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Matrix Elements for the Nuclear Photoeffect*

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A phenomenological formulation of the interaction of nuclear systems with the transverse electromagnetic field is developed, and expressions for the nuclear photoeffect matrix elements are obtained. The development is based on the differential charge conservation law and the hypothesis that the charge and current density operators for a nuclear system can be expressed in terms of nucleon variables only, even though virtual mesons may play a role in the interaction of the system with the electromagnetic field. The charge and current density operators are expressed as the sum of one-particle, two-particle, etc., terms in analogy with the phenomenological treatment of potential interactions between nucleons. The treatment is sufficiently general to include velocity dependence of interaction charges and currents, but is essentially nonrelativistic. Methods of explicitly constructing the most general forms of the charge and current density operators are given, taking into account general invariance and symmetry conditions to which they are subject. A new feature found in the present treatment is the possible existence of interaction effects in the charge density operator which imply that long-wavelength electric dipole matrix elements can be affected by interactions between nucleons (contrary to recent statements in the literature). The effects of the principles of charge symmetry and charge independence on nuclear-electromagnetic interactions are discussed. The form in which the photoeffect matrix elements are presented is such as to allow computation of all multipoles in an already summed form.

INTRODUCTION

THE problem of unraveling the structural and dynamic properties of nuclear systems through the study of their interaction with electromagnetic radiation has proved to be more difficult and more complicated than the corresponding problem in atomic physics. The source of the difficulties is twofold: Firstly, since the electromagnetic interactions of nucleons are inseparable from their mesonic interactions, and since we are not in possession of a satisfactory meson theory, we do not know the precise form of the interaction of a nuclear system with the electromagnetic field. Secondly, we do not have very satisfactory models of nuclear structure on which we can base quantitative calculations. In quantum mechanical terminology, we may state these difficulties as follows: We are hindered in the calculation of radiative transition probabilities by the lack of reasonably good wave functions to represent the initial and final states of the nuclear system and also by a lack of knowledge of the proper interaction Hamiltonian to be employed in calculating the matrix elements for such transitions.

While explicit calculations of the electromagnetic effects of meson-nucleon interactions have been made with some of the currently popular forms of meson theory, the results have not proved quantitatively dependable. The reaction to this unsatisfactory situation has been a greater concentration on (1) analyzing the available experimental data for such universal features—selection rules, for example—which might be expected to be present irrespective of the detailed form of the meson-nucleon interaction, and (2) developing phenomenological formulations of the interaction of nuclear systems and the electromagnetic field which have sufficient breadth to include at least many of the characteristic features arising from meson effects.[1] The contents of this paper belongs to the second class of these endeavors.

The philosophy behind this latter approach to the problem may be formulated as follows. Let us assume for the moment that we possessed a completely satisfactory theory of the interaction of nucleons, mesons, and the electromagnetic field. Within such a theory it should be permissible to inquire as to the value of certain transition probabilities relating to transitions in which no real mesons are present in either the initial or final state of the system. The transition, viewed from the vantage point of quantum perturbation theory, would involve transitions through intermediate states in which *virtual* mesons are present. The final transition probability will not have any direct reference to the dynamical variables referring to these mesons and should be specifiable in terms referring only to the initial and final states of the nuclear system and the electromagnetic field. One should then be able to determine an equivalent interaction operator involving only nucleonic and radiation-field dynamical variables whose matrix elements for any such transition are the same as those calculated with direct reference to the mediation of the meson field. Actually this procedure can be formalized through the application of canonical transformations to the original Hamiltonian for the system of nucleons, meson field, and electromagnetic field, such that to any order in the meson field coupling the intermediary role of the meson field in a transition of the type described is eliminated. The specific manner in which this procedure may be executed is now well known.[2]

Once the above possibility is admitted, one can

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[1] R. G. Sachs, Phys. Rev. **74**, 433 (1948); R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1950); Blanchard, Avery, and Sachs, Phys. Rev. **78**, 292 (1950); J. H. D. Jensen and M. Goeppert-Mayer, Phys. Rev. **85**, 1040 (1952).

[2] See, for example, S. Borowitz and W. Kohn, Phys. Rev. **76**, 18 (1949).

approach the problem in another manner. From the invariance and symmetry properties of the original Hamiltonian one can infer the existence of associated invariance and symmetry properties for the equivalent Hamiltonian from which meson variables have been eliminated. It is then possible to ask: within the bounds prescribed by these invariance and symmetry properties, what is the most general form which the resultant equivalent interaction between the nuclear system and the electromagnetic field can take? It is to at least a partial answer to this question that we here devote ourselves. The reason for seeking only a partial rather than a complete answer is easily evident to anyone who approaches this problem. One encounters immediately exceedingly difficult problems which have not yet permitted a satisfactory resolution. To quote an example, one should obtain a completely Lorentz-covariant formulation of the equivalent Hamiltonian, yet, to our knowledge, it has not yet been possible to construct a completely Lorentz-invariant equation for the interaction of even two particles without introducing explicitly an intermediary field through which the interaction is propagated. Thus we have been forced to bypass the requirements of relativity in our treatment and hence to limit the applicability of the resultant theory to such cases where the nucleons move with nonrelativistic velocities.

However, beyond these difficulties of a methodological character, there is an additional problem in application of the theory arising from the tremendous manifold of possibilities which arise in the investigation of this problem. If a complete treatment of all these possibilities were necessary to a satisfactory conclusion of the problem we have set ourselves, one would soon be forced to concede that there is no more virtue in a phenomenological approach to this problem than in an extensive program of meson theoretical calculations.

It is therefore obvious that one must employ discretion in selecting from this manifold of possibilities those which might be expected to be of dominant importance. Such selections based on conjecture could easily turn out to be false, and in such a situation we can only hope that empirical evidence will draw our attention to their falsity at an early stage. As an example of the application of such a selection principle, we may refer to the analogous phenomenological theory of the *potential* interactions between nucleons, where one makes the simplifying assumption that velocity-dependent and many-body potentials are of subordinate importance to two-body, velocity-independent interactions for nonrelativistic velocities of the nucleons involved. In the same manner, while we shall attempt to keep our phenomenological treatment of electromagnetic interactions of nucleons sufficiently broad to include the possibilities of velocity dependence and a many-body character for these interactions, in preliminary applications of the formalism one might assume that such complicating aspects of the problem are of secondary importance.

The work which follows is very closely related to that recently published by Sachs and Austern[3] and covers much the same ground. The principal differences arise in connection with the starting point. Sachs and Austern begin basically with the condition of gauge invariance for the Hamiltonian, while we prefer to begin with the differential charge conservation law.[4] The latter approach has perhaps some advantage in the way of physical perspicuity, but basically the formulations are equivalent. The method of Sachs and Austern lends itself readily to consideration of interactions of arbitrarily high order in the electromagnetic field. Our method can be extended to such cases but we restrict our present considerations only to terms linear in the electromagnetic field. We go somewhat further than these authors in the direction of studying explicit forms for the electromagnetic interaction. We may note also that the treatment of Sachs and Austern employs a restricted definition of gauge invariance which involves the assumption that the charge density associated with a nucleon is a point charge. By an appropriate generalization[5] of their definition, one can take account of the real possibility that the charge distribution associated with nucleons is spatially extended. However, as a consequence of their restriction to this special case they are led to the conclusion that the *long-wavelength*, electric multipole matrix elements are unambigously determined by gauge invariance alone and are unaffected by the interactions between nucleons.[6] Our treatment recognizes the possibility of spatial extension of the charge distributions associated with nucleons from the start and shows that *interaction effects may affect even the long-wavelength, electric dipole matrix elements*. The possible importance of this conclusion with respect to

[3] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); N. Austern and R. G. Sachs, Phys. Rev. **81**, 710 (1951).

[4] G. J. Kynch, Phys. Rev. **81**, 1060 (1951).

