are chosen for both the 3.22- and 3.35-Mev levels since shell model considerations indicate that (−) parity levels would not occur at such low energies of excitation.

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Upper Limits on the Neutrino Mass from the Tritium Beta Spectrum*

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The shape of the tritium beta spectrum near the end point has been investigated in a spherical electrostatic integral spectrograph with particular reference to the possible effects of a nonzero neutrino mass. It is shown that the source thickness of 100 micrograms/cm^2 may be satisfactorily taken into account in the last kilovolt of the spectrum, upon which the results are based. An upper limit to the neutrino mass of 500, 250, and 150 electron volts is found for the Dirac, Majorana, and Fermi forms, respectively, of the beta interaction.

INTRODUCTION

THE effect of a nonzero neutrino mass upon the shape of the beta spectrum is very marked within a distance of several neutrino rest masses from the end point of the spectrum,[1] and is hence best investigated with a spectrum of end point as small as possible. The present investigation uses tritium for this purpose. Since the original report on this work,[2] the results of a similar investigation by Langer and Moffat[3] have appeared; their results are in essential agreement with our mass limits, although we prefer to theirs our method of taking into account the effects of spectrograph resolution (see below). References to earlier work on tritium appear in reference 3.

The spectrograph used has been reported upon elsewhere;[4] electrons from a source at a center of hemispherical symmetry pass through a retarding field and those having more than a certain energy go to a collector thus giving an integral spectrum. The momentum resolution of 1/3 percent is an essential ingredient of the neutrino mass limits which can be set.

EFFECT OF SOURCE THICKNESS

In a spectrum of end point 18 kilovolts, source effects constitute a formidable problem which, however, diminishes if, as in the present instance, one is interested primarily in the last few kilovolts of the spectrum. It is shown in what follows that a thick source converts a differential spectrum to a simple integral spectrum near the end point, and that this process and a subsequent integration by the spectrograph do not blur the characteristic features which arise from finite neutrino mass. The source used in these measurements consisted of tritium soaked up in 100 micrograms/cm^2 of zirconium which in turn had been deposited on a tungsten button.[5]

We base our conclusions on the suitability of thick sources for our purpose upon two lines of evidence: the behavior of monoenergetic beams of electrons in passing through thin foils, as summarized, for example, by Klemperer;[6] and the shape of conversion lines from sources of varying thicknesses, as observed and discussed by Owen and Primakoff.[7]

As to the first line of evidence: the work of Klemperer at 7–13 kilovolts[6] and of White and Millington above 100 kilovolts[6,8] are quantitatively consistent in indicating the existence of a "universal straggling curve" for the emergent electron spectrum when monoenergetic electrons are incident upon a foil. This universal straggling curve has several noteworthy features, as follows. The energy is peaked at a value less than the incident energy by an amount ΔV proportional to the thickness of the foil; at 18 kilovolts for Al absorber and not too badly for other substances, ΔV (kilovolts) $\approx 6\times 10^3 t$ (g/cm^2); the width of the peak at half maximum value is nearly equal to ΔV; the peak has a long low-energy tail, the average energy and the larger part

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of the half-width lying below the peak; the area under the peak is approximately constant, apparently diminishing no faster than does the most probable energy.

The above statements are based primarily on observations of electrons traveling in beams normal to the foils; but a spread of incident and emergent angles should change only the numerical value of ΔV in the above. Thus the spectrum $F(x, \epsilon)$ obtained from monochromatic electrons of energy E_1 starting at a depth x in the source and suffering an energy loss $\epsilon = E_1 - E$ in emerging will have the same general features as those just noted. We may thus write

$$F(x, \epsilon) = (x_0/x)G(x_0\epsilon/x\epsilon_0) = (x_0/x)G(y). \qquad (1)$$

Here $y \equiv x_0\epsilon/x\epsilon_0$ and ϵ_0 is the most probable energy loss for electrons coming from some arbitrary depth x_0. (The ratio ϵ_0/x_0 is significant; x_0 or ϵ_0, where they appear alone, are arbitrary normalization factors.) Since

$$\int_0^\infty F(x, \epsilon)d\epsilon = \epsilon_0 \int_0^\infty G(y)dy,$$

the right-hand integral being independent of x, it is clear that the above expression assumes no attenuation in the number of electrons coming from depth x; for $\epsilon \ll E$ this will be a fair assumption.

