| $T_\zeta=-1$ | $[\frac{3}{2},\frac{3}{2}]_i[(5/2),-(5/2)]_j$ | $[\frac{3}{2},\frac{1}{2}]_i[(5/2),-\frac{3}{2}]_j$ | $[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-\frac{1}{2}]_j$ | $[\frac{3}{2},-\frac{3}{2}]_i[(5/2),\frac{1}{2}]_j$ |
|---|---|---|---|---|
| $[4,-1]_{ij}$ | $\sqrt{(1/56)}$ | $\sqrt{(15/56)}$ | $\sqrt{(15/28)}$ | $\sqrt{(5/28)}$ |
| $[3,-1]_{ij}$ | $\sqrt{\frac{1}{8}}$ | $\sqrt{(49/120)}$ | $-\sqrt{(1/60)}$ | $-\sqrt{(9/20)}$ |
| $[2,-1]_{ij}$ | $\sqrt{(5/14)}$ | $\sqrt{(1/42)}$ | $-\sqrt{(25/84)}$ | $\sqrt{(9/28)}$ |
| $[1,-1]_{ij}$ | $\sqrt{\frac{1}{2}}$ | $-\sqrt{(3/10)}$ | $\sqrt{(3/20)}$ | $-\sqrt{(1/20)}$ |

| $T_\zeta=-2$ | $[\frac{3}{2},\frac{1}{2}]_i[(5/2),-(5/2)]_j$ | $[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-\frac{3}{2}]_j$ | $[\frac{3}{2},-\frac{3}{2}]_i[(5/2),-\frac{1}{2}]_j$ |
|---|---|---|---|
| $[4,-2]_{ij}$ | $\sqrt{(3/28)}$ | $\sqrt{(15/28)}$ | $\sqrt{(5/14)}$ |
| $[3,-2]_{ij}$ | $\sqrt{(5/12)}$ | $\sqrt{(1/12)}$ | $-\sqrt{\frac{1}{2}}$ |
| $[2,-2]_{ij}$ | $\sqrt{(10/21)}$ | $-\sqrt{(8/21)}$ | $\sqrt{(1/7)}$ |

| $T_\zeta=-3$ | $[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-(5/2)]_j$ | $[\frac{3}{2},-\frac{3}{2}]_i[(5/2),-\frac{3}{2}]_j$ |
|---|---|---|
| $[4,-3]_{ij}$ | $\sqrt{\frac{3}{8}}$ | $\sqrt{\frac{5}{8}}$ |
| $[3,-3]_{ij}$ | $\sqrt{\frac{5}{8}}$ | $-\sqrt{\frac{3}{8}}$ |

Addition of more than two vectors may be performed in steps. For instance, for a system of one nucleon and two pions,

$$(1/2)_1\times(1)_3\times(1)_4=(1/2)_1\times\{(2)_{34}\&(1)_{34}\&(0)_{34}\}$$
$$=\{(5/2)_{134}\&(3/2)_{134}\}\&\{(3/2)'_{134}\&(1/2)'_{134}\}\&(1/2)_{134},$$

where for instance $(3/2)'_{134}$ differs from $(3/2)_{134}$ by the fact that the isobaric spins of the two pions are parallel in $(3/2)$, but form a total pion isobaric spin 1 in $(3/2)'$.

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# Scattering of Mesons by Complex Nuclei

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The result of the previous calculation on the scattering of mesons by a single nucleon has been extended to the case of multiple scattering by nucleons in nuclear matter. Methods familiar in the dispersion theory in physical optics are used, and the index of refraction of meson wave has been formally calculated. The absorption and diffraction scattering cross sections are computed and relation with the results of the recent experimental observation is discussed.