[5] The definition of gauge invariance usually quoted is that for a particle carrying a point charge. However, it is quite inadequate to describe the gauge invariance of theories describing particles with which there is associated an extended charge distribution. A more general definition than that employed by Sachs and Austern in Eqs. (7) and (8) of their paper [Phys. Rev. **81**, 705 (1951)] consists in replacing their expression g by

$$g=\frac{i}{\hbar c}\int \rho(\mathbf{x})G(\mathbf{x},t)d\mathbf{x},$$

where $\rho(\mathbf{x})$ is the charge density operator for the system and may involve other variables referring to the nucleons than their position alone—their spins for example. It should be remembered that gauge invariance of a theory requires only that after a gauge transformation on the electromagnetic potentials, there exist a unitary transformation which restores the Hamiltonian to its original form. No limitations need be imposed on the form of the unitary transformation.

[6] With respect to short-wavelength electric multipole matrix elements, see, however, J. G. Brennan and R. G. Sachs, Phys. Rev. **88**, 824 (1952). It should be noted that A. J. F. Siegert in an early consideration of this question [Phys. Rev. **52**, 787 (1937)] was cognizant of the possibility of modifications of the electric dipole matrix element at long wavelengths by interaction effects and states carefully the conditions under which the so-called "Siegert Theorem" would be expected to be valid.

the breakdown of certain selection rules is pointed out at an appropriate place.

We now turn from the question of the methodology involved in solving the problem we have set forth above to some practical considerations of its application. It is well known that in many applications of the theory of electromagnetic interactions, the practical solutions of the problems involved is facilitated by an expansion of the electromagnetic field into multipoles. This is particularly true when one is dealing with radiation whose wavelength is long compared to the dimensions of the radiating system, since in such cases the contributions of higher multipoles to any radiative process which occurs is much smaller than the contribution arising from the lowest multipole order in which the transition can occur. The latter is determined by the angular momentum and parity changes of the material system involved in the radiative transition, and it is just this fact which makes spectroscopy a valuable tool by which one can obtain information about the states of a material system. Now, since the ratio of system dimensions to photon wavelength is the parameter which measures both the relative importance of higher multipole contributions as well as the importance of retardation effects,[7] as long as one is in the long-wavelength region it makes little difference whether one employs a rigorous multipole expansion based on irreducible representations of the rotation group or a simple expansion in inverse powers of the wavelength (or direct powers of the wave vector) whose terms are related to reducible representations of the rotation group.[3] The error involved in terminating the latter expansion at some term is always of the same order as that arising in the termination of the multipole expansion at the corresponding term. Either series becomes less useful as one moves to shorter wavelengths where either many multipoles or many terms in an expansion in powers of the wave vector of the photon must be retained.

Now if one is interested in formulating a radiation theory which will be useful even when the wavelength of the photon is shorter or of the order of the system dimensions, it is useful to avoid formulating it explicitly in terms of an expansion either in multipoles or powers of the wave vector since such an expansion is then only slowly convergent. We may offer as an advantage of the formalism which we develop below the fact that it is not based on such an expansion and therefore (if no other limitations intervene) it can equally well be applied to the interactions which involve short-wavelength photons as long-wavelength photons. Wherever one wishes to employ a multipole or wave vector expansion one may do so by the appropriate expansion of certain exponentials occurring in our formulation. Our formulation still employs a pseudoseparation into electric and magnetic interactions exactly of the form of Sachs and Austern[3] but each of these is given in a summed form. The utility of this formulation will be demonstrated in a paper on the photodisintegration of the deuteron which will appear shortly.

[7] We use the term retardation here in a special sense to describe all but the first nonvanishing term in an expansion of a given *multipole* matrix element in powers of the wave vector.

PRELIMINARY CONSIDERATIONS

We consider the emission or absorption of a photon by a nuclear system consisting of Z protons and $(A-Z)$ neutrons. The nucleons will be treated nonrelativistically and each nucleon will be characterized by a position vector $\mathbf{x}_n$, a momentum $\mathbf{p}_n$, a spin vector $\boldsymbol{\sigma}_n$, and an isotopic spin vector $\boldsymbol{\tau}_n$. Since the treatment is nonrelativistic, we may introduce the center-of-mass coordinate $\mathbf{R}$,

$$\mathbf{R}=\sum_n \mathbf{x}_n/A,$$

and relative coordinates $\mathbf{r}_n$

$$\mathbf{r}_n=\mathbf{x}_n-\mathbf{R},$$

as well as the momenta conjugate to these coordinates

$$\mathbf{P}=\sum_n \mathbf{p}_n$$

representing the total momentum of the system, and

$$\boldsymbol{\pi}_n=\mathbf{p}_n-\mathbf{P}/A.$$

The Hamiltonian for the system may then be written in the form

$$H_m=T_0+H,$$

where

$$T_0=P^2/2AM$$

is the kinetic energy associated with the motion of the center of mass and H is the internal energy of the nuclear system. The latter is a function of the $\boldsymbol{\pi}_n$, $\mathbf{r}_n$, $\boldsymbol{\sigma}_n$, and $\boldsymbol{\tau}_n$ and may be written in the form

$$H=T+V,$$

with

$$T=\sum_n \pi_n^2/2M.$$

We now turn our attention to the operators $\rho(\mathbf{x})$ for the electric charge density of the system and $\mathbf{J}(\mathbf{x})$ for the current density. These will be required to satisfy the differential conservation law for charge, which in our case will take the form

$$\mathrm{div}\mathbf{J}(\mathbf{x})+i[H_m, \rho(\mathbf{x})]/\hbar c=0. \tag{1}$$

The fundamental premise on which the following development is based is the assumption that these operators can be represented completely in terms of variables describing the nucleons. Since some of the charges and currents in the nucleus are associated with the exchange of charged virtual particles (mesons) between nucleons, it is by no means obvious that one can eliminate the variables relating to these particles from the expression for the charge and current density. However, on the basis of the discussion in the introduction one might expect that such a representation should be possible at least to the same degree to which one can phenomenologically represent the potential interaction between nucleons in terms of the variables describing the nucleons alone; this is exactly what one attempts to do in constructing a phenomenological theory of exchange and interaction currents in nuclei, and we shall proceed on the assumption that such a representation is possible.

It is quite clear that $\rho(\mathbf{x})$ and $\mathbf{J}(\mathbf{x})$ will depend only on the relative separation of the point $\mathbf{x}$ from the positions of the nucleons. Hence in this case one may define what might be called the external convection current density of the nuclear system as

$$\mathbf{J}^C(\mathbf{x})=[\mathbf{P}\rho(\mathbf{x})+\rho(\mathbf{x})\mathbf{P}]/2AM, \tag{2}$$

and it will have the property that

$$\mathrm{div}\mathbf{J}^C(\mathbf{x})+i[T_0,\rho(\mathbf{x})]/\hbar c=0. \tag{3}$$

Hence, if we divide the total current density into the external convection current density and the internal current density $\mathbf{j}(\mathbf{x})$,

$$\mathbf{J}(\mathbf{x})=\mathbf{J}^C(\mathbf{x})+\mathbf{j}(\mathbf{x}). \tag{4}$$

Then combining Eqs. (3) and (1) we obtain a differential conservation law involving only internal coordinates of the nuclear system,

$$\mathrm{div}\mathbf{j}(\mathbf{x})+i[H,\rho(\mathbf{x})]/\hbar c=0. \tag{5}$$

While we do not know the exact forms for either $\rho(x)$ or $\mathbf{J}(\mathbf{x})$ in view of the participation of charge-exchange effects, we will later effect a further decomposition of both operators into a part about whose form we are quite certain, and a remainder, about whose form we do not have much information. We now pass to the problem of the interaction of the nuclear system with the radiation field.

