi_{\mathbf{k}_p,\mathbf{k}}$ giving a complete description of the collision between a proton of momentum $\hbar\mathbf{k}_p$ and a neutron of momentum $\hbar\mathbf{k}$, while $\varphi_{\frac{1}{2}(\mathbf{k}_p-\mathbf{k})}$ describes the same collision, but in the center-of-mass coordinate system. The function $g_t(\mathbf{k}_p)$ is the Fourier transform of $G_t(\mathbf{r}_p)$, for $r_p\geq r_0$.

It is convenient also to represent $G_{t'}'(\mathbf{r}_n)$ by its Fourier transform, for $r_n\geq r_0$,

$$g_{t'}'(\mathbf{k}_n)\equiv\int_{r_n\geq r_0} d^3r_n e^{-i(\mathbf{k}_n\cdot\mathbf{r}_n)}G_{t'}'(\mathbf{r}_n). \quad (4)$$

Inserting (3) and (4) into (1), we find directly:

$$\int d^3k_p g_{t'}'^*(\mathbf{k}_p+\mathbf{k}-\mathbf{k}')g_t(\mathbf{k}_p) \times\langle\mathbf{k}'-\tfrac{1}{2}(\mathbf{k}_p+\mathbf{k})|r_{np}|\tfrac{1}{2}(\mathbf{k}_p-\mathbf{k})\rangle, \quad (5)$$

the matrix element of the r matrix being defined as

$$\langle\mathbf{q}'|r_{np}|\mathbf{q}\rangle\equiv\int d^3r e^{-i(\mathbf{q}'\cdot\mathbf{r})}V_{np}\varphi_{\mathbf{q}}(\mathbf{r}).$$

The r matrix, as in Chew's work, is to be obtained phenomenologically from the two-body scattering experiments.

The integral of Eq. (5) leads directly to the cross section. It can be evaluated once the r matrix is known, the functions g_t and $g_{t'}'$, by assumption, being known for the outside region of the nucleus. Now a phenomenological derivation of $\langle\mathbf{q}'|r_{np}|\mathbf{q}\rangle$ is obtained from the free-scattering cross sections through the relation

$$\sigma_{np}=(2\pi/\hbar)M^2|\langle\mathbf{q}'|r_{np}|\mathbf{q}\rangle|^2. \quad (6)$$

Using (6) we deduce from the data that $\langle\mathbf{q}'|r_{np}|\mathbf{q}\rangle$ is a very slowly varying function of $\mathbf{q}$ and $\mathbf{q}'$. With this fact we simplify (5) by taking out the matrix element of r_{np}

[4] G. Chew, Phys. Rev. **80**, 196 (1950).

from under the integral sign, approximating it by its value at $k_p=0$.

A first observation on the free n-p scattering cross section[5] is that it is hardly at all a function of angle. Thus for the free scattering we can write the matrix element in the simpler form, $r_{np}(q)$. Although this form may seem inadequate for our discussion of "bound" n-p scattering, which involves an off-energy-shell matrix element of r_{np}, closer inspection shows that for outgoing protons at small forward angles, where $\mathbf{k}'\approx\mathbf{k}$, our process actually is close to the energy shell. In addition to this, we have been able to derive a semi-phenomenological form which shows that r_{np} changes slowly as it leaves the energy shell.

The energy variation of $r_{np}(q)$ follows from (6) as being proportional to $(\sigma_{np})^{\frac{1}{2}}$. In (5), if k_p were small compared to k and k' then we could immediately take out the matrix element from the integral. Now k and k' pertain to energies of, say, 14 Mev. Aside from some oscillations which are introduced by the cutoffs, the ranges of important arguments in g_t and $g_{t'}'$ are determined by the binding energies of the initial and final nuclei, about 8 Mev. Now suppose the magnitude of $\mathbf{k}_p$ is related to the binding energy, and suppose $\mathbf{k}_p$ is permitted to swing through all angles. In this case the energy of the equivalent free-scattering experiment from which $r_{np}(q)$ must be found for (5) will range over about 3–30 Mev, a region in which σ_{np} changes by a factor of 7, so $r_{np}(q)$ by a factor $\sqrt{7}$. Most of even this variation of r_{np} occurs when $\mathbf{k}_p$ is within about 30° of being parallel to $\mathbf{k}$. Since such a localized angle variation is unimportant if g_t and $g_{t'}'$ are not very rapid functions of angle, we merely limit ourselves to nuclear states of low orbital angular momentum, and safely approximate $k_p\approx 0$ in the matrix element of r_{np}. Equation (5) thus becomes approximately[6]