## I. INTRODUCTION

THE interactions of high-energy particles with complex nuclei have been discussed theoretically by several authors in the recent years. Fernbach, Serber, and Taylor[1] considered nuclear matter as a continuous optical medium characterized by an index of refraction and an absorption coefficient with respect to the incoming neutron waves. Recently a detailed quantum-mechanical theory of the interaction of high-energy $\pi$ mesons with nuclear matter based on the optical model has been given by Watson.[2] He introduced phenomenological interactions for the scattering and absorption of $\pi$ mesons by individual nucleon into a many-body Schrödinger equation and showed that the solution of the equation has the structure of a multiply scatter wave.

In this paper an attempt is made to discuss the interaction of $\pi$ mesons with complex nuclei in a manner similar to that of Fernbach, Serber, and Taylor for high-energy neutrons. While the index of refraction there was obtained by simply ascribing a uniform potential well for the nuclear matter, in our case we pay a special attention on the calculation of the index of refraction[3] by the methods familiar in physical optics. We consider a nuclear matter of a large extent and assume that the mesons with which we are concerned are of sufficiently low energy that their wavelengths are large compared with the average internucleon distance.

For the scattering of a meson wave by the individual nucleon we make use of the results of the previous paper on the subject.[4] The fact that the nucleons within a

[1] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[2] K. M. Watson, Phys. Rev. **89**, 575 (1953).

[3] See M. Lax, Revs. Modern Phys. **23**, 287 (1951), for the extensive literature on the subject.
[4] W. W. Wada, Phys. Rev. **88**, 1032 (1952). This will be referred to as I. Also D. Feldman, Phys. Rev. **88**, 890 (1952).

nucleus are bound should not make the scattering amplitude differ appreciably from that by a free nucleon, since the energy of the incoming meson is so much larger than the binding energy.

Under these circumstances it is a comparatively simple matter to deduce a wave equation for the multiply scattered meson field, from which one obtains a complex propagation vector and a complex index of refraction. In order to discuss the whole of the interaction of mesons with the nuclear matter, however, one must add the true absorption coefficient[5] to the imaginary part of the index of refraction, since the former is known to be the most important effect in the attention of the meson beam.

## II. OPTICAL MODEL AND THE AVERAGE POLARIZATION MOMENT

According to I, Eq. (13), the field of the scattered meson by a free nucleon may be written as follows:

$$\varphi_\alpha{}^s(\mathbf{r})=i(2\sqrt{\pi})^{-1}fV(k)k\{(\boldsymbol{\sigma}_\omega\cdot\mathbf{n}_s)\tau_{0,\alpha}+(\boldsymbol{\sigma}_0\cdot\mathbf{n}_s)\tau_{\omega,\alpha}\}r^{-1}\exp[ikr]+\text{c.c.}\quad(1)$$

Here $\boldsymbol{\sigma}_0$ and $\boldsymbol{\sigma}_\omega$ are the undisturbed and the gyrating (with frequency $\omega$) components of the spin vector of the nucleon, respectively. $\boldsymbol{\tau}_0$ and $\boldsymbol{\tau}_\omega$ are the corresponding components of the isotopic spin vector. $\alpha$ runs from 1 to 3 corresponding to the three charge states of the meson. $\mathbf{n}_s$ is a unit vector in the coordinate space in the direction of the scattered field. $k=(\omega^2-\mu^2)^{\frac{1}{2}}$. For quantities $f$, $V(k)$, $\omega$, and $\mu$ see I, Sec. II.

Working with complex functions, Eq. (1) may be written

$$\varphi_\alpha{}^s(\mathbf{r})=\operatorname{div}_r(\mathbf{M}_\alpha r^{-1}\exp[ikr]),\quad(2)$$

where

$$\mathbf{M}_\alpha=\eta(2\sqrt{\pi})^{-1}fV(k)(\boldsymbol{\sigma}_\omega\tau_{0,\alpha}+\boldsymbol{\sigma}_0\tau_{\omega,\alpha}),\quad(3)$$

The parameter $\eta$ is introduced at this point to express the effect of binding of individual nucleon on the scattering amplitude.