In the presence of interaction of the nuclear system with the transverse electromagnetic field, the Hamiltonian for the combined system will have the form

$$\mathcal{H}=H_m+H_r+H_i. \tag{6}$$

We have already discussed the form of H_m. H_r represents the Hamiltonian of the free radiation field,

$$H_r=\frac{1}{8\pi}\int[\mathcal{E}^2+\mathcal{H}^2]d\mathbf{x},$$

where

$$\boldsymbol{\mathcal{E}}(\mathbf{x})=-\partial\mathbf{A}(\mathbf{x})/c\partial t,$$

$$\boldsymbol{\mathcal{H}}(\mathbf{x})=\mathrm{curl}\mathbf{A}(\mathbf{x}),\quad[\mathrm{div}\mathbf{A}(\mathbf{x})=0].$$

With the transverse field expanded in plane waves in a volume V,

$$\mathbf{A}(\mathbf{x})=\sum_{\mathbf{k},\epsilon}\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}[A_{\mathbf{k},\epsilon}+A_{-\mathbf{k},\epsilon}^*]\boldsymbol{\varepsilon}e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{7}$$

it takes the form

$$H_r=\tfrac{1}{2}\sum_{\mathbf{k},\epsilon}[A_{\mathbf{k},\epsilon}A_{\mathbf{k},\epsilon}^*+A_{\mathbf{k},\epsilon}^*A_{\mathbf{k},\epsilon}]\hbar kc,$$

where $A_{\mathbf{k},\epsilon}$ and $A_{\mathbf{k},\epsilon}^*$ are the usual destruction and creation operators, respectively, for a photon of momentum $\mathbf{k}$ and unit (electric) polarization vector $\boldsymbol{\varepsilon}$. The transversality condition is $\boldsymbol{\varepsilon}\cdot\mathbf{k}=0$.

The term H_i represents the interaction of the nuclear system with the transverse electromagnetic field. In order to describe the absorption or emission of one photon by the nuclear system, it is sufficient to consider only terms linear in the vector potential of the transverse field; the interaction can then be written as

$$H_i=-\int\mathbf{J}(\mathbf{x})\cdot\mathbf{A}(\mathbf{x})d\mathbf{x}. \tag{8}$$

On introducing the expansion of $\mathbf{A}$ in plane waves this becomes

$$H_i=-\sum_{\mathbf{k},\epsilon}\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}(A_{\mathbf{k},\epsilon}+A_{-\mathbf{k},\epsilon}^*)\int\mathbf{J}(\mathbf{x})\cdot\boldsymbol{\varepsilon}e^{i\mathbf{k}\cdot\mathbf{x}}d\mathbf{x}. \tag{9}$$

MATRIX ELEMENTS FOR EMISSION AND ABSORPTION OF A PHOTON

We shall construct the matrix element for the absorption of a photon of momentum $\hbar\mathbf{k}$ while the nuclear system makes a transition from an internal state represented by the wave function ψ_a to a final internal state represented by the wave function ψ_b. The matrix element will be computed in the reference frame in which the total momentum is zero. The matrix element for the emission of a photon of momentum $\hbar\mathbf{k}$ while the nuclear system makes a transition from the state b to the state a will be given by the complex conjugate of the absorption matrix element.

In the center-of-mass system, the initial and final wave functions of the nuclear system, in the case of photon absorption, will be given by

$$\Phi_a=V^{-\frac{1}{2}}e^{-i\mathbf{k}\cdot\mathbf{R}}\psi_a(\mathbf{r}_n),\quad\Phi_b=V^{-\frac{1}{2}}\psi_b(\mathbf{r}_n).$$

On inserting $\mathbf{J}^C(\mathbf{x})+\mathbf{j}(\mathbf{x})$ for $\mathbf{J}(\mathbf{x})$ in (9) and evaluating the matrix element, one finds that because of the transversality condition $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$, the external convection-current density makes no contribution and the required matrix element is given by

$$\mathcal{T}_{ba}=-\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}\left(b\left|\int\mathbf{j}(\mathbf{r})\cdot\boldsymbol{\varepsilon}e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}\right|a\right), \tag{10}$$

where we have introduced the variable $\mathbf{r}=\mathbf{x}-\mathbf{R}$.

We shall now separate this matrix element into two parts which we shall refer to as the *electric* matrix element and the *magnetic* matrix element though these do not correspond precisely to the usual division into electric and magnetic multipoles. The separation is based on the identity

$$\boldsymbol{\varepsilon} e^{i\mathbf{k}\cdot\mathbf{r}} = \int_0^1 \{\operatorname{grad}(\boldsymbol{\varepsilon}\cdot\mathbf{r}e^{is\mathbf{k}\cdot\mathbf{r}}) - is\mathbf{r}\times[\mathbf{k}\times\boldsymbol{\varepsilon}]e^{is\mathbf{k}\cdot\mathbf{r}}\}ds. \quad (11)$$

Introducing this into Eq. (10), we have

$$\mathcal{T}_{ba} = \mathcal{T}_{ba}{}^{e} + \mathcal{T}_{ba}{}^{m}, \quad (12)$$

where

$$\mathcal{T}_{ba}{}^{e} = -\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}\left(b\left|\int_0^1\int \mathbf{j}(\mathbf{r})\cdot\operatorname{grad}(\boldsymbol{\varepsilon}\cdot\mathbf{r}e^{is\mathbf{k}\cdot\mathbf{r}})d\mathbf{r}ds\right|a\right), \quad (13)$$

$$\mathcal{T}_{ba}{}^{m} = i\left(\frac{2\pi\hbar c}{kV}\right)\left(b\left|\int_0^1\int \mathbf{j}(\mathbf{r})\cdot[k\mathbf{r}\times\boldsymbol{\varepsilon}']e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}sds\right|a\right). \quad (14)$$

Here $\boldsymbol{\varepsilon}'$ represents the unit magnetic polarization vector $[\mathbf{k}\times\boldsymbol{\varepsilon}]/k$.

The expression for the electric matrix element may be simplified by an integration by parts and the use of the differential charge conservation law (5),

$$\begin{aligned}\mathcal{T}_{ba}{}^{e} &= \left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}\left(b\left|\int_0^1\int \operatorname{div}\mathbf{j}(\mathbf{r})\boldsymbol{\varepsilon}\cdot\mathbf{r}e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}ds\right|a\right)\\ &= -\frac{i}{\hbar c}\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}\left(b\left|\left[H, \int_0^1\int \boldsymbol{\varepsilon}\cdot\mathbf{r}\rho(\mathbf{r})e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}ds\right]\right|a\right)\\ &= -\frac{i}{\hbar c}(\epsilon_b-\epsilon_a)\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}\\ &\quad\times\left(b\left|\int_0^1\int \boldsymbol{\varepsilon}\cdot\mathbf{r}\rho(\mathbf{r})e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}ds\right|a\right), \end{aligned} \quad (15)$$

where ϵ_a and ϵ_b represent the internal energies of the nuclear system in the initial and final states ($H\psi_a=\epsilon_a\psi_a$, $H\psi_b=\epsilon_b\psi_b$). We may note that conservation of energy for the transition requires

$$\hbar kc = (\epsilon_b-\epsilon_a)+\hbar^2k^2/2AM. \quad (16)$$

The last term is generally negligible for photons of energy less than 100 Mev. To keep in mind this difference, however, we shall write $\hbar k'c=\epsilon_b-\epsilon_a$. Then

$$\mathcal{T}_{ba}{}^{e} = -ik'\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}(b|\boldsymbol{\varepsilon}\cdot\mathfrak{P}|a), \quad (17)$$

$$\mathcal{T}_{ba}{}^{m} = -ik\left(\frac{2\pi\hbar c}{kV}\right)^{\frac{1}{2}}(b|\boldsymbol{\varepsilon}'\cdot\mathfrak{M}|a), \quad (18)$$

where

$$\mathfrak{P} = \int_0^1\int \mathbf{r}\rho(\mathbf{r})e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}ds, \quad (19)$$

$$\mathfrak{M} = \int_0^1\int \mathbf{r}\times\mathbf{j}(\mathbf{r})e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}sds. \quad (20)$$

It will be noted that for $\mathbf{k}=0$, $\mathfrak{P}$ is just the electric dipole moment of the nuclear system while $\mathfrak{M}$ is just the magnetic dipole moment of the nuclear system.

CHARGE AND CURRENT DENSITY OPERATORS—GENERAL CONSIDERATIONS

To apply the results obtained in the last section one must have further information concerning the specific forms of the charge and current density operators for the nuclear system. The succeeding three sections are devoted to the question of constructing the most general nonrelativistic forms for these operators on the basis of the assumptions outlined earlier. In the present section we will discuss briefly certain general properties of these operators.

Certain generally accepted invariance conditions restrict the form which the charge and current density operators must possess. Actually we have already made use of some of these in our previous development but it is now necessary to consider them in more detail.