$$r_{np}(k/2)\int d^3k_p g_{t'}'^*(\mathbf{k}_p+\mathbf{k}-\mathbf{k}')g_t(\mathbf{k}_p). \qquad (7)$$

This is the matrix element for our n-p process.

The second factor of (7) gives the form of the angular distribution. It is most easily treated by returning to coordinate space, where it becomes

$$\int_{r_p\geqslant r_0} d^3r_p \exp(i\mathbf{Q}\cdot\mathbf{r}_p)G_t(\mathbf{r}_p)G_{t'}'^*(\mathbf{r}_p), \qquad (8)$$

and $\mathbf{Q}\equiv\mathbf{k}-\mathbf{k}'$.

On expanding $\exp(i\mathbf{Q}\cdot\mathbf{r}_p)$ in spherical harmonics and integrating over angles, (8) becomes

$$\sum_l i^l\{(2l_n+1)(2l_p+1)\}^{\frac{1}{2}}C_{l_nl_p}(l,0;0,0)\times C_{l_nl_p}(l,0;-m_n,m_p)\int_{r_0}^{\infty} r_p^2dr_p \times j_l(Qr_p)f_t(r_p)f_{t'}'^*(r_p), \qquad (9)$$

where the C's are Clebsch-Gordan coefficients,[7] f_t and $f_{t'}'$ are the radial parts of the wave functions, $Q=|\mathbf{k}-\mathbf{k}'|$, and j_l is the spherical Bessel function. Now for $r_p\geqslant r_0$ the functions r_pf_t and $r_pf_{t'}'$ are "exponentially" decreasing, since they correspond in general to bound states. The variation of the Bessel function is, in general, slow compared with this decrease (the momentum difference $\mathbf{Q}$ being small at small angles of scattering), and the integral of (9) may therefore be approximated by

$$j_l(Qr_0)\int_{r_0}^{\infty} r_p^2dr_pf_t(r_p)f_{t'}'(r_p)=\text{(say)}\quad \epsilon(r_0)j_l(Qr_0).$$

When the sum and average over initial and final states is carried out, we obtain therefore for the differential cross section:

$$\sigma\propto|\langle|r_{np}|\rangle|^2=|r_{np}(k/2)|^2\times\epsilon^2(r_0)\times(2l_p+1)\times\sum_l C_{l_nl_p}^2(l,0;0,0)\{j_l(Qr_0)\}^2, \qquad (10)$$

where l is restricted to the values $l_n+l_p\geqslant l\geqslant|l_n-l_p|$, and, to conserve parity, can take only all odd or even values in this range.

The angular distribution given by (10) is analogous to that obtained in the case of deuteron stripping, although the straightforward physical interpretation which the parameter l has in stripping is not applicable here. However, the spherical Bessel functions do introduce peaks which are characteristic of the particular l values allowed, and, as in stripping, there is no interference between different l values. Unlike the deuteron case, there is no form factor which suppresses the maxima from higher l values. Still it is not necessary to study all the maxima, as the position of the first maximum is adequate for determining the lowest allowed value of l, i.e., $|l_n-l_p|$. Thus l_n, the orbital angular momentum of the captured neutron, is given by

$$l_n=l_p\pm l_{\min}, \qquad (11)$$

the negative sign being omitted if $l_{\min}>l_p$. Thus, information concerning the spins and parities of the final state should be obtainable in much the same way as is done for deuteron stripping reactions.

