

FIG. 1. Oscilloscope pictures of 50-kc/sec nuclear resonance absorption vs static magnetic field. Field excursion 0.2 gauss. Top line:  $\text{Li}^7$  resonance (lost in noise). Middle line:  $\text{Li}^7$  resonance enhanced by electron saturation. Bottom line: Proton resonance in glycerin sample.

indicated an alternating field of about 4 gauss. The nuclear resonance was detected using a 50-kc/sec crystal controlled oscillator and a twin- $T$  bridge, the 50-kc/sec signal being converted to 600 kc/sec and detected in a communications receiver. The signal was observed on an oscilloscope or with a 30-cps lock-in amplifier. The rf tank coil of 1 turn, the 270-turn nuclear resonance coil, and the solenoid were oriented mutually perpendicular, and the array was placed in a copper box to shield the detection apparatus from rf radiation. The 84-Mc/sec oscillator could be switched on or off without disturbing the bridge balance.

The accompanying oscilloscope photographs (Fig. 1) summarize the results. The top line shows the appearance of the ordinary lithium nuclear resonance, which is so weak at these frequencies as to be completely lost in noise. The second line was photographed after the electron saturating oscillator was turned on. The  $\text{Li}^7$  resonance now appears strongly. For comparison, the proton line in glycerin (also at 50 kc/sec) is shown in line three. In all three cases the amplifier gain settings and degree of bridge balance were identical. The very weak glycerin resonance, however, is produced by a sample of eight times the number of nuclei as the lithium. The enhancement of the lithium resonance, therefore, confirms Overhauser's theory strikingly.

On the basis of comparison with the glycerin resonance, we estimate the nuclear population difference to be increased nearly 100-fold in our experiment. This figure is somewhat smaller than the maximum amount predicted, showing that either complete saturation was not achieved, or that other nuclear relaxation processes short-circuit the alignment partially. One process we believe to be important is the relaxation produced by self-diffusion of the nuclei.<sup>2</sup> We have also observed a small enhancement in sodium under conditions of incomplete saturation. With both metals the saturating fields produced intense heating of the samples.

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Norberg on nuclear relaxation times in lithium were helpful in searching for the nuclear resonance. We wish to thank Dr. Richard M. Brown for several useful suggestions concerning low-frequency nuclear resonance.

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<sup>4</sup> Carver, Holcomb, and Slichter (to be published).

<sup>5</sup> A. W. Overhauser, *Phys. Rev.* **89**, 689 (1953); **91**, 476 (1953); *Phys. Rev.* (to be published).

## Phase-Shift Analysis of High-Energy $p$ - $p$ Scattering Experiments\*

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It has often been suggested<sup>1</sup> that the isotropy of the differential cross section for  $p$ - $p$  scattering up to 350 Mev<sup>2,3</sup> as well as its magnitude (at 240 Mev this is  $4.97 \pm 0.43$  mb/sterad,<sup>2</sup> and  $\sigma$  is fairly constant with energy from 150-350 Mev<sup>3</sup>) might be fitted with  $s$  and  $p$  waves only.<sup>4</sup> Wolfenstein and Ashkin<sup>5</sup> have proposed including in such an analysis the left-right asymmetry, observed at Rochester, of  $2e = 9.8 \pm 5$  percent in the final angular distribution of 240-Mev protons scattered successively by two (effective) hydrogen targets.<sup>6</sup> Such a fit can in fact be made for the 240-Mev data with a number of different combinations of phase shifts, as will be shown.

In terms of the phase shifts  $\delta_L^j$  the above facts mean the following, if we neglect all phase shifts for  $L > 1$ :

- (1) The isotropy of the angular distribution requires that the coefficient of  $\cos^2\theta$  in  $\sigma(\theta)$  vanish:

$$0 = \sin^2\delta_1^0 + 3 \sin^2\delta_1^1 + 5 \sin^2\delta_1^2 - \sin^2(\delta_1^0 - \delta_1^2) - (9/4) \sin^2(\delta_1^1 - \delta_1^2). \quad (1)$$

- (2) In this case, the differential cross section in the center-of-mass system is given by

$$k^2\sigma(k, \theta) = \sin^2\delta_0^0 + \sin^2\delta_1^0 + 3 \sin^2\delta_1^1 + 5 \sin^2\delta_1^2. \quad (2)$$

- (3) The general theory of polarization effects in scattering problems has been discussed by Wolfenstein and Ashkin,<sup>7</sup> and the application to the double scattering experiment has been considered by Goldfarb and Feldman,<sup>8</sup> and by Swanson.<sup>9</sup> One finds that the observed asymmetry,

$$2e = [I(\phi=0) - I(\phi=\pi)] / I(\phi=\pi/2),$$

where  $I$  is the proton intensity from the second target at azimuthal angle  $\phi$ , is<sup>7</sup>

$$2e = 2P(k_A, \theta_A)P(k_B, \theta_B). \quad (3)$$

Here  $P(k, \theta)$  is the percentage polarization<sup>7</sup> of one of the scattered protons induced by the collision of two initially unpolarized proton

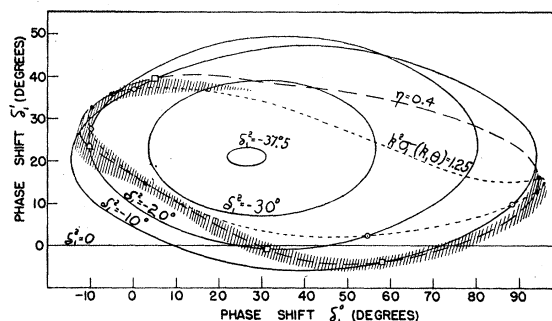


FIG. 1. Phase-shift diagram.