(1) *Invariance with respect to translations.*—The first invariance principle which we shall apply requires that, if the nuclear system is subject to a certain spatial translation, the charge and current density distributions shall undergo the same translation. Since these quantities are functions of $\mathbf{x}$ and of the spatial coordinates of the nucleons $\mathbf{x}_n$, the satisfaction of this invariance condition requires that they be a function of these variables only through the combinations $\mathbf{x}-\mathbf{x}_n$.

(2) *Invariance with respect to rotations.*—Similarly we require that if the nuclear system undergoes a rigid rotation, then the charge and current distributions associated with the system shall undergo the same rotation. This requires that the charge density be a *scalar* function of the vectors $\mathbf{x}-\mathbf{x}_n$ and $\boldsymbol{\sigma}_n$, while the current density operator shall be a *vector* function formed from these same vectors.

(3) *Invariance with respect to space inversion.*—Invariance of the equations for the nuclear system on passing from a right-handed to a left-handed coordinate system (implying conservation of parity) requires that under the transformation $\mathbf{x}_n\rightarrow\mathbf{x}_n$, $\mathbf{x}\rightarrow-\mathbf{x}$ the charge density operator remain invariant while the current density operator reverse sign. This means that the charge density operator must be a true scalar (as opposed to a pseudoscalar) function while the current density operator must be a true (polar) vector function (as opposed to a pseudovector or axial vector function).

(4) *Invariance with respect to time reversal.*—The invariance of the nuclear system with respect to time reversal requires that under the transformation $\mathbf{p}_n\rightarrow\mathbf{p}_n$, $\boldsymbol{\sigma}_n\rightarrow-\boldsymbol{\sigma}_n$, $\tau_n{}^y\rightarrow-\tau_n{}^y$, the charge density operator remain invariant while the current density operator reverses sign.[4]

(5) *Symmetry with respect to all nucleons.*—Since all nucleons are treated as indistinguishable particles it is necessary that the charge and current density operators be invariant under the permutation of dynamical

variables referring to any pair of nucleons, or in other words, that these operators be completely symmetric functions of all the nucleon variables.

(6) *"Superselection Principle" for electric charge.*—It is rather strange that while our whole treatment of electromagnetic interactions of nuclei is based on differential conservation law for electric charge, this is still not sufficient to guarantee that the total charge of a nuclear system remains constant. The reason for this is that the current density operator is so unrestricted as to include possibly terms corresponding to a removal of the electric charge from a proton, say, to infinity. To overcome this difficulty it is necessary to postulate a "superselection" principle[8] for electric charge, by requiring that all observables (in the usual sense of Dirac) commute with the operator for the total charge of the nuclear system. In particular, then, the charge and current density operators as well as the Hamiltonian for the system will be required to commute with the total charge operator for the nuclear system, $\frac{1}{2}e\sum_n(1+\tau_n^z)$. While it may appear that this is an arbitrary assumption to make, and that it would be better to impose the restriction that the integral of the normal component of the current density over a sufficiently large closed surface enclosing the nuclear system shall vanish, the latter alternative has some undesirable features. To illustrate, suppose that the part of the current density operator which is independent (that is, of zeroth order) of the electromagnetic potentials contains a term which does not commute with the total charge operator. This would manifest itself in our treatment only when we came to study differential charge conservation of the parts of the charge and current density operator which are of *first order* in the electromagnetic potentials, and it would be only after studying these that we would find that it would be necessary to delete this term from the zeroth-order part of the current density operator as a consequence of our alternative condition. It is more convenient to simply impose the above superselection principle.

CHARGE AND CURRENT DENSITY OPERATORS—ONE-PARTICLE TERMS

We now give consideration to the explicit forms of the charge and current density operators for the nuclear system. It is convenient here to proceed in analogy with the phenomenological treatment of internucleonic potential interactions and consider the charge and current density operators to be expressed as the sum of one-particle, two-particle, etc., terms,

$$\begin{aligned}\rho(\mathbf{x})&=\rho_{(1)}(\mathbf{x})+\rho_{(2)}(\mathbf{x})+\rho_{(3)}(\mathbf{x})+\cdots,\\ \mathbf{J}(\mathbf{x})&=\mathbf{J}_{(1)}(\mathbf{x})+\mathbf{J}_{(2)}(\mathbf{x})+\mathbf{J}_{(3)}(\mathbf{x})+\cdots.\end{aligned} \tag{21}$$

Here $\rho_{(1)}(\mathbf{x})$ and $\mathbf{J}_{(1)}(\mathbf{x})$ are assumed to consist themselves of a symmetric sum over all nucleons of terms each involving variables referring to only one nucleon,

$$\rho_{(1)}(\mathbf{x})=\sum_n\rho_n(\mathbf{x}),\quad \mathbf{J}_{(1)}(\mathbf{x})=\sum_n\mathbf{J}_n(\mathbf{x}). \tag{22}$$

$\rho_{(2)}(\mathbf{x})$ and $\mathbf{J}_{(2)}(\mathbf{x})$ are assumed to consist of a symmetric sum over all nucleons of terms each involving variables referring to two nucleons only,

$$\rho_{(2)}(\mathbf{x})=\sum_{n<n'}\rho_{nn'}(\mathbf{x}),\quad \mathbf{J}_{(2)}(\mathbf{x})=\sum_{n<n'}\mathbf{J}_{nn'}(\mathbf{x}), \tag{23}$$

etc. It will be convenient to refer to all terms other than the one-particle terms as *interaction* terms. In the present section we limit our considerations to the one-particle terms only.

Consider a single nucleon at rest ($\mathbf{p}_n=0$) characterized by the dynamical variables $\mathbf{x}_n$, $\boldsymbol{\sigma}_n$, $\boldsymbol{\tau}_n$. The charge density at the point $\mathbf{x}$ associated with this nucleon can be written as a function of $\mathbf{x}$ and the above variables. Translational invariance requires that it be a function of $\mathbf{x}$ only in the combination $\mathbf{x}-\mathbf{x}_n$, while rotational invariance and invariance under space inversion requires that it be a scalar function of $\mathbf{x}-\mathbf{x}_n$ and $\boldsymbol{\sigma}_n$. One easily finds that the only such functions are spherically symmetric functions of the separation $\mathbf{x}-\mathbf{x}_n$. If one further assumes the "superselection rule" for electric charge, then the most general form for the charge density operator must be

$$\rho_n(\mathbf{x})=e[\tau_n{}^PU^P(|\mathbf{x}-\mathbf{x}_n|)+\tau_n{}^NU^N(|\mathbf{x}-\mathbf{x}_n|)], \tag{24}$$

where U^P and U^N are arbitrary functions of their argument and normalized by the conditions

$$\int U^P(\mathbf{x})d\mathbf{x}=1,\quad \int U^N(\mathbf{x})d\mathbf{x}=0, \tag{25}$$

to ensure that the total charge on a proton is e and the total charge on a neutron is zero.

By the application of similar arguments to the current density operator (including now the condition that the current density change sign under time reflection), one concludes that the most general form of the current density operator corresponding to a nucleon at rest is

$$\mathbf{J}_n(\mathbf{x})=\frac{e\hbar}{2Mc}\operatorname{curl}\{\mu^P\tau_n{}^P\boldsymbol{\sigma}_nS^P(|\mathbf{x}-\mathbf{x}_n|)+\mu^N\tau_n{}^N\boldsymbol{\sigma}_nS^N(|\mathbf{x}-\mathbf{x}_n|)\}, \tag{26}$$

where S^P and S^N are again arbitrary functions of their argument but normalized to

$$\int S^P(x)d\mathbf{x}=\int S^N(\mathbf{x})d\mathbf{x}=1 \tag{27}$$

to ensure that the magnetic moment of a proton is μ^P nuclear magnetons, and the magnetic moment of a neutron is μ^N nuclear magnetons.

To determine the corresponding operators for a nucleon in motion with a momentum $\mathbf{p}_n$, we perform a

[8] Wick, Wightman, and Wigner, Phys. Rev. 88, 101 (1952).