For the above picture to be valid we require an initial nucleus in which there are some relatively lightly bound protons, all in the same orbital momentum state, surrounding a main core in which the protons are sub-

[5] R. K. Adair, Revs. Modern Phys. 22, 249 (1950).

[6] One further justification for dropping $\mathbf{k}_p$ in the matrix element of r_{np} is that it cancels identically in the Born approximation limit of the exchange part of the matrix element. Production of protons in the forward direction, the direction of interest in this paper, is most likely to be by exchange scattering.

[7] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Appendix A.

stantially more tightly bound. In this event we consider the reaction to proceed by one of the outer protons being replaced by a neutron with orbital angular momentum appropriate for formation of the particular state of the final nucleus under consideration, contributions from protons within the core being suppressed by the effects which were previously discussed.

On the independent particle model an initial nucleus well suited for the above picture should be one in which the protons of the (in general incompleted) last shell are all of the same orbital angular momentum. In this event the value l_p appropriate to this shell is employed in (11). For any one nucleus in which protons occupy more than one subshell of the last major shell, the energy differences are sufficiently small so that, on energetic grounds alone, all the subshells are apt to participate in an (n,p) reaction. Nevertheless, even in such cases, the formation of at least the ground state of the final nucleus, and perhaps the first excited state or two, might involve the replacement of one of the protons in the last subshell by a neutron, hence for these few final states the appropriate value of l_p to employ in (11) would also be known.

Finally, it might be remarked that, because of the use of free waves in the impulse approximation, we do not expect Eq. (10) to give accurate angular distributions. We wish merely to suggest that (n,p) angular distributions can give information about the properties of nuclear levels, and to use (10) to obtain a qualitative idea of the way in which the distributions depend on the orbital angular momentum change l. If the need arises, a more accurate theory can be developed.

In order to confirm that (10) is indeed qualitatively correct, we have considered a simple model, in which the incident neutron beam is permitted to interact with the core. In this model it is assumed that, in the incident neutron beam, all partial neutron waves l' for $l' \leqslant kr_0$ are absorbed by the nucleus, all other partial waves being unaffected. This should overestimate the effects of neutron scattering. Thus, the neutron wave

$$e^{i(\mathbf{k}\cdot\mathbf{r}_n)} = \sum_{l'} i^{l'}[4\pi(2l'+1)]^{\frac{1}{2}} j_{l'}(kr_n) Y_{l',0}(\theta_n),$$

was replaced in the matrix element (1) by the sum

$$\sum_{l'} i^{l'}[4\pi(2l'+1)]^{\frac{1}{2}} f_{l'}(kr_n) Y_{l',0}(\theta_n),$$

where

$$f_{l'} = j_{l'}, \quad \text{if } l' > kr_0$$
$$= \tfrac{1}{2}(j_{l'} + i n_{l'}), \quad \text{if } l' \leqslant kr_0,$$

$n_{l'}$ being the irregular spherical Bessel function. Also, as the previous discussion showed that the $(n\text{-}p)$ potential itself gave little angular dependence, this potential was assumed to have very small range.

A comparison of the results of this calculation with those of formula (10) is shown in Fig. 1, in which all cases refer to a nucleus of $A=34$ and a neutron energy of 14 Mev.

