Hyperfine Structure of the Spectra of Dysprosium, Cobalt, Vanadium, Manganese, and Lanthanum

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Study of the hyperfine structure of the spectra of Dy I and D II yielded the result that the nuclear spins of Dy^{161} and Dy^{163} are very probably 7/2 and their nuclear magnetic moments are approximately equal. The quadrupole moments of Co, V, Mn, and La were deduced by hyperfine structure investigation to be $Q(Co^{59}) = (0.5\pm0.2)\times10^{-24}$ cm², $Q(V^{51}) = (0.3\pm0.2)\times10^{-24}$ cm², $Q(Mn^{55}) = (0.4\pm0.2)\times10^{-24}$ cm², $Q(La^{139}) = (0.9\pm0.1)\times10^{-24}$ cm². The eccentricities of odd-proton nuclei were calculated from the existing data, and they were plotted *versus* the proton number. It was found that they attain large maxima in light and heavy elements.

I. HYPERFINE STRUCTURE OF THE SPECTRA OF Dy I AND Dy II

THE hyperfine structure (hfs) of the spectrum of dysprosium was studied, using a water-cooled hollow cathode discharge tube and a Fabry-Perot etalon.

Unfortunately no classification of the spectrum of Dy I or Dy II is available. Natural dysprosium consists of seven isotopes:[1] Dy^{164} (28.18 percent), Dy^{163} (24.97 percent), Dy^{162} (25.53 percent), Dy^{161} (18.88 percent), Dy^{160} (2.294 percent), Dy^{158} (0.0902 percent) and Dy^{156} (0.0524 percent), where a number in parentheses represents the isotopic abundance. The abundances of Dy^{158} and Dy^{156} are so small that their existence will be neglected in the discussion of the hfs.

The hfs of the line Dy I λ6259 is shown in Fig. 1. The intensity ratio of the components A to a is equal to the ratio of the abundance of Dy^{163} to Dy^{161} within experimental error, and this indicates that the nuclear spins of Dy^{163} and Dy^{161} must be equal. Approximate equality of the distances AB and ab shows that the magnetic moments of Dy^{163} and Dy^{161} are approximately equal.

There are many lines of Dy I in which the displacement effect of Dy^{164} against Dy^{162} is of the order of 0.035 cm⁻¹. When 0.035 cm⁻¹ is assumed to be the isotopic effect due to the transition $6s-6p$, the order of magnitude is very reasonable.[2]

Meggers[3] published a constant difference (828.3 cm⁻¹) between Dy II lines among which strong lines and especially the strongest line λ3968 are included. This line appears even when the condition of the discharge is unfavorable for the production of the Dy II spectrum. Thus we may assume that the final level of this line belongs to the ground multiplet or nearby multiplet of Dy II. The theoretical electron configuration of Dy II is given by Meggers as $4f^95d6s^2$. It is, therefore, expected that the ground state of Dy II has a sufficiently high J value. We may assume that the final level of Dy II λ3968 has also a sufficiently high J value. The observed hfs of λ3968 is shown in Fig. 1 and is in harmony with the assumption of the high J values of the levels involved.

The central strong component of λ3968 corresponds to the even isotopes, the remaining components belonging to the odd isotopes. The number of components and the intensity distribution in λ3968 show that the spin of Dy^{163} and Dy^{161} is very probably 7/2.

II. QUADRUPOLE MOMENTS OF Co, V, Mn AND La

In order to determine the quadrupole moments of Co, V, Mn, and La, the hfs of the spectra of Co I, V I, Mn I, La I, and La II were studied, using a liquid-air cooled hollow cathode discharge tube and a Fabry-Perot etalon. Each of these elements consists of only one isotope and the interpretation of the hfs is simple. Previous authors found that the nuclear spins of Co, V, and La are 7/2 and that of Mn is 5/2.