The spectrum, which we call $K(\epsilon)$, produced by initially monoenergetic electrons of energy E_1 coming from all depths in a source of infinite thickness, will be given by

$$K(\epsilon) = \int_0^\infty F(x, \epsilon)dx = x_0 \int_0^\infty G(y)dy/y.$$

Thus to this approximation $K(\epsilon)$ is independent of ϵ—that is,

$$K(\epsilon) = 0 \text{ for } \epsilon < 0; \quad K(\epsilon) = \text{constant for } \epsilon \geqslant 0.$$

Of course $K(\epsilon)$ is also a function of E_1, but not to a degree relevant here.

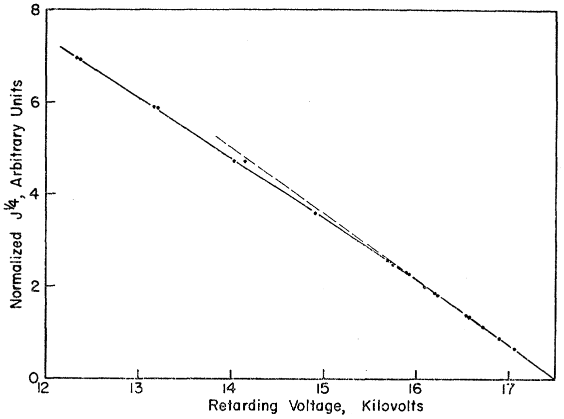

FIG. 1. Kurie-type fourth-root plot of observed collector current (as normalized for Coulomb factor and variation of electron momentum) as function of retarding voltage.

We will actually have $K(\epsilon)$ falling as ϵ increases, due to effects of electron attenuation and of noninfinite source thickness; we will expect to have $K(\epsilon) \ll K(0)$ when ϵ is of the order of the average energy loss in the source thickness (1 kilovolt for 100 micrograms/cm^2).

The feature of $K(\epsilon)$ which is most independent of approximation is the discontinuity at $\epsilon = 0$, and this is also the most relevant feature for our purposes. This behavior is borne out, for 100-microgram/cm^2 sources, by the direct observations of $K(\epsilon)$ by Owen and Primakoff[7] in measuring the shape of the conversion electron lines from sources of thickness from 40 to 840 micrograms/cm^2 for conversion lines from 50 to 500 kilovolts. For sources up to 250, but not 500, micrograms/cm^2, $K(\epsilon)$ appears in their results to be discontinuous at $\epsilon = 0$ except for the effects of spectrograph resolution; and the "width" of the lines at the base, extrapolated to 100 micrograms/cm^2 and 20 kilovolts, is about one kilovolt. As with the electron beams[6,8] discussed above, these conversion electrons were traveling normal to the source layer; as above, a spread in emergent angles should not affect the basic behavior at $\epsilon = 0$.

Electrons backscattered from the tungsten source backing produce an effect somewhat like that of an additional source thickness and will make minor modifications in $K(\epsilon)$ only for $\epsilon > 0$.

If the continuous differential beta spectrum under investigation is given, for zero neutrino mass, by $N(E_1) \sim (E_0 - E_1)^2$ (a good approximation near the end point E_0), the differential spectrum from an infinitely thick source will to the approximation that $K(\epsilon)$ is a constant for $\epsilon \geqslant 0$ be given by

$$M(E) = \int_E^{E_0} N(E_1)K(E_1 - E)dE_1 \sim (E_0 - E)^3.$$

The spectrum observed in an integral spectrograph, that is, the collector current at retarding voltage E, will then, near the end point, be given by

$$J(E) = \int_E^{E_0} M(E)dE \sim (E_0 - E)^4. \qquad (2)$$

The spectrum $J(E)$ is referred to hereafter as the "double integral spectrum."