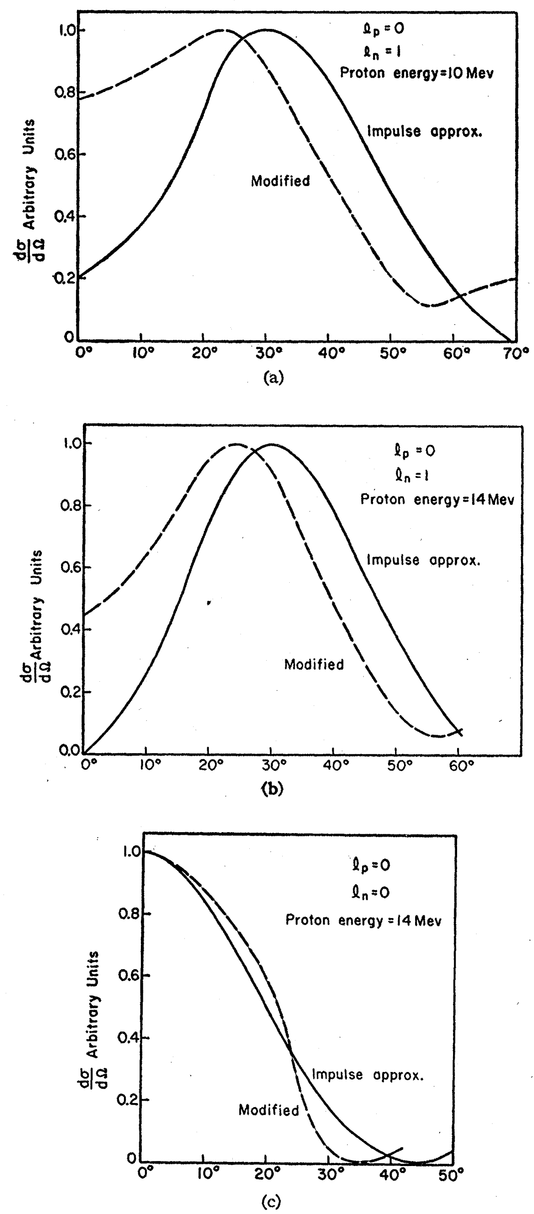

FIG. 1. Some typical (n,p) angular distributions. The solid curves are calculated from Eq. (10), while the dotted curves show the results of an alternative calculation which overestimates the effects of neutron scattering. The curves all apply to the case of 14-Mev neutrons incident on a nucleus with $A=27$.

III. TRANSITIONS INVOLVING MANY STATES

In this section we consider briefly those cases where there are no particular reasons for picking out special orbitals for the neutron and proton. Here the shape of the differential cross section arising from surface interactions still has restrictions, and these are shown to be determined by the change of nuclear spin, and by any change of parity.

We expand the antisymmetrical wave function $\Psi(J_X, M_X)$ of the initial nucleus (spin J_X and projection M_X) with respect to a complete set of one-particle states for the αth proton, and with respect to the energy eigenstates of the system composed of the remaining nucleons; i.e.,

$$\Psi(J_X, M_X) = \sum_{s,\,t,\,m_s,\,\mu_p} A_{st}{}^{\alpha}(j_s, m_s, j_p, \mu_p) \times\psi_s{}^{\alpha}(j_s, m_s)G_t(j_p, \mu_p; \mathbf{r}_p{}^{\alpha}), \quad (12)$$

where the $\psi_s{}^{\alpha}(j_s, m_s)$ are the antisymmetrical wave functions of the states s of the nucleus with the αth proton absent. These states have total spins designated by j_s and projections m_s. The spatial coordinates of the αth proton are given by $r_p{}^{\alpha}$ and the total angular momentum of this particle is represented by j_p (with projection μ_p). Thus $\mathbf{J}_X = \mathbf{j}_s + \mathbf{j}_p$.

In the expression (12) the antisymmetrization of $\Psi(J_X, M_X)$ exhibits itself in certain properties of the $A_{st}{}^{\alpha}$. It is, of course, equally possible to perform the expansion in terms of wave functions $\psi_s{}^{\alpha'}$ and $G_t(\mathbf{r}_p{}^{\alpha'})$ where the coordinates of a different particle (the α'th) have been chosen for the single particle wave function. Then, because of the antisymmetry of Ψ, we will have

$$A_{st}{}^{\alpha'} = \pm A_{st}{}^{\alpha} = (\text{say}) \pm A_{st}(j_s, m_s, j_p, \mu_p).$$