The hfs of the Co I spectrum was studied by several authors.[4–6] Rasmussen[6] detected for the first time a

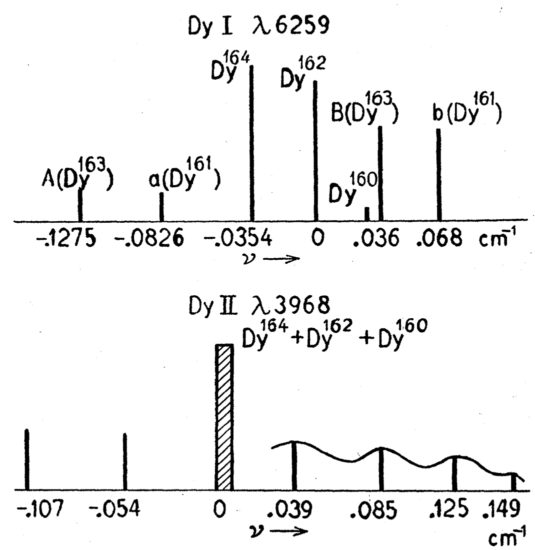

FIG. 1. Hfs of the lines Dy I λ6259 and Dy II λ3968.

[1] M. G. Inghram and D. C. Hess, quoted by G. T. Seaborg and I. Perlman, Revs. Modern Phys. 20, 585 (1948).
[2] See, for example, Fig. 2 of the work of K. Murakawa and J. S. Ross, Phys. Rev. 83, 1272 (1951).
[3] W. F. Meggers, Revs. Modern Phys. 14, 96 (1942).
[4] K. R. More, Phys. Rev. 46, 470 (1934).
[5] H. Kopfermann and E. Rasmussen, Z. Physik 94, 58 (1935).
[6] E. Rasmussen, Z. Physik 102, 229 (1936).

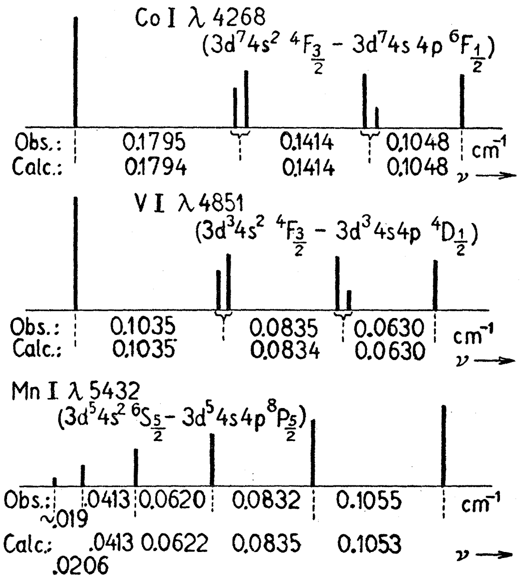

FIG. 2. Hfs of the lines Co I λ4268, V I λ4851 and Mn I λ5432.

deviation from the cosine interval rule in the term $3d^74s^2\,{}^4F_{\frac{3}{2}}$. In the present work the line λ4268 ($3d^74s^2\,{}^4F_{\frac{3}{2}}-3d^74s4p\,{}^6F_{\frac{1}{2}}$) was studied, and the result is given in Fig. 2. Rasmussen got the intervals 0.179, 0.140, and 0.105 cm^{-1} in the same line. The hfs splitting of the initial term must be very small. Denoting the interval factor and the quadrupole constant of the final level by A and B, respectively, and the interval factor of the initial level by δ, we get from the observed hfs the values: $A-\frac{1}{12}\delta=35.53\times10^{-3}$ cm^{-1}, $B=0.015\times10^{-3}$ cm^{-1}. The structure of λ4268 that was calculated using these values of $A-\frac{1}{12}\delta$ and B is also shown in Fig. 2.

Assuming an LS coupling for the final level and using the usual procedure,[7] we get the value of the quadrupole moment:

$$Q(\text{Co}^{59})=(0.5\pm0.2)\times10^{-24}\ \text{cm}^2.$$

The hfs of the V I spectrum was studied by Kopfermann and Rasmussen,[8] but they detected no deviation from the cosine interval rule in any term of V I. The present authors studied the line V I λ4851, and the result is shown in Fig. 2. Using the same notation as in the case of Co I λ4851, we get from the observed hfs the values:

$$A-\tfrac{1}{12}\delta=20.81\times10^{-3}\ \text{cm}^{-1},\quad B=-0.0046\times10^{-3}\ \text{cm}^{-1}.$$