Lorentz transformation (and after discarding relativistic terms, that is, terms quadratic in the momentum) obtain the same form (24) for the charge density operator and

$$\mathbf{J}_n(\mathbf{x})=\frac{e\hbar}{2Mc}\operatorname{curl}\{\mu^P\tau_n{}^P\boldsymbol{\sigma}_n S^P(|\mathbf{x}-\mathbf{x}_n|)+\mu^N\tau_n{}^N\boldsymbol{\sigma}_n S^N(|\mathbf{x}-\mathbf{x}_n|)\}+\frac{e}{2Mc}[\mathbf{p}_n\rho_n(\mathbf{x})+\rho_n(\mathbf{x})\mathbf{p}_n] \quad (28)$$

for the current density operator. The complete one-particle charge and current density operators for the whole nuclear system is then obtained by substituting in (22)

$$\begin{aligned}
\rho_{(1)}(\mathbf{x})&=\sum_n e[\tau_n{}^PU^P(|\mathbf{x}-\mathbf{x}_n|)+\tau_n{}^NU^N(|\mathbf{x}-\mathbf{x}_n|)],\\
\mathbf{J}_{(1)}(\mathbf{x})&=\mathbf{J}_{(1)}{}^C(\mathbf{x})+\mathbf{j}_{(1)}{}^c(\mathbf{x})+\mathbf{j}_{(1)}{}^s(\mathbf{x}),\\
\mathbf{J}_{(1)}{}^C(\mathbf{x})&=e[\mathbf{P}\rho_{(1)}(\mathbf{x})+\rho_{(1)}(\mathbf{x})\mathbf{P}]/2AMc,\\
\mathbf{j}_{(1)}{}^c(\mathbf{x})&=\sum_n e[\boldsymbol{\pi}_n\rho_n(\mathbf{x})+\rho_n(\mathbf{x})\boldsymbol{\pi}_n(\mathbf{x})]/2Mc, \qquad (29)\\
\mathbf{j}_{(1)}{}^s(\mathbf{x})&=\sum_n\frac{e\hbar}{2Mc}\operatorname{curl}\{\mu^P\tau_n{}^P\boldsymbol{\sigma}_n{}^PS^P(|\mathbf{x}-\mathbf{x}_n|)+\mu^N\tau_n{}^N\boldsymbol{\sigma}_nS^N(|\mathbf{x}-\mathbf{x}_n|)\},
\end{aligned}$$

where we have separated the one-particle current density into its external convection current ($\mathbf{J}^C$), its internal convection current ($\mathbf{j}^c$), and spin current ($\mathbf{j}^s$) parts. One easily verifies that

$$\begin{aligned}
\operatorname{div}\mathbf{J}_{(1)}{}^C(\mathbf{x})+i[T_0,\rho_{(1)}(\mathbf{x})]/\hbar c&=0,\\
\operatorname{div}\mathbf{j}_{(1)}{}^c(\mathbf{x})+i[T,\rho_{(1)}(\mathbf{x})]/\hbar c&=0, \qquad (30)\\
\operatorname{div}\mathbf{j}_{(1)}{}^s(\mathbf{x})&=0.
\end{aligned}$$

The functions U^P and U^N obviously describe the spatial distribution of the charge density associated with isolated protons and neutrons, respectively, while the functions S^P and S^N describe the spatial distribution of the spin current densities associated with isolated protons and neutrons, respectively. To the best of our present knowledge all of these functions rapidly approach zero when their arguments exceed values of the order of a meson Compton wavelength. Beyond this we have little knowledge concerning their functional form; any specific meson theory, however, uniquely prescribes their form.

CHARGE AND CURRENT DENSITY OPERATORS—INTERACTION TERMS

We now turn our attention to the examination of the interaction terms $\rho^i(\mathbf{x})$ and $\mathbf{J}^i(\mathbf{x})$ in the charge and current density operators and to the methods of constructing them in their most general form. We may note first that in view of (30) and (1) they will satisfy the differential conservation law,

$$\operatorname{div}\mathbf{J}^i(\mathbf{x})+i\{[T_0+T,\rho^i(\mathbf{x})]+[V,\rho_{(1)}(\mathbf{x})+\rho^i(\mathbf{x})]\}/\hbar c=0. \quad (31)$$

If we write the interaction energy between nucleons as the sum of two-, three-, $\cdots$ particle terms,

$$V=V_{(2)}+V_{(3)}+\cdots, \quad (32)$$

with

$$V_{(2)}=\sum_{n<n'}V_{nn'},\quad V_{(3)}=\sum_{n<n',<n''}V_{nn'n''}, \quad (33)$$

etc., then Eq. (31) can be split up into separate equations for the two-, three-, $\cdots$ particle parts of the current density operator.

We shall consider in some detail only the two-particle terms in the charge and current density operators; construction of higher terms proceeds in close analogy. We begin by considering a system of two nucleons whose total momentum vanishes ($\mathbf{p}_n+\mathbf{p}_{n'}=0$). The operator for the charge density associated with this system will be a scalar function formed from the quantities $\mathbf{x}-\mathbf{x}_n$, $\mathbf{x}-\mathbf{x}_{n'}$, $\mathbf{p}_n-\mathbf{p}_{n'}$, $\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_{n'}$, $\boldsymbol{\tau}_n$, $\boldsymbol{\tau}_{n'}$. The problem of constructing the most general form possible is straightforward but lengthy[9] and not worth our present consideration. Let us assume that this most general function has been determined and is designated by $\rho_{nn'}(\mathbf{x})$. The two-particle term in the current density operator, in contradistinction to the one-particle case, must now satisfy the two particle part of Eq. (31), which in our present case may be written as

$$\operatorname{div}\mathbf{J}_{nn'}(\mathbf{x})+i\{[(\mathbf{p}_n-\mathbf{p}_{n'})^2/4M,\rho_{nn'}(\mathbf{x})]+[V_{nn'},\rho_n(\mathbf{x})+\rho_{n'}(\mathbf{x})+\rho_{nn'}(\mathbf{x})]\}/\hbar c=0. \quad (34)$$

The general solution of this equation will consist of any special solution plus the curl of an arbitrary function of $\mathbf{x}$, with the limitation, of course, that the whole solution be a two-particle function and satisfy the required symmetry and invariance properties.

One method of obtaining a formal solution to Eq. (34) is to split $\mathbf{J}_{nn'}(\mathbf{x})$ into *longitudinal* (irrotational) and *transverse* (solenoidal) parts,

$$J_{nn'}(\mathbf{x})=\operatorname{grad}\chi_{nn'}(\mathbf{x})+\operatorname{curl}\mathbf{m}_{nn'}(\mathbf{x}). \quad (35)$$

Substituting into Eq. (41) we obtain

$$\nabla^2\chi_{nn'}(\mathbf{x})=-i\{[(\mathbf{p}_n-\mathbf{p}_{n'})^2/4M,\rho_{nn'}(\mathbf{x})]+[V_{nn'},\rho_n(\mathbf{x})+\rho_{n'}(\mathbf{x})+\rho_{nn'}(\mathbf{x})]\}/\hbar c=Q(\mathbf{x}), \quad (36)$$

with the solution (which vanishes at infinity)

$$\chi_{nn'}(\mathbf{x})=\frac{1}{4\pi}\int\frac{Q(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}'. \quad (37)$$

[9] The methods employed for constructing general functions of the required form are exemplified in E. P. Wigner, Phys. Rev. **51**, 106 (1937); L. Eisenbud and E. P. Wigner, Proc. Nat. Acad. Sci. **27**, 281 (1941); R. K. Osborn and L. L. Foldy, Phys. Rev. **79**, 795 (1950).

The function $\mathbf{m}_{nn'}(\mathbf{x})$ is an arbitrary function, which because of the symmetry and invariance conditions to be imposed on it must be an axial vector function of $\mathbf{p}_n-\mathbf{p}_{n'}$, $\mathbf{x}-\mathbf{x}_n$, $\mathbf{x}-\mathbf{x}_{n'}$, $\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_{n'}$, $\boldsymbol{\tau}_n$, $\boldsymbol{\tau}_{n'}$, which changes sign under time reflection and commutes with the total charge operator $\tau_n{}^P+\tau_{n'}{}^P$. Again the construction of the most general form of $\mathbf{m}_{nn'}$ satisfying these conditions is a straightforward but lengthy procedure which we shall omit.