If $K(\epsilon)$ is replaced by a function which jumps discontinuously from zero at $\epsilon = 0$ and then falls linearly to zero at some value $\epsilon = \epsilon_1$, the error in Eq. (2) at $E_0 - E = \epsilon_1$ is only several percent. Thus, all told, for the source used in this experiment one would expect to observe a non-linearity in $J^{1/4}(E)$ at distances below the end point of the order of a kilovolt. Empirically a plot of observed $J^{1/4}(E)$ (further normalized in the manner of Kurie plots), as shown in Fig. 1 for a range 8 kilovolts below the end point, shows a fall off from a straight line beyond a point 1.5 kilovolts below the end point. On the basis of this behavior we feel justified in interpreting the more detailed data near the end point in terms of a comparison with theoretical curves for $J^{1/4}$, which are calculated for various neutrino masses, and which are normalized to be tangent to the experimental data 1 kilovolt below the extrapolated end point. (Normalization at 1.5 kilovolts would probably have been safe and would have given smaller mass limits.)

THEORETICAL SHAPE OF DOUBLE-INTEGRAL SPECTRUM

As summarized by Langer and Moffatt,[3] various hypotheses concerning the nature of the neutrino give electron spectra which for nonzero neutrino mass differ just in the region near the end point. To an accuracy sufficient for discussion of anatomy but not for final calculation, we may neglect Coulomb effect and variation of electron momentum; the various differential spectra $N(z)$ may be represented by

$$N_0 \sim z(z^2-1)^{1/2},$$
$$N_\pm \sim (z\pm 1)(z^2-1)^{1/2},$$

where

$$z \equiv (E_0-E)/\mu c^2,$$
$$\mu \equiv \text{neutrino mass}.$$

The spectra N_0, N_+, N_- are respectively associated with the names of Majorana, Fermi, and Dirac and correspond to emission of a negatron simultaneously with emission of a Majorana neutrino (one having only positive energy states), emission of a positive-energy neutrino, and absorption of an antineutrino.

The principal features of the differential spectrum $N(z)$ and the double integral spectrum

$$J(z)=\int_0^z\int_0^z N(z)dz$$

are noted below:

$$z\gg 1, \quad N_0^{1/2}\sim J_0^{1/4}\sim z; \quad N_\pm^{1/2}\sim J_\pm^{1/4}\sim (z\pm 1/2);$$
$$z-1\ll 1, \quad N_0^{1/2}\sim N_+^{1/2}\sim (z-1)^{1/4}; \quad N_-^{1/2}\sim (z-1)^{3/4}$$
$$J_0^{1/4}\sim J_+^{1/4}\sim (z-1)^{5/8}; \quad J_-^{1/4}\sim (z-1)^{7/8}.$$

Thus Kurie-type plots of both the differential and double integral spectra (that is, plots of $N^{1/2}$ and $J^{1/4}$) have the same general features—at the end point a vertical tangent, far from the end point a straight line extrapolating to the true end point plus 1, 3/2, 1/2 neutrino masses for Majorana, Fermi, and Dirac neutrinos, respectively. It should be noted that the Dirac case (N_-) has a differential spectrum (but not $N_-^{1/2}$ or $J_-^{1/4}$ Kurie plots) with zero rather than infinite slope at the end point, hence less easily detectable mass.

But the principal conclusion from the above is that a loss of the characteristic nonzero mass behavior is not produced by going from differential to double integral spectra.

EFFECTS OF FINITE RESOLUTION

The resolution characteristic of the spectrometer is indicated in Fig. 2, which indicates the apparent differential energy distribution obtained by differentiating the integral spectrum which was observed for monoenergetic electrons. (The disappearance of the low-energy tail which appeared in the resolution characteristic of reference 4 resulted from the use, in the present resolution measurement, of a diode source which was well centered and of the same size as the tritium source.)

The problem of deducing the true beta spectrum from the observed spectrum, given the known resolution characteristics of the spectrometer, is an old one. For the purpose, procedures have been developed[7] for correcting the experimental points, the correction depending upon energy derivatives of the observed energy distribution and upon observed resolution characteristics. These procedures do not seem to be suited to the present problem, for several reasons. Thus we are interested in the region just below the end point, where the scatter of individual points is great enough to make evaluation of derivatives of the observed spectrum practically meaningless unless one wishes to prejudge the result by assuming a straight-line Kurie plot (zero neutrino mass). But what is more relevant, a correction based upon one or two observed spectrum derivatives implicitly assumes the validity of a power series expansion of the true spectrum, whereas for the finite-mass spectra N_0 and N_+ the first and higher, and for N_- the second and higher, derivatives are infinite at the true end point.