TABLE I. Some phase shifts (in degrees) at 213 Mev consistent with isotropy,  $\sigma(\theta) = 4.97$  mb/sterad,  $\eta = 0.40 \pm 0.12$ .

$\delta_1^2$	$\delta_1^0$	$\delta_1^1$	$\delta_0^0$	$\eta$
-10	50	-5.4	46	0.340
	60	-3.5	37	0.418
	70	-0.3	29	0.481
-15	35	-3.6	51	0.327
	45	-3.2	41	0.462
	-10	32.4	12	0.449
-20	30	-0.6	42	0.387
	-10	26.8	13	0.475
-25	20	3.8	30	0.368

beams;  $k, \theta$  are the c.m. momentum and scattering angle; and  $A, B$  refer to the first and second collisions, respectively.  $P$  is given by:<sup>8</sup>

$$\eta \equiv [k^2 \sigma(k, \theta) / \sin 2\theta] P(k, \theta) = 3 \sin \delta_1^0 \sin \delta_1^2 \sin(\delta_1^0 - \delta_1^2) + (9/2) \sin \delta_1^1 \sin \delta_1^2 \sin(\delta_1^1 - \delta_1^2). \quad (4)$$

In the double scattering experiment, the protons were scattered through  $27^\circ$  in the lab system in both collisions. When the appropriate values of  $k, \theta$  for the two collisions are used along with the values quoted above for  $\sigma(k, \theta)$  and  $2e$ , one obtains from Eqs. (3) and (4) the value  $(\eta_A \eta_B)^{1/2} = 0.40 \pm 0.12$ .<sup>10</sup> This must be the same as the value of  $\eta$  at some energy intermediate to those of the first and second collisions: 240 Mev and 186 Mev respectively.<sup>11</sup>

The phase shifts consistent with the data are shown in Fig. 1, which is a plot on the  $\delta_1^0, \delta_1^1$  plane of:

(1) Values of  $\delta_1^0, \delta_1^1$  for fixed  $\delta_1^2$  which satisfy the isotropy condition (1) give the solid curves. These are the "contour lines" for constant  $\delta_1^2$  of the isotropy surface in the  $\delta_1^0, \delta_1^1, \delta_1^2$  space.

(2) The surface defined by  $k^2 \sigma_1 = \sin^2 \delta_1^0 + 3 \sin^2 \delta_1^1 + 5 \sin^2 \delta_1^2 = 1.25$  intersects the isotropy surface in a curve whose projection

on the  $\delta_1^0, \delta_1^1$  plane is the short dashed curve (---). The open circles show its intersections with the  $\delta_1^2$  contour lines.

(3) The surface defined by  $\eta = 0.40$  intersects the isotropy surface in a curve whose projection on the  $\delta_1^0, \delta_1^1$  plane is shown as a dashed (---) curve. The squares show its intersections with the  $\delta_1^2$  contour lines. The solid circles show its intersections with the  $k^2 \sigma_1 = 1.25$  curve.

(4) The shaded region shows the phases consistent with  $\eta = 0.41 \pm 0.12$ .  $\sigma = 4.97 \pm 0.43$  mb/sterad (i.e.,  $k^2 \sigma_1 \leq k^2 \sigma$ ) and these are interpreted as referring to 213 Mev.

Some particular values of the phases are listed in Table I, which also gives values of  $\delta_0^0$  which are calculated using Eq. (2) and the experimental cross section. Equally valid solutions may be obtained by replacing all  $\delta_L^J$  by  $-\delta_L^J$ .

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<sup>6</sup> Oxley, Cartwright, Rouvina, Baskir, Klein, Ring, and Skillman, Phys. Rev. **91**, 419 (1953).

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<sup>8</sup> L. J. B. Goldfarb and D. Feldman, Phys. Rev. **85**, 1099 (1952).

<sup>9</sup> D. R. Swanson, Phys. Rev. **89**, 740 (1953).

<sup>10</sup> Berkeley gives a value of the cross section  $3.7 \pm 0.35$  mb/sterad. This value leads to  $(\eta_A \eta_B)^{1/2} = 0.30 \pm 0.09$ .

<sup>11</sup> Oxley *et al.* give another value for the asymmetry:  $2e = 8.5 \pm 2.2$  percent obtained from a carbon-carbon measurement. This value need not satisfy Eq. (3) in view of the energy degradation and so was not used. The error arising from energy degradation could be eliminated in future experiments by making three measurements for the asymmetry with energies in the first and second collisions  $E_A, E_B; E_B, E_C$ ; and  $E_A, E_C$ . From Eqs. (3) and (4) we see that this gives  $\eta_A \eta_B, \eta_B \eta_C$ , and  $\eta_A \eta_C$ ; so  $\eta_A, \eta_B$ , and  $\eta_C$  can all be determined.