To obtain the corresponding forms for the charge and current density operators when the two nucleons have a total momentum $\mathbf{p}_n+\mathbf{p}_{n'}$, we again make a Lorentz transformation and, after dropping relativistic terms, we find that the charge density operator is still given by the same expression for $\rho_{nn'}(\mathbf{x})$ while the current density operator is given by

$$\mathbf{J}_{nn'}(\mathbf{x})=e[(\mathbf{p}_n+\mathbf{p}_{n'})\rho_{nn'}(\mathbf{x})+\rho_{nn'}(\mathbf{x})(\mathbf{p}_n+\mathbf{p}_{n'})]/4Mc+\text{grad}\chi_{nn'}(\mathbf{x})+\text{curl}\mathbf{m}_{nn'}(\mathbf{x}). \quad (38)$$

The two-particle parts of the charge and current density operators for the entire nuclear system are then obtained by substitution in (23) with the results,

$$\begin{aligned}
\rho_{(2)}(\mathbf{x})&=\sum_{n<n'}\rho_{nn'}(\mathbf{x}),\\
\mathbf{J}_{(2)}(\mathbf{x})&=\mathbf{J}_{(2)}{}^C(\mathbf{x})+j_{(2)}(\mathbf{x}),\\
\mathbf{J}_{(2)}{}^C(\mathbf{x})&=c[\mathbf{P}\rho_{(2)}(\mathbf{x})+\rho_{(2)}(\mathbf{x})\mathbf{P}]/2AMc,\\
\mathbf{j}_{(2)}(\mathbf{x})&=\text{grad}\chi_{(2)}(\mathbf{x})+\text{curl}\mathbf{m}_{(2)}(\mathbf{x})\\
&\quad+e\sum_{n<n'}[(\boldsymbol{\pi}_n+\boldsymbol{\pi}_{n'})\rho_{nn'}(\mathbf{x})\\
&\qquad+\rho_{nn'}(\mathbf{x})(\boldsymbol{\pi}_n+\boldsymbol{\pi}_{n'})]/4Mc,\\
\chi_{(2)}(\mathbf{x})&=\sum_{n<n'}\chi_{nn'}(\mathbf{x}),\\
\mathbf{m}_{(2)}(\mathbf{x})&=\sum_{n<n'}\mathbf{m}_{nn'}(\mathbf{x}),
\end{aligned} \quad (39)$$

where again a separation of the external convection current density $\mathbf{J}_{(2)}{}^C(\mathbf{x})$ has been made. The generalization of this procedure to three-, four-, $\cdots$ particle terms is obvious.

Before proceeding, it is necessary to express a warning relative to our method of solution of Eq. (34). While simple and correct, it is nevertheless deceptive in one aspect. We have intimated that the function $\mathbf{m}_{nn'}$, apart from the necessary symmetry and invariance conditions which it must satisfy, is completely arbitrary. This is mathematically correct but physically unrealistic. We are quite certain that the charge and current distributions associated with nucleons are closely localized in the regions where the nucleons are themselves situated and vanish rapidly (probably at least exponentially) as one moves away from these regions. Now the longitudinal part of the two-particle current density operator that we have found falls off relatively slowly with distance from the nucleons, in fact, only as the inverse cube of the distance at large distances. Hence, if the total two-particle current density is to fall off exponentially, it is necessary the $\mathbf{m}_{nn'}$ satisfy an asymptotic condition such that the solenoidal current density cancels the longitudinal current density to terms of exponential order at large distances. This means that $\mathbf{m}_{nn'}$ is not as arbitrary as indicated but, to yield a physically reasonable current distribution, must have an appropriate asymptotic form. Unfortunately, there does not appear to be any simple alternative treatment of Eq. (34) which yields an explicit solution and yet avoids this difficulty. Hence we must content ourselves with the treatment we have given and keep in mind this condition which $\mathbf{m}_{nn'}$ must satisfy, or else forsake an explicit representation of the two-particle current density operator.

An important conclusion which we may draw from the considerations in this section is immediately apparent on examining Eq. (34) or Eq. (37). We note that even if there exists no two-particle contribution to the charge density operator, whenever the one-particle charge density operator fails to commute with the interaction energy operator for the nucleons, one will have two-particle terms in the current density operator. A specific case of the latter situation arises whenever charge-exchange forces operate between nucleons, since in this case the one-particle charge density operator will fail to commute with the interaction energy operator. Since charge exchange forces correspond to the transfer of charge from one nucleon to another, differential charge conservation requires that currents flow in the intervening region between the two nucleons, and the two-particle contributions to the current density operator are the mathematical manifestation of the necessity for such currents.

SPECIFIC FORMULAS FOR PHOTOEFFECT MATRIX ELEMENTS

Having now obtained more specific formulas for the charge and current density operators of the nuclear system, these may be substituted in Eqs. (19) and (20) to obtain more specific formulas for the photoeffect matrix elements. In particular we may write for the one-particle contributions,

$$\mathfrak{P}_{(1)}=e\sum_n\int_0^1\int\mathbf{r}\{\tau_n{}^PU^P(|\mathbf{r}-\mathbf{r}_n|)+\tau_n{}^NU^N(|\mathbf{r}-\mathbf{r}_n|)\}e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}ds, \quad (40)$$

$$\begin{aligned}\mathfrak{M}_{(1)}{}^c=\frac{e}{2Mc}\sum_n\int_0^1\int\{&\mathbf{r}\times\boldsymbol{\pi}_n[\tau_n{}^PU^P(|\mathbf{r}-\mathbf{r}_n|)\\
&+\tau_n{}^NU^N(|\mathbf{r}-\mathbf{r}_n|)]\}e^{is\mathbf{k}\cdot\mathbf{r}}\\
&+e^{is\mathbf{k}\cdot\mathbf{r}}[\tau_n{}^PU^P(|\mathbf{r}-\mathbf{r}_n|)\\
&+\tau_n{}^NU^N(|\mathbf{r}-\mathbf{r}_n|)]\mathbf{r}\times\boldsymbol{\pi}_nd\mathbf{r}sds,\end{aligned} \quad (41)$$

$$\mathfrak{M}_{(1)}{}^{s}=\frac{e\hbar}{2Mc}\sum_{n}\int_{0}^{1}\int \mathbf{r}\times\text{curl}\{\mu^{P}\tau_{n}{}^{P}\boldsymbol{\sigma}_{n}S^{P}(|\mathbf{r}-\mathbf{r}_{n}|)+\mu^{N}\tau_{n}{}^{N}\boldsymbol{\sigma}_{n}S^{N}(|\mathbf{r}-\mathbf{r}_{n}|)\}e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}sds$$

$$=\frac{e\hbar}{2Mc}\sum_{n}\int \boldsymbol{\sigma}_{n}\{\mu^{P}\tau_{n}{}^{P}S^{P}(|\mathbf{r}-\mathbf{r}_{n}|)+\mu^{N}\tau_{n}{}^{N}S^{N}(|\mathbf{r}-\mathbf{r}_{n}|)\}e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}. \quad (42)$$

If, further, the linear dimensions of the regions over which the functions U^{P}, U^{N}, S^{P}, and S^{N} are appreciable are small compared to the wavelength of the photon involved in the transition, one can carry out approximately the integrations over $\mathbf{r}$ and obtain the simpler formulas,

$$\mathfrak{P}_{(1)}\simeq e\sum_{n}\int_{0}^{1}\mathbf{r}_{n}\tau_{n}{}^{P}e^{is\mathbf{k}\cdot\mathbf{r}_{n}}ds, \quad (43)$$

$$\mathfrak{M}_{(1)}{}^{c}\simeq\frac{e}{2Mc}\sum_{n}\int_{0}^{1}\tau_{n}{}^{P}\{\mathbf{r}_{n}\times\boldsymbol{\pi}_{n}e^{is\mathbf{k}\cdot\mathbf{r}_{n}}+e^{is\mathbf{k}\cdot\mathbf{r}_{n}}\mathbf{r}_{n}\times\boldsymbol{\pi}_{n}\}sds, \quad (44)$$

$$\mathfrak{M}_{(1)}{}^{s}\simeq\frac{e\hbar}{2Mc}\sum_{n}(\tau_{n}{}^{P}\mu^{P}+\tau_{n}{}^{N}\mu^{N})\boldsymbol{\sigma}_{n}e^{i\mathbf{k}\cdot\mathbf{r}_{n}}. \quad (45)$$

From the gamma-ray energies in which we are particularly interested (0–100 Mev) these latter formulas should represent quite good approximations. They would be exact, of course, if the charge density and spin-current density distributions associated with the individual nucleons extend only over infinitesimal regions.