It therefore seems much preferable in the present instance to start out with a postulated spectrum $N(E)$, characterized by one of the three possible analytical forms and by a definite neutrino mass, calculate how the non-ideal resolution will distort this spectrum, and then compare the properly normalized result [say $M(E)$] with the raw experimental points.

Since it is apparent from the work of Owen and

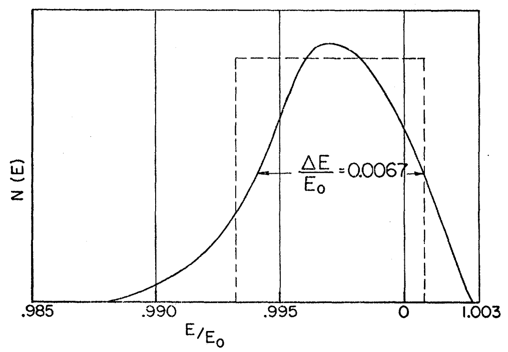

FIG. 2. Resolution characteristic of spectrograph; differential spectrum for monoenergetic electrons of energy E_0, as deduced by differentiation of observed integral spectrum. Dotted line indicates the resolution characteristic assumed for certain calculations discussed in the text.

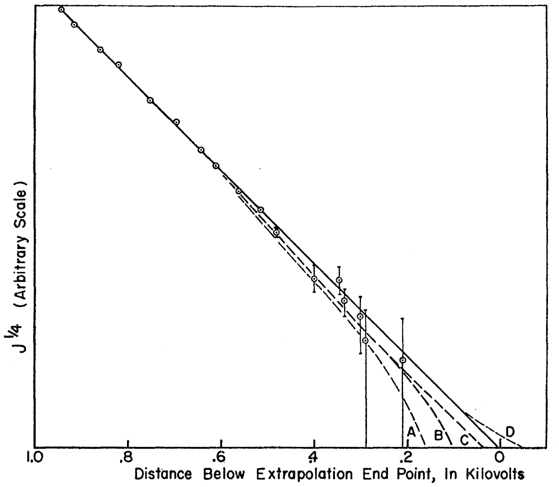

FIG. 3. Fourth root of tritium current plotted against kilovolts below end point. Dotted curves represent curves predicted on the basis of measured resolution and for various neutrino masses and interactions. Majorana, Fermi, and Dirac interactions indicated by (0) (+) (−), respectively. Neutrino mass μ in electron volts.

Curve A: $\mu=250$ (+), 350 (0).
Curve B: $\mu=150$ (+), 200 (0).
Curve C: $\mu=500$ (−).
Curve D: $\mu=0$ (0, +, −).

Primakoff[7] that the exact shape of the resolution characteristic is important, determining not only the magnitude but also the sign of the correction to be made in the usually applicable standard procedure, it appears that one needs to carry out a tedious numerical integration over the resolution characteristic, and this was done. However, in the case under consideration we are concerned primarily with the shape of $M(E)$, and not so much with displacement of $M(E)$ from $N(E)$ to higher or lower energies; for the shape of $M(E)$, the width of the resolution characteristic is the primary factor, and one may simplify things considerably by using a square characteristic of the proper width. Thus as a check, we have calculated $M(E)$ for $\mu=250$ electron volts by simple analytic procedures using a square characteristic 135 volts wide (indicated in Fig. 2 for comparison); the two methods give results indistinguishable on the scale of Fig. 3. (This latter figure shows the most relevant of the calculated double-integral spectra.) In accordance with the discussion of source thickness effects above, all these curves (as well as the experimental data, as discussed below) have been normalized to tangency with a common straight line 1 kilovolt below the end point.

For both the differential and double-integral spectra the resolution width has the effect near the end point of reversing the sign of that negative curvature of the Kurie plot which is characteristic of the nonzero neutrino mass. (This effect becomes more striking when normalization is further from the end point than in Fig. 3.) As an example, for N_- with resolution width equal to the neutrino mass and hence equal to twice the difference between true and extrapolated end points, the reversal of curvature extends for two neutrino masses from the extrapolated end point and thus destroys the most characteristic consequence of a finite mass.