As in the case of Co, the value of B yields

$$Q(\text{V}^{51})=(0.3\pm0.2)\times10^{-24}\ \text{cm}^2.$$

The hfs of the Mn I spectrum was studied by several authors,[9–13], but none of the previous authors detected any deviation from the cosine interval rule in any term of Mn I. The present authors studied the hfs of Mn I λ5432 ($3d^54s^2\,{}^6S_{\frac{5}{2}}-3d^54s4p\,{}^8P_{\frac{5}{2}}$), and the result is shown in Fig. 2. It is well known that the final level is of an LS-coupling type, so that it has a vanishing interval factor and a vanishing quadrupole constant. The hfs of λ5432 therefore gives directly the hfs of the initial level $3d^53s4p\,{}^8P_{\frac{5}{2}}$. Denoting the interval factor and the quadrupole constant of the initial level by A and B respectively, we get from the observed hfs the values:

$$A=20.90\times10^{-3}\ \text{cm}^{-1},\quad B=0.005\times10^{-3}\ \text{cm}^{-1}.$$

Assuming an LS coupling for the initial level, we get

$$Q(\text{Mn}^{55})=(0.4\pm0.2)\times10^{-24}\ \text{cm}^2.$$

Recently Javan, Silvey, and Townes[14] deduced $Q(\text{Mn}^{55})=0.5\times10^{-24}$ cm^2 by means of microwave spectroscopy. This is in good agreement with the value deduced here.

It would be desirable to check the value of Q by studying the hfs of the line Mn I λ5395 ($3d^54s^2\,{}^6S_{\frac{5}{2}}-3d^54s4p\,{}^8P_{7/2}$). However, unfortunately the hfs obtained by different spacers were not sufficiently consistent, so that such an attempt has been abandoned for the time being.

The hfs of La I spectrum was studied by Anderson,[15] but he found no deviation from the cosine interval rule in any level of La I.

The hfs of the line La I λ5106 [$5d^26s\,{}^4F_{\frac{3}{2}}-5d^2({}^3F)6p\,{}^4D_{\frac{1}{2}}$][16] obtained in the present work is shown in Fig. 3. The deviation from the cosine interval rule in $5d^26s\,{}^4F_{\frac{3}{2}}$ is considerable, and the interval factor A and the quadrupole constant B of this level are calculated to be

$$A=-16.7\times10^{-3}\ \text{cm}^{-1},\quad B=0.011\times10^{-3}\ \text{cm}^{-1}.$$

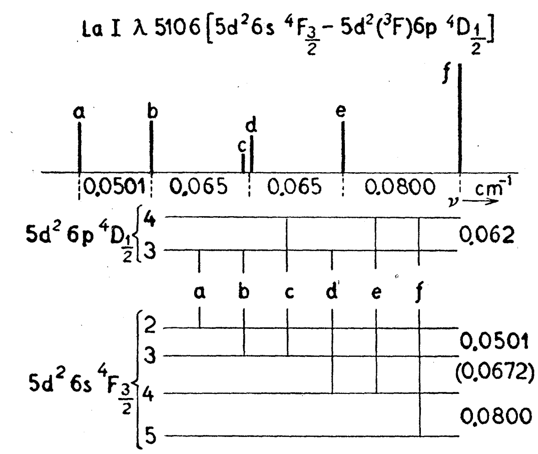

FIG. 3. Hfs of the line La I λ5106 and the transition scheme.

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[13] R. A. Fisher and E. R. Peck, Phys. Rev. **55**, 270 (1939).
[14] Javan, Silvey, Townes, and Grosse, Phys. Rev. **98**, 222 (1953).
[15] O. E. Anderson, Phys. Rev. **46**, 473 (1934).
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Using the formula published by Crawford,[17] the quadrupole moment is calculated to be

$$Q(\text{La}) = (0.95 \pm 0.20) \times 10^{-24}\ \text{cm}^2.$$

The line La II λ4809 ($5d6s\,{}^3D_1 - 5d4f\,{}^3P_0$) was observed to consist of three components: 0 (3), 0.137 (4), 0.305 (5) cm^{-1}, where a number in parentheses represents the relative intensity. The interval factor A and the quadrupole constant B of $5d6s\,{}^3D_1$ deduced from the hfs are

$$A = -38.1 \times 10^{-3}\ \text{cm}^{-1}, \quad B = 0.057 \times 10^{-3}\ \text{cm}^{-1}.$$

Using the formula published by Casimir,[18] we get the quadrupole moment:

$$Q(\text{La}) = (0.91 \pm 0.10) \times 10^{-24}\ \text{cm}^2.$$

Summarizing the results obtained from the hfs of the La I and La II spectra, we can conclude that

$$Q(\text{La}^{139}) = (0.9 \pm 0.1) \times 10^{-24}\ \text{cm}^2.$$

Regularities in the quadrupole moments of atomic nuclei were discussed by Gordy[19] and Townes[20] some time ago. More values have been found since then, and it will not be out of place to discuss the regularities again here.