Without making explicit assumptions about the interaction parts of the charge density and current density, we can do little more than indicate their contributions to the matrix elements by substituting $\rho^{i}(\mathbf{r})$ and $\mathbf{j}^{i}(\mathbf{r})$ in (19) and (20), respectively. However, we may note that since the general solution of Eq. (31) for $\mathbf{J}^{i}(\mathbf{r})$ will contain a term of the form $\text{curl}\mathbf{m}(\mathbf{r})$, at least the contribution of this term to the matrix element can be somewhat simplified,

$$\int_{0}^{1}\int \mathbf{r}\times\text{curl}\mathbf{m}(\mathbf{r})e^{is\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}sds=\int \mathbf{m}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}. \quad (46)$$

Before leaving this section we should mention an alternative and exceedingly simple method of dealing with the matrix element (10) but again a method which is deceptive in that it tends to conceal rather than reveal the difficulties with the interaction current density. This treatment is based directly on Eq. (5) without making any attempt to split $\mathbf{j}(\mathbf{r})$ into one-particle and interaction parts. We note that an explicit solution for $\mathbf{j}(\mathbf{r})$ can be obtained in the form,

$$\mathbf{j}(\mathbf{r})=\text{grad}\chi(\mathbf{r})+\text{curl}\mathbf{m}(\mathbf{r}), \quad (47)$$

$$\chi(\mathbf{r})=\frac{i}{4\pi\hbar c}\int\frac{[H,\rho(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}', \quad (48)$$

with $\mathbf{m}(\mathbf{r})$ an arbitrary vector function. We may now note that the longitudinal part of $\mathbf{j}(\mathbf{r})$ when substituted in (10) gives a vanishing contribution,

$$\int \boldsymbol{\varepsilon}\cdot\text{grad}\chi(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}=-\int\chi(\mathbf{r})\,\text{div}[\boldsymbol{\varepsilon}e^{i\mathbf{k}\cdot\mathbf{r}}]d\mathbf{r}=0,$$

while the transverse part simply gives us a term of the form (46). The deception involved in this treatment again arises, as in our earlier treatment, through the fact that for differential charge conservation alone to be satisfied it is possible for $\mathbf{m}(\mathbf{r})$ to be completely arbitrary except for certain symmetry and invariance conditions; but, in order that the complete current density operator have a physically reasonable form, it is necessary that $\mathbf{m}(\mathbf{r})$ actually satisfy some special asymptotic conditions. Again this is a consequence of the fact that the irrotational part of the current density operator as given by (48) falls off only slowly with distance from the nuclear system, while the total current density should fall off rapidly; hence $\mathbf{m}(\mathbf{r})$ must have an appropriate asymptotic form for this to be the case. The difficulty is more serious here in that even the ordinary one-particle part of the convection current density has been split up into irrotational and solenoidal parts.

THE LONG-WAVELENGTH APPROXIMATION

In many practical problems involving the photoeffect, conditions are such that one may make a further approximation to the photoeffect matrix elements in which higher multipoles than the electric dipole and magnetic dipole together with retardation effects may be neglected. This will be the case when the wavelength of the photon involved in the transition is sufficiently long that $e^{i\mathbf{k}\cdot\mathbf{r}}$ does not vary appreciably over the region in which the charge and current density matrix elements are appreciable. In such cases we may replace the exponentials $e^{i\mathbf{k}\cdot\mathbf{r}}$ and $e^{is\mathbf{k}\cdot\mathbf{r}}$ occurring in the matrix elements by unity. The one-particle contributions to the matrix elements then simply become

$$\begin{aligned}\mathfrak{P}_{(1)}&=e\textstyle\sum_{n}\tau_{n}{}^{P}\mathbf{r}_{n},\\ \mathfrak{M}_{(1)}{}^{c}&=(e/Mc)\textstyle\sum_{n}\tau_{n}{}^{P}[\mathbf{r}_{n}\times\boldsymbol{\pi}_{n}],\\ \mathfrak{M}_{(1)}{}^{s}&=(e\hbar/2Mc)\textstyle\sum_{n}\boldsymbol{\sigma}_{n}(\tau_{n}{}^{P}\mu^{P}+\tau_{n}{}^{N}\mu^{N}).\end{aligned} \quad (49)$$

Not only does this approximation simplify the treatment of the one-particle parts of the matrix elements, but it considerably reduces the arbitrariness in the interaction or many-particle parts. To illustrate this point let us consider the two-particle contributions to the electric (dipole) matrix element. If we envisage that $\rho_{(2)}(\mathbf{r})$ is substituted for $\rho(\mathbf{r})$ in Eq. (19), the exponential replaced by unity, and the integration over s performed, then what is required are matrix elements of the integral

$$\int \mathbf{r}\rho_{(2)}(\mathbf{r})d\mathbf{r}.$$

Now by the application of invariance and symmetry arguments, one can make some definite assertions about the form which the result of this integral must take. First it will be the sum of terms each a function of dynamical variables referring to two and only two nucleons. For simplicity we shall consider only velocity-independent terms, so that each term then is a function only of the variables $\mathbf{r}_n$, $\mathbf{r}_{n'}$, $\boldsymbol{\sigma}_n$, $\boldsymbol{\sigma}_{n'}$, $\boldsymbol{\tau}_n$, $\boldsymbol{\tau}_{n'}$. Furthermore, translational invariance requires that it be a function of $\mathbf{r}_n$ and $\mathbf{r}_{n'}$ only through the relative separation $\mathbf{r}_{nn'}=\mathbf{r}_n-\mathbf{r}_{n'}$. Rotational invariance requires that each term transform as a vector, and invariance with respect to space inversions requires that this vector be a polar vector. Invariance with respect to time reversal requires that this vector be invariant under the operation of time reversal. The superselection rule for charge requires that it commute with the total charge operator $e\sum_n \tau_n^P$. Furthermore, it must be symmetric under exchange of the particle coordinates. By established methods[9] one then finds that the most general (velocity-independent) form that this integral can have will be a linear combination of the linearly independent terms listed below:

$$\mathbf{P}_{\mathrm{I}}=e\sum_{n<n'} G_{\mathrm{I}}(r_{nn'})(\tau_n^z-\tau_{n'}^z)\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{II}}=e\sum_{n<n'} G_{\mathrm{II}}(r_{nn'})(\tau_n^z-\tau_{n'}^z)(\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_{n'})\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{III}}=e\sum_{n<n'} G_{\mathrm{III}}(r_{nn'})(\tau_n^z-\tau_{n'}^z)\times[(\boldsymbol{\sigma}_n\cdot\mathbf{r}_{nn'})\boldsymbol{\sigma}_{n'}+(\boldsymbol{\sigma}_{n'}\cdot\mathbf{r}_{nn'})\boldsymbol{\sigma}_n],$$

$$\mathbf{P}_{\mathrm{IV}}=e\sum_{n<n'} G_{\mathrm{IV}}(r_{nn'})(\tau_n^z-\tau_{n'}^z)\times[3(\boldsymbol{\sigma}_n\cdot\mathbf{r}_{nn'})(\boldsymbol{\sigma}_{n'}\cdot\mathbf{r}_{nn'})/r_{nn'}^2-(\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_{n'})]\mathbf{r}_{nn'}, \tag{P}$$