The incident field is expressed by (see I, Sec. III)

$$\varphi_\alpha{}^I(\mathbf{r})=\chi P_{I\alpha}\exp[ik(\mathbf{n}_I\cdot\mathbf{r})-i\omega t]+\text{c.c.}\quad(4)$$

Here $\chi$ is the amplitude of the incident field and $P_{I\alpha}$ is the $\alpha$ component of the unit vector $\mathbf{P}_I$ in charge space describing the charge state of the incident meson. The vector field of the incident meson wave is obtained by taking the gradient of $\varphi_\alpha{}^I(\mathbf{r})$. The amplitude of the incoming vector field is then given by

$$\mathbf{F}_\alpha{}^{(0)}=i\chi k\mathbf{n}_I P_{I\alpha}.\quad(5)$$

Introducing Eq. (5) and $\boldsymbol{\sigma}_\omega$ and $\boldsymbol{\tau}_\omega$ [see I, Eq. (12)] into (4), one obtains

$$\begin{aligned}M_\alpha{}^a=-i2f^2\eta[V(k)]^2\omega^{-1}(1-R^2)^{-1}[&\epsilon^{abc}\tau_{0,\alpha}\tau_{0,\beta}\sigma_0{}^bF_\beta{}^{(0)c}\\&+iR\{\sigma_0{}^a\sigma_0{}^b\tau_{0,\alpha}\tau_{0,\beta}F_\beta{}^{(0)b}-\tau_{0,\alpha}\tau_{0,\beta}F_\beta{}^{(0)a}\}\\&+\epsilon_{\alpha\beta\gamma}\tau_{0,\beta}\sigma_0{}^a\sigma_0{}^bF_\gamma{}^{(0)b}+iR\{\sigma_0{}^a\sigma_0{}^b\tau_{0,\alpha}\tau_{0,\beta}F_\beta{}^{(0)b}\\&-\sigma_0{}^a\sigma_0{}^bF_\alpha{}^{(0)b}\}].\end{aligned}\quad(6)$$

[5] Brueckner, Serber, and Watson, Phys. Rev. **84**, 258 (1951).

For the definition of $R$, see I, Sec. III. Roman letters refer to the components in the coordinate space and Greek letters to those in the charge space, respectively. $\epsilon_{123}=\epsilon^{123}=1$, $\epsilon_{132}=\epsilon^{213}=-1$, etc. One may call $M_\alpha{}^a$ the "polarization moment" of the nucleon, because it is proportional to the vector field.

Assuming that nucleons are uniformly distributed throughout the nuclear matter, we may now go from the discrete nucleon distribution to that of continuum by introducing the notion of "average polarization moment" defined in the vicinity of a coordinate point $\mathbf{r}$ as follows:

$$\langle M_\alpha{}^a\rangle(\mathbf{r})=\sum_{dv(\mathbf{r})}M_\alpha{}^a(\mathbf{r})/Ndv(\mathbf{r}).\quad(7)$$

Here $dv(\mathbf{r})$ designates a "physically small volume" in the vicinity of $\mathbf{r}$ and $N$ the density of nucleons in the nuclear matter. By the "physically small volume" one implies a volume element smaller than the wavelength but large enough to contain in it some number of scattering centers.

Since $\boldsymbol{\sigma}_0$ and $\boldsymbol{\tau}_0$ are classical vectors in our theory, we may assume that they are oriented completely at random throughout the nuclear matter. Then, under the assumption of uniform distribution of nucleons the volume average implied in (7) may be taken as the average over all directions of $\boldsymbol{\sigma}_0$ and $\boldsymbol{\tau}_0$ in coordinate and charge space. Then we obtain

$$\langle M_\alpha{}^a\rangle(\mathbf{r})=\gamma F_\alpha{}^{(0)a}(\mathbf{r}),\quad(8)$$

where

$$\gamma=-(8/9)f^2\eta[V(k)]^2\omega^{-1}(1-R^2)^{-1}R.\quad(9)$$

## III. DERIVATIVE OF THE COHERENT WAVE EQUATION

From Eq. (2) the field vector for the scattered wave at point $\mathbf{r}$ arising from a nucleon at $\mathbf{r}'$ may be written