Similarly, the antisymmetrized wave function of the final nucleus can be written

$$\Psi(J_Y, M_Y) = \sum B_{s't'}{}^{\lambda}(j_{s'}, m_{s'}, j_n, \mu_n) \times\psi_{s'}{}^{\lambda}(j_{s'}, m_{s'})G_{t'}{}'(j_n, \mu_n; \mathbf{r}_n{}^{\lambda}), \quad (13)$$

where λ designates the particular neutron coordinates chosen for the single particle wave functions. Then again,

$$B_{s't'}{}^{\lambda'} = \pm B_{s't'}{}^{\lambda} = (\text{say}) \pm B_{s't'}(j_{s'}, m_{s'}, j_n, \mu_n).$$

In analogy to (1) we now compute the matrix element of the interaction, $\sum_\alpha V_{n\lambda, p\alpha}$ and obtain

$$\pm Z^{\frac{1}{2}} \sum_{s,\,t,\,t'} A_{st}(j_s, m_s, j_p, \mu_p)B_{st'}(j_s, m_s, j_n, \mu_n) \times\langle e^{i(\mathbf{k}'\cdot\mathbf{r}_p)}G_{t'}(j_n, \mu_n)|V_{np}|\Psi_t(\mathbf{k}, \mathbf{r}_n, j_p, \mu_p)\rangle. \quad (14)$$

The symmetry properties of A and B are used in deriving (14). There are no transitions between different states of the nucleus represented by ψ, i.e., $s=s'$, $j_s = j_{s'}$; hence

$$\mathbf{j}_p - \mathbf{j}_n = \mathbf{J}_X - \mathbf{J}_Y. \quad (15)$$

The cross section is proportional to the absolute square of (14).

Now $\langle e^{i(\mathbf{k}'\cdot\mathbf{r}_p)}G_{t'}(j_n, \mu_n)|V_{np}|\Psi_t(\mathbf{k}, \mathbf{r}_n, j_p, \mu_p)\rangle$ is a linear combination of matrix elements of the type $\langle e^{i(\mathbf{k}'\cdot\mathbf{r}_p)}\psi(l_n, m_n)|V_{np}|\Psi(\mathbf{k}, \mathbf{r}_n, l_p, m_p)\rangle$ already considered, where $l_n = j_n \pm \frac{1}{2}$ and $l_p = j_p \pm \frac{1}{2}$. Each of these we have seen to be a sum of the form (9), where l is restricted to the possible magnitudes of the vector $l_p - l_n$. But $l_p - l_n = \mathbf{j}_p - \mathbf{j}_n + \mathfrak{S}$, where $\mathfrak{S}$ is a vector of unit magnitude; so, by (15), $l_p - l_n = \mathbf{J}_X - \mathbf{J}_Y + \mathfrak{S}$. The possible values of l are therefore given by

$$J_X + J_Y + 1 \geqslant l \geqslant |\mathbf{J}_X + \mathbf{J}_Y + \mathfrak{S}|_{\min}. \quad (16)$$

Conservation of parity yields the further restriction that l can take only odd or even values, according to whether or not there is a change of nuclear parity.

In the cases, therefore, where there are no special reasons for choosing particular orbitals for the neutron and proton, the angular distribution from surface interactions will still exhibit a series of peaks in the forward direction [arising from the same factor $j_l(Qr_0)$] except that now the selection rules governing these peaks are somewhat less stringent. Thus $l_{\min}$ (obtained from the first peak) is the first even (no) or odd (yes) integer not less than the minimum value of $|\mathbf{J}_X + \mathbf{J}_Y + \mathfrak{S}|$. If we write $|J_X - J_Y| = \Delta J$, a peak directly forward indicates $\Delta J = 0$ or 1 (no). If there is no peak directly forward, then either $\Delta J \geqslant 0$ (yes) or $\Delta J \geqslant 2$ (no). If $l_{\min}$ is observed to be 3, $\Delta J = 3$ or 4 (yes), and so on.

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