In general, it would seem that one should approach with care, and with due regard to the considerations just outlined, any process of deducing upper limits to the neutrino mass which are smaller than the resolution width. (It would be more exact here to say "apparent neutrino mass," which is μ, $\mu/2$, $3\mu/2$ for the spectra N_0, N_-, N_+, respectively.) This philosophy is at variance with that of two recent neutrino-mass investigations,[3,9] in each of which the quoted upper limit on difference between true and extrapolated end point is about one-quarter of the resolution width.

EXPERIMENTAL METHOD AND RESULTS

The collector current in the spectrograph was amplified by an FP54 circuit, the output of which led to a dc photocell-galvanometer amplifier, the over-all gain of the combination being of the order of 10^{12}. The output of the second amplifier actuated an Esterline-Angus recorder. Customary procedure was to alternate ten-minute periods with the collector connected and disconnected to the amplifier inputs; in optimum operation the zero (collector disconnected) trace showed only small statistical fluctuations superimposed on a secular drift which was very constant over many hours.

Measurements made with no source in the spectrograph indicated two types of background current: a retarding voltage independent background caused by paralleling the collector resistance to ground with the FP54 grid resistor; and a strong field emission current (between the spectrograph grids) which increased slowly with voltage in a manner which could be expressed best as a quadratic increase with voltage for six kilovolts above the tritium end point, with an extrapolated zero one kilovolt below the tritium end point. At the tritium end-point voltage the strong field current was hardly perceptible. With a tritium source in the spectrograph the same strong-field current above the end point was observed, and the observed current for the last kilovolt of the spectrum was therefore corrected by subtracting the extrapolated strong field current for this region. At 300 volts below the end point the strong field current subtracted was one-third of the total, and if it had not been subtracted the points in Fig. 3 at 0.3 and 0.4 kev would have been raised by 11 and 3 percent, respectively. These amounts being small compared to the flags in the corresponding points in Fig. 3, neglect of the strong-field current would not have had a significant result.

The fourth root of the resulting current, normalized for the very small effect of the Coulomb factor and variation of electron momentum, is shown as a function

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[9] G. C. Hanna and B. Pontecorvo, Phys. Rev. **75**, 983 (1949).

of voltage in Fig. 3, for the run in which the current amplifier output showed least background fluctuation. The flags on individual points represent rms fluctuations calculated from cases where the same point was repeated, and agree with rms fluctuations in recorder traces. Below 0.25 kev in the notation of Fig. 3 the fluctuations are larger than the tritium current.

The principal uncertainty in the zero neutrino mass absolute end-point value as deduced from such data as Figs. 1 and 3 arises from the uncertainty in the value of the 500-megohm resistor used for voltage measurement. This was very constant over a day's run, when corrected for the observed thermal drift due to voltage changes; but several months after the data on tritium had been taken it was discovered that the resistor possessed an instantaneous voltage coefficient, that is, a non-thermal dependence of resistance on voltage, which was an order of magnitude above the manufacturer's quoted value. (Hence the difference between the end point below and that of reference 2.) This does not give rise to any significant residual uncertainty in conclusions based on the spectrum shape near the end point; but the uncertainty in the final end-point value, 17.6±0.4 kilovolts, arises primarily from the possibility of a secular drift in the voltage coefficient of the resistor.

DISCUSSION

Also shown in Fig. 3 are the $J^{1/4}$ plots predicted theoretically for various masses and interactions on the basis of the resolution characteristic of Fig. 2. As already noted, the theoretical curves are normalized to be tangent to the experimental curve one kilovolt below the end point; by choosing a point this near the end point, as seemed to be indicated by Fig. 1, the effect of any neutrino mass has unfortunately been reduced, as will be seen by the fact that the difference between true and extrapolated end points is less than that which would have resulted from extrapolation using a tangent at a greater distance below the end point.

In Fig. 3, curves B, C, D seem consistent with the experimental data but curve A seems not consistent; it therefore seems likely that the neutrino mass measured in electron volts is not greater than 500(−), 200(0), 150(+) for the Dirac, Majorana, and Fermi interactions as denoted by (−), (0), (+), respectively. The values obtained by Hanna and Pontecorvo[9] [1000(−)], and Langer and Moffat[3] [250(−)] fall in the same region and would not be increased in order of magnitude by use of the method of resolution correction which, as discussed above, we think preferable to the one which these authors used.

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