$$\mathbf{F}_\alpha(\mathbf{r})=\Omega(\mathbf{r},\mathbf{r}')\mathbf{M}_\alpha(\mathbf{r}'),\quad(10)$$

where $\Omega(\mathbf{r},\mathbf{r}')$ is the following linear operator:

$$\begin{aligned}&\Omega(\mathbf{r},\mathbf{r}')=(\boldsymbol{\nabla}\operatorname{div})_rW(\mathbf{r},\mathbf{r}'),\\&W(\mathbf{r},\mathbf{r}')=\rho^{-1}\exp[ik\rho],\quad\rho=|\mathbf{r}-\mathbf{r}'|.\end{aligned}\quad(11)$$

Note that

$$(\Delta_r+k^2)W(\mathbf{r},\mathbf{r}')=0.\quad(12)$$

When a meson wave goes through the nuclear matter, the field at a nucleon $q$ may be regarded as the sum of the external field and the field scattered by all other nucleons $q'$,

$$\mathbf{F}_\alpha(q)=\mathbf{F}^{(0)}(q)+\sum_{q'}{}'\,\Omega(q,q')\mathbf{M}(q').$$

This type of equation constitutes the basis for multiple scattering theory.[2,3] In our case of continuum approach we generalize this equation by replacing $\sum_{q'}{}'$ by a volume integration which extends over the entire nuclear matter but excludes a small volume in the vicinity of the nucleon at $\mathbf{r}$. Then, we obtain

$$F_\alpha{}^a(\mathbf{r})=F_\alpha{}^{(0)}(\mathbf{r})+N\int_{s(r)}^{V} d\mathbf{r}'\Omega(\mathbf{r},\mathbf{r}')\langle M_\alpha{}^a\rangle(\mathbf{r}'). \quad (13)$$

Here, the lower limit $S(\mathbf{r})$ designates a small sphere whose center is located at $\mathbf{r}$. The upper limit $V$ designates the entire nuclear volume. The radius of the sphere $s(r)$ will later be reduced to zero in the limit. The so-called self-consistent field method[3] implies that we replace $F_\alpha{}^{(0)a}(\mathbf{r})$ in Eq. (8) by $F_\alpha{}^a(\mathbf{r})$ given by Eq. (13),

$$\langle M_\alpha{}^a\rangle(\mathbf{r}) =\gamma\left[F_\alpha{}^{(0)a}(\mathbf{r})+N\int_{s(r)}^{V} d\mathbf{r}'\Omega(\mathbf{r},\mathbf{r}')\langle M_\alpha{}^a\rangle(\mathbf{r}')\right]. \quad (14)$$

This is an integral equation for $\langle M_\alpha{}^a\rangle(\mathbf{r})$. For the purpose of obtaining a wave equation for $\langle M_\alpha{}^a\rangle(\mathbf{r})$, we make use of the following equations:[6]

$$\frac{\partial}{\partial x_i}\int_{s(r)}^{V} d\mathbf{r}'W(\rho)\langle M_\alpha{}^a\rangle(\mathbf{r}') =\int_{s(r)}^{V} d\mathbf{r}'\frac{\partial}{\partial x_i}W(\rho)\langle M_\alpha{}^a\rangle(\mathbf{r}'), \quad (15)$$

$$\frac{\partial^2}{\partial x_i\partial x_j}\int_{s(r)}^{V} d\mathbf{r}'W(\rho)\langle M_\alpha{}^a\rangle(\mathbf{r}') =\int_{s(r)}^{V} d\mathbf{r}'\frac{\partial^2 W(\rho)}{\partial x_i\partial x_j}\langle M_\alpha{}^a\rangle(\mathbf{r}')-\frac{4\pi}{3}\langle M_\alpha{}^a\rangle(\mathbf{r})\delta_{ij}. \quad (16)$$