Wageningen and de Boer[21] calculated the quadrupole moments of some nuclei and verified that the intrinsic quadrupole moment Q_0 that was introduced by Bohr[22] is more adequate than the experimentally found quadrupole moment Q for use in the calculation. Under the assumption of spin-orbit coupling, Q_0 is expressed by the equation:

$$Q_0 = Q\frac{(I+1)(2I+3)}{I(2I-1)}. \tag{1}$$

Then the relation between Q_0 and the eccentricity ϵ is given by

$$\epsilon = (r_1^2 - r_2^2)/(r_1^2 + r_2^2) = 5Q_0/(4ZR^2), \tag{2}$$

where Z is the charge number (=proton number), R is the radius of the nucleus, and the nucleus is assumed to have the form of a spheroid with semi-axes r_1 and r_2. ϵ is positive for a prolate spheroid and negative for an oblate spheroid.

Using the values of Q listed in the review articles of Mack[23] and Klinkenberg,[24] values of ϵ of odd-proton nuclei were calculated and plotted *versus* the proton number Z (see Fig. 4).[25]

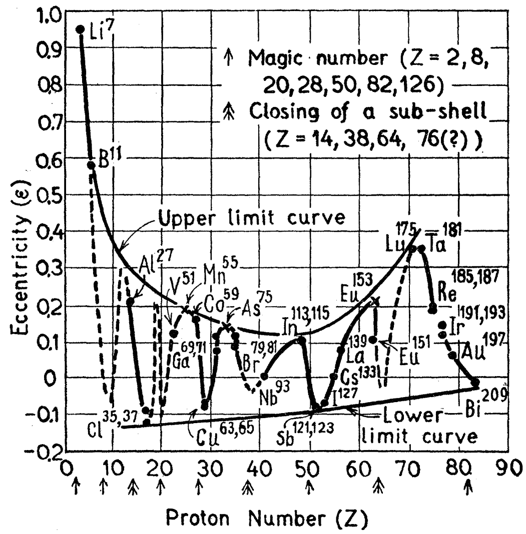

FIG. 4. Plot of eccentricity *versus* proton number.

Each of the nuclei Na^{23}, Mn^{55}, As^{75}, and Eu^{153} is anomalous in that the magnetic moment deviates to some extent from that predicted by the Mayer-Jensen shell model. However, in the present work it was assumed that the shell model is still valid for these nuclei and their Q_0 values were calculated by Eq. (1). In Fig. 4 the values of ϵ for these nuclei are plotted by x.

Figure 4 shows that two curves can be drawn in such a way that all the observed ϵ values lie between them. The upper-limit curve is flat for medium proton numbers but steep for small and large proton numbers. All of the above-mentioned anomalous nuclei except Na are observed to have ϵ lying on the upper limit curve. No data for Na are available, but it is probable that ϵ of Na also lies on the upper-limit curve, and it is predicted that $\epsilon(\text{Na}^{23}) = 0.3$, $Q(\text{Na}^{23}) = +8.4 \times 10^{-26}\ \text{cm}^2$.

It will be useful to plot ϵ *versus* neutron number for odd-neutron nuclei also, but since the available data for such nuclei are still meager, such an attempt has been abandoned for the time being.

Note added in proof.—Recently K. Murakawa and S. Suwa [J. Phys. Soc. Japan (to be published)] have shown that $Q(\text{Pr}^{141}) = -0.05 \times 10^{-24}\ \text{cm}^2$, $\epsilon(\text{Pr}^{141}) = -0.005$, and according to the recent article of D. R. Inglis [Revs. Modern Phys. **25**, 390 (1953)] $Q(\text{Li}^7)$ is probably negative. Figure 4 of the present paper, therefore, requires correction with respect to Pr^{141} and Li^7.

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