$$\mathbf{P}_{\mathrm{V}}=e\sum_{n<n'} G_{\mathrm{V}}(r_{nn'})(\tau_n^x\tau_{n'}^y-\tau_n^y\tau_{n'}^x)[\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_{n'}]\times\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{VI}}=e\sum_{n<n'} G_{\mathrm{VI}}(r_{nn'})(\tau_n^z+\tau_{n'}^z)[\boldsymbol{\sigma}_n\times\boldsymbol{\sigma}_{n'}]\times\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{VII}}=e\sum_{n<n'} G_{\mathrm{VII}}(r_{nn'})(1+\tau_n^z\tau_{n'}^z)[\boldsymbol{\sigma}_n\times\boldsymbol{\sigma}_{n'}]\times\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{VIII}}=e\sum_{n<n'} G_{\mathrm{VIII}}(r_{nn'})(1-\tau_n^z\tau_{n'}\tau^z)[\boldsymbol{\sigma}_n\times\boldsymbol{\sigma}_{n'}]\mathbf{r}_{nn'},$$

$$\mathbf{P}_{\mathrm{IX}}=e\sum_{n<n'} G_{\mathrm{IX}}(r_{nn'})(\tau_n^x\tau_{n'}^x+\tau_n^y\tau_{n'}^y)[\boldsymbol{\sigma}_n\times\boldsymbol{\sigma}_{n'}]\times\mathbf{r}_{nn'}.$$

The functions G_N are arbitrary functions of the indicated separation distance of the two nucleons involved. One would expect that these functions rapidly approach zero as the separation distance between the nucleons increases much beyond the observed range of nuclear forces.

The fact that such interaction contributions to the electric dipole moment operator for a nuclear system can be constructed phenomenologically suggests that such terms may very well arise from a more detailed meson-theoretical study of this problem. It is likely that these terms are very much smaller than the one-particle terms and their presence would consequently be difficult to detect experimentally in most ordinary transitions. However, the selection rules for these interaction terms differ from those of the one-particle terms in some circumstances and this may lead to a possibility for their experimental detection. Thus, consider a nuclear model for a self-conjugate nucleus in which the total orbital angular momentum L, the total spin angular momentum S, and the charge parity are constants of the motion. Then the usual selection rules for a long-wavelength electric dipole transition in such a nucleus (based on the one-particle terms in the electric dipole moment operator) are: (1) $\Delta J=0, \pm 1$ ($0\rightarrow 0$ forbidden) where J is the total angular momentum, (2) space parity change: yes, (3) $\Delta L=0, \pm 1$ ($0\rightarrow 0$ forbidden), (4) $\Delta S=0$, and (5) charge parity change: yes. On the other hand, the interaction terms in the electric dipole moment operator, while preserving, of course, the first two selection rules, can relax the remaining rules to (3′) $\Delta L=0, \pm 1, \pm 2, \pm 3$, ($0\rightarrow 0$ forbidden), (4′) $\Delta S=0, \pm 1, \pm 2$, and (5′) charge parity change: yes or no. Of course, in practical cases L, S, and the charge parity operator are not exact, and perhaps not even approximate, constants of the motion so that these selection rules lose some of their value.

Of more practical significance than interaction terms in the electric dipole moment operator are the interaction terms in the magnetic dipole moment operator, particularly since the very existence of exchange forces between nucleons necessitates the existence of such terms. In fact the subject of exchange and interaction magnetic moments of nuclei is of such importance that we have relegated its discussion to a separate paper.

CHARGE SYMMETRY AND CHARGE INDEPENDENCE

The results obtained in the preceding section concerning the phenomenological form of the electric dipole moment operator for a nuclear system is illustrative of the difficulties, mentioned in the introduction, concerning the manifold of possible interactions arising in a phenomenological treatment. Already nine linearly independent forms were obtained for this operator, even with the restriction to velocity-independent terms. Including velocity-dependent terms would greatly increase the number of possibilities, and even a casual examination of the problem of determining the general form for the two-body charge density operator would indicate that the number of linearly independent terms in this operator would be tremendous. It is hence imperative for the practical utility of these results to recognize any further symmetry principles or invariance properties which the charge and current density operators might possess in order to reduce the manifold of possibilities.

While we know of no further rigorous invariance properties which these operators possess, there seem to exist some approximate invariance principles which can play a useful role in identifying which terms in a phenomenological expression for the charge and current density operators may have dominant magnitude and which a subordinate magnitude. These principles appear to have their origin in certain symmetry properties of the coupling of mesons to nucleons.

The first of these, for which there is a considerable amount of experimental support, is the *charge symmetry* principle.[10] It may most simply be stated in the form that meson theories (neglecting electromagnetic interactions) are invariant under the simultaneous replacement of protons by neutrons, neutrons by protons, positive mesons by negative mesons, and negative mesons by positive mesons. In more precise terms, it states that the meson-nucleon Hamiltonian (again neglecting electromagnetic interactions) commutes with the charge-parity operator[11] (that is, the operator corresponding to a 180° rotation about an axis in the x-y plane in charge or isotopic spin space). The application of this result to electromagnetic interactions arises from the fact that in these theories, the charge and current density operators are each composed of a contribution from the nucleons plus a contribution from the mesons, and the latter contribution anticommutes with the charge parity operator. Since this last property is preserved under the canonical transformations which eliminate the virtual mesonic interactions to any order, it is to be expected that the contribution to the charge and current density operators arising from virtual mesons will anticommute with the charge parity operator. The nucleonic contribution has no correspondingly simple properties. As a consequence, these facts, in themselves, do not give any specific limitations on the form of our phenomenological charge and current density operators.

However, there is some scanty experimental evidence to indicate that nucleonic terms in the charge and current density operators are relatively small in states of the system in which virtual mesons are present. These experimental indications are the approximate equality in magnitude (but oppositeness in sign) of the anomalous magnetic moments of the neutron and proton and the exchange moment contribution to the magnetic moments of H^3 and He^3, the smallness of any exchange moment contributions to the magnetic moments of H^2 and Li^6, and certain systematics in the deviations of the magnetic moments of odd-even nuclei from the Schmidt values.[12] If we accept this evidence we may draw the following conclusions: (1) That part of the *interaction* charge and current density operators which commutes with the charge parity operator is about an order of magnitude smaller than the part which anticommutes with the charge parity operator. (2) The same statement holds for the *one-particle* contributions to the charge and current density operators, once one has subtracted the normal contributions to these operators from free nucleons. This statement implies that the following two relations hold to an approximation of the order of 10 percent:

$$\begin{aligned} U^P(\mathbf{x}-\mathbf{x}_1)&=\delta(\mathbf{x}-\mathbf{x}_1)-U^N(\mathbf{x}-\mathbf{x}_1),\\ S^P(\mathbf{x}-\mathbf{x}_1)&=\delta(\mathbf{x}-\mathbf{x}_1)-S^N(\mathbf{x}-\mathbf{x}_1). \end{aligned} \tag{50}$$

The application of the first conclusion to the two-particle velocity-independent contributions to the electric dipole moment operator derived in the preceding section would allow us to conclude that the terms VI–IX are an order of magnitude smaller than the terms I–V.

The second invariance principle, the so-called *charge independence* principle, includes charge symmetry as a special case, but provides that the meson theory (again apart from electromagnetic interactions) is invariant under all rotations in charge or isotopic spin space. In this case the conclusion one can draw is that the nucleonic contributions to the charge and current density operators transform as the sum of an invariant and the z component of a vector under rotations in charge space, while the meson contributions transform as the z component of a vector under such rotations. This fact alone places no restrictions on the one-particle terms in our phenomenological formulation, though it does impose restrictions on the interaction parts. In particular, for our general two-particle velocity-independent contributions to the electric dipole moment operator, it requires the relation

$$G_{\mathrm{IX}}(r_{nn'})=G_{\mathrm{VII}}(r_{nn'})-G_{\mathrm{VIII}}(r_{nn'}).$$

However, again, if one assumes that the nucleonic contributions to the charge and current density operators in states in which virtual mesons are present play a subordinate role, one is led again to the relations discussed under the charge symmetry principle.

It is to be hoped that the accumulation of further experimental evidence may soon allow us to evaluate the exact degree of validity of the charge symmetry and charge independence principles and the exact magnitude of nucleon recoil terms.

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