By means of Eq. (16), Eq. (14) now becomes

$$\left(1-\frac{4\pi}{3}N\gamma\right)\langle\mathbf{M}_\alpha\rangle(\mathbf{r}) =\gamma\mathbf{F}_\alpha{}^{(0)}(\mathbf{r})+N\gamma(\nabla\,\mathrm{div})_r\int_{s(r)}^{V} d\mathbf{r}'W(\rho)\langle\mathbf{M}_\alpha\rangle(\mathbf{r}'). \quad (17)$$

Since $\mathrm{curl}_r\mathbf{F}^{(0)}(\mathbf{r})=0$, we obtain $\mathrm{curl}_r\langle\mathbf{M}_\alpha\rangle(r)=0$ from Eq. (17). Applying operator $(\Delta+k^2)$ on Eq. (17) the first term on the right-hand side drops out. Using Eq. (16) and the vanishing curl condition, one then obtains the following wave equation for the average polarization moment:

$$\left[\Delta_r+\frac{1-(4\pi/3)N\gamma}{1+(8\pi/3)N\gamma}k^2\right]\langle\mathbf{M}_\alpha\rangle(\mathbf{r})=0. \quad (18)$$

This is a modified Gordon-Klein equation with index of refraction $n$ given by

$$n^2=\frac{1-(4\pi/3)N\gamma}{1+(8\pi/3)N\gamma}. \quad (19)$$

This equation may be called the analog of the Lorentz-Lorenz relation in the electromagnetic case. It can be shown easily that the total scalar field inside the nuclear matter constructed in analogy to Eq. (13),

$$\varphi_\alpha(\mathbf{r})=\varphi_\alpha{}^{(0)}(\mathbf{r})+N\int_{s(r)}^{V} d\mathbf{r}'\,\mathrm{div}_r W(\rho)\langle\mathbf{M}_\alpha\rangle(\mathbf{r}'),$$

also satisfies the wave equation similar to (18), namely

$$(\Delta_r+n^2k^2)\varphi_\alpha(\mathbf{r})=0. \quad (20)$$

## IV. COMPLEX PROPAGATION VECTOR

Writing $\gamma=\gamma_1+i\gamma_2$, where $\gamma_1$ and $\gamma_2$ are both real, and introducing $R=\xi+i\zeta$ [see I, Eq. (12) and (14)] into $\gamma_1$ and $\gamma_2$, one obtains

$$\gamma_1=-\delta\frac{\xi(1-\xi^2-\zeta^2)}{(1-\xi^2-\zeta^2)^2+4\zeta^2},\quad \gamma_2=-\delta\frac{\zeta(1+\xi^2+\zeta^2)}{(1-\xi^2-\zeta^2)^2+4\zeta^2},$$

where $\delta=(8/9)(f\mu)^2\eta[V(k)]^2\mu^{-3}(\mu\omega^{-1})$. Note that $\delta$ has the dimension of a volume.

Decomposing the modified propagation vector $k'$ into real and imaginary parts,

$$k'=nk=k(S_1+iS_2), \quad (21)$$

one obtains

$$S_1=(l^2+m^2)^{\frac14}\cos(\tfrac12\tan^{-1}(m/l)),$$
$$S_2=(l^2+m^2)^{\frac14}\sin(\tfrac12\tan^{-1}(m/l)),$$

where $l$ and $m$ are real and imaginary parts of $n^2$. In Table I we have computed $\lambda$ (de Broglie wavelength), $l$, $m$, $S_1$ and $S_2$ at various energies of the incoming $\pi$ mesons. In computing these numbers we used $(f\mu)^2=0.32$, $[V(k)]^2=1.6$ (see I, Sec. V), $\mu=0.71\times10^{13}$ cm$^{-1}$, and the density of nucleons $N=0.87\times10^{38}$ cm$^{-3}$. Although the value of the parameter $\eta$ (Sec. II) which designates the influence of binding of each nucleon in the nuclear matter on the scattering amplitude is not well known,[7] it may be considered to be of the order of unity since the binding energy per nucleon is very much smaller than the energy of the mesons involved. Tentatively we took $\eta=1$ for our calculation. If one takes $\mu^{-1}=1.45\times10^{-13}$ cm as the average internucleon separation, the number of nucleons per cube of the wavelength turns out to be about 125 at 85-Mev kinetic energy of $\pi$ mesons. This number goes down to about 30 in the vicinity of 200 Mev. We are under the

TABLE I. Wavelength, index of refraction, and propagation vectors.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\omega\mu^{-1}$ | 1 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 |
| $\lambda\mu$ | $\infty$ | 6.4 | 5.0 | 4.2 | 3.7 | 3.2 | 2.9 |
| $l$ | 1.28 | 1.45 | 1.30 | 0.67 | 0.26 | 0.47 | 0.48 |
| $m$ | 0 | 0.52 | 1.12 | 1.12 | 1.08 | 0.84 | 0.69 |
| $S_1$ | 1.13 | 1.22 | 1.23 | 0.99 | 0.83 | 0.85 | 1.03 |
| $S_2$ | 0 | 0.21 | 0.45 | 0.56 | 0.66 | 0.49 | 0.42 |

[6] H. Hoek, doctoral dissertation, University of Leiden (1939); L. Rosenfeld, *Theory of Electrons* (Interscience Publishers, Inc., New York, 1951).

[7] N. C. Francis and K. M. Watson, Phys. Rev. **89**, 328 (1953).

assumption that these numbers are sufficiently large to justify our continuous medium approach to the multiple scattering problem. It may be worth noting that the imaginary part of the propagation vector goes through a single maximum in the vicinity of the resonance maximum in the scattering cross section of mesons by a free nucleon (see I, Sec. V). The real part, on the other hand, goes through a maximum and a minimum in the same vicinity, as is expected.

### V. DISCUSSION

From Eq. (21) it is clear that while the coherent waves constitute the forward beam of the mesons traveling through the nuclear matter with effective index of refraction $S_1$, it is continuously being attenuated by the imaginary part of the index of refraction $S_2$. The power attenuation coefficient is $2k_2=2kS_2$. A physical interpretation of the energy dissipation implied by this attenuation effect may be given in the following way. Classically speaking, as the field strikes at the individual nucleon, the spin and the isotopic spin of the nucleon absorb the energy from the incident radiation and go into gyrational motions. It is possible then to consider that the energy thus absorbed by the nucleon is not given back to the radiation field, but rather passed on to the nuclear matter due to collision and binding of the nucleons. In this manner one may regard the imaginary part of the index of refraction as the cause for the inelastic scattering of mesons by the nuclear matter. This is to be distinguished from the true absorption of mesons by two or more nucleons,[5] which is known to be the real cause for the removal of mesons from the beam. The absorption of mesons will in general excite the nuclear systems (formation of stars, etc.), which effect may be regarded as a further attenuation of the forward coherent wave.

Writing $k'=k+k''$ [see Eq. (21)], the change in the propagation vector inside a nucleus becomes $k''=k(s_1-1)+iks_2=k_1+ik_2$. If one designates the true absorption coefficient by $a$, then $k''=k_1+i(k_2+a/2)=k_1+iK$, where $2K=2k_2+a$ is the total power attenuation coefficient. Then, according to Fernbach, Serber, and Taylor,[1] the angular distribution of the wave scattered by a sphere endowed with material constants $K$ and $k_1$ is given by

$$f(\theta)=k\int_0^R [1-e^{(-K+2ik_1)s}]J_0(k\rho\sin\theta)\rho d\rho,$$

where $2s$ is the distance through the sphere that the portion of the wave at a distance $\rho$ from a line through the center of the sphere traverses. Hence, $S^2=R^2-\rho^2$. $R$ is the radius of the sphere $(=1.37\times10^{-13}\times A^{\frac{1}{3}}$ cm). We disregard here also the surface refraction effect, since it is of the order $k_1^2(kR)^{-1}$.

At 100 Mev, $k=1.25\mu$, which gives $kR=1.22\times A^{\frac{1}{3}}$. Thus, at this energy for comparatively heavy nuclei it may be a reasonable approximation to suppose that $kR\gg1$, then the above formula becomes

$$f(\theta)=\tfrac{1}{2}k\sum_{l=0}^{l+\frac{1}{2}<kR}(2l+1)(1-e^{(-K+2ik_1)s_l})P_l(\cos\theta),\quad(22)$$

where $S_l=[k^2R^2-(l+\tfrac{1}{2})^2]^{\frac{1}{2}}/k$.

TABLE II. Absorption and diffraction cross sections.

| | | |
|---|---|---|
| $\eta=1$ ($k_1=-0.011\times10^{13}$ cm$^{-1}$, $k_2=0.595\times10^{13}$ cm$^{-1}$, $a=0.250\times10^{13}$ cm$^{-1}$) | | |
| | $A=64$ | $A=208$ |
| $\sigma_a/\pi R^2$ | 0.968 | 0.973 |
| $\sigma_d/\pi R^2$ | 0.797 | 0.820 |
| $\eta=0.5$ ($k_1=0.035\times10^{13}$ cm$^{-1}$, $k_2=0.308\times10^{13}$ cm$^{-1}$, $a=0.250\times10^{13}$ cm$^{-1}$) | | |
| | $A=64$ | $A=208$ |
| $\sigma_a/\pi R^2$ | 0.912 | 0.926 |
| $\sigma_d/\pi R^2$ | 0.590 | 0.622 |

The total absorption and diffraction scattering cross sections, $\sigma_a$ and $\sigma_d$, that arise from Eq. (22) are given in Eqs. (5) and (6) of the paper of Fernbach *et al.*[1] $\sigma_a$ involves not only the true absorption but also the inelastic scattering discussed before. We have calculated $\sigma_a$ and $\sigma_d$ for two nuclei ($A=64$ and 208). The value for the true absorption coefficient $a$ is not very well known, but we take a plausible value $0.25\times10^{13}$ cm$^{-1}$, corresponding to the mean free path of $4\times10^{-13}$ cm. $\sigma_a$ and $\sigma_d$ are calculated for two different values of $\eta$ [see Eq. (13)] at 100-Mev kinetic energy for mesons.

The absorption cross section is smaller than the geometrical area, as is expected. The diffraction cross section may be either larger or smaller depending on the amount of phase shift involved in traversing the nucleus. Both $\sigma_a$ and $\sigma_d$ increase with larger nuclear size. These numbers should of course be taken with reservation because of the general nature of our classical approach to the problem.

The recent experimental observations[8] on the total absorption and inelastic scattering cross section ($\sigma_a$) of $\pi$ mesons in the vicinity of 100-Mev kinetic energy by various elements show that the absorption plus inelastic cross section is closely equal to the geometrical area of the nucleus. If one takes the true absorption coefficient alone for $K$, then for $a=0.250\times10^{13}$ cm$^{-1}$, $\sigma_a/\pi R^2$ turns out to be 0.80 and 0.82 for the two nuclei. If one doubles the value, i.e., $a=0.50\times10^{13}$ cm$^{-1}$, then the true absorption alone gives 0.90 and 0.92 for $\sigma_a/\pi R^2$. However, the mean free path becomes rather short ($2\times10^{-13}$ cm). Thus, it appears that both absorption and inelastic coefficients are equal to or somewhat larger than these numbers given in Table II. In the experiments mentioned the diffraction scattering is concentrated at such small angles with the beam that its contribution to the attenuation is considered very small.

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[8] Chedester, Isaacs, Sachs, and Steinberger, Phys. Rev. **82**, 958 (1951); G. Bernardini and F. Levy, Phys. Rev. **84**, 610 (1951); Martin, Anderson, and Yodh, Phys. Rev. **85**, 486 (1952).