PHYSICAL REVIEW VOLUME 92, NUMBER 1 OCTOBER 1, 1953

Pions from Production of Baryons by Protons

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(Received March 16, 1953)

When matter is bombarded by very high-energy nucleons, over 600 Mev in the laboratory system, part of the pions produced may be decay products of excited nucleons of ordinary spin and isobaric spin $\frac{3}{2}$ ("baryons") produced by the incident nucleons. Assuming charge independence, we discuss the ratios of the cross sections for various possible processes creating one or more pions.

1. INTRODUCTION

SEVERAL authors[1–3] have recently discussed the relative probabilities for production of positive, neutral, and negative pions by nucleon-nucleon interaction, using the principle of spherical symmetry in isobaric space,[4] but assuming that the pions are produced directly by interaction of the nucleons. On the other hand there is now some evidence[5–7] of a nucleon isomer, as predicted by strong-coupling theory.[8] The properties of this isomer are much different from those of the nucleon, its ordinary spin and its isobaric spin both being $\frac{3}{2}$, so that it supposedly has four isobaric states b^{++}, b^{+}, b^{0}, and b^{-}, instead of the two isobaric states p and n of the nucleon. Therefore, we shall in the following adopt for this nucleon isomer a name of its own, *baryon* (=heavy particle), indicating its slightly heavier mass (estimated at around $2385m_e$). We shall treat the baryon as an excited state of the same *nucleoid* (=system of a bare nucleon core with surrounding pion cloud), of which the nucleon is the ground state. This excited state is very unstable, causing a widening of the energy level and uncertainty in the baryon rest mass of perhaps $10m_e$, corresponding to a life time of about 10^{-22} sec.[9] This life time may be just long enough to make a baryon move out of the pion cloud with which it may have been created, before it disintegrates into a nucleon and a pion. In that case, the production of a baryon and its decay will be independent processes, and the cross section for production of a certain kind of pions from baryon decay will be given by the product of the probability of baryon production, and the fraction of the baryons produced that will disintegrate into the pion desired. This fraction is determined by the branching ratios between the various disintegration possibilities. In the present paper, we shall derive relations between these cross sections arising from the postulate of charge independence.[4]

2. NOTATION

By indices 1, 2 we shall denote the interacting two nucleoids. Pions are indicated by indices 3, 4, $\cdots$. The complete state vector should be antisymmetric in 1 and 2, and symmetric in 3, 4, $\cdots$. The nucleon and baryon states of the nucleoid can be distinguished by their isobaric spin functions. By $[T, T_\zeta]$ we shall indicate states of isobaric spin T with "charge component" T_ζ. The charge is $q=(T_\zeta+\frac{1}{2})e$ for particles with half-odd isobaric spin, and it is $q=T_\zeta e$ for particles with integral isobaric spin. To avoid here a minus sign, the proton instead of the neutron has been ascribed positive T_ζ.[10]

In vector addition of isobaric spins of systems $i, j, \cdots$ the normalized isobaric spin functions of the combined system corresponding to the various possible values of T and T_ζ are linear combinations of the products of the normalized isobaric spin functions of the component systems, with coefficients as listed in the Appendix. Conversely these same coefficients (taken this time from one same column in the tables of the Appendix) may be used in the expansion of each such product in terms of the isobaric-spin functions of the combined system.

Let ψ_i be the space- and ordinary-spin function of the incident nucleon, and ψ_t of the target nucleon. The initial wave function Ψ is then, if pn means protons incident on target neutrons, and if $\psi^s=(\psi_{i1}\psi_{t2}+\psi_{t1}\psi_{i2})/\sqrt{2}$, $\psi^a=(\psi_{i1}\psi_{t2}-\psi_{t1}\psi_{i2})/\sqrt{2}$;

$$\left.\begin{aligned}
\Psi_{pp}&=\{\psi_{i1}[\tfrac{1}{2},\tfrac{1}{2}]_1\psi_{t2}[\tfrac{1}{2},\tfrac{1}{2}]_2-\psi_{t1}[\tfrac{1}{2},\tfrac{1}{2}]_1\psi_{i2}[\tfrac{1}{2},\tfrac{1}{2}]_2\}/\sqrt{2}=\psi^a[1,1]_{12};\\
\Psi_{pn}&=\{\psi_{i1}[\tfrac{1}{2},\tfrac{1}{2}]_1\psi_{t2}[\tfrac{1}{2},-\tfrac{1}{2}]_2-\psi_{t1}[\tfrac{1}{2},-\tfrac{1}{2}]_1\psi_{i2}[\tfrac{1}{2},\tfrac{1}{2}]_2\}/\sqrt{2}\\
&=(\sqrt{\tfrac{1}{2}})\psi^a[1,0]_{12}+(\sqrt{\tfrac{1}{2}})\psi^s[0,0]_{12};\\
\Psi_{np}&=(\sqrt{\tfrac{1}{2}})\psi^a[1,0]_{12}-(\sqrt{\tfrac{1}{2}})\psi^s[0,0]_{12};\\
\Psi_{nn}&=\psi^a[1,-1]_{12}.
\end{aligned}\right\}\quad(2.1)$$

[1] A. M. L. Messiah, Phys. Rev. **86**, 430 (1952).
[2] J. M. Luttinger, Phys. Rev. **86**, 571 (1952).
[3] Van Hove, Marshak, and Pais, Phys. Rev. **88**, 1211 (1952).
[4] This principle has been called the "charge independence hypothesis" by N. Kemmer, Proc. Cambridge Phil. Soc. **34**, 354 (1938). Its combination with the assumption that pions have isobaric spin 1 (as contrasted to 0) was called the "symmetrical theory" (as contrasted to the "neutral theory") by H. A. Bethe, Phys. Rev. **55**, 1261 (1939). K. M. Watson and K. A. Brueckner, Phys. Rev. **83**, 1 (1951), have generalized the charge independence hypothesis, postulating invariance of all nuclear and mesonic interactions under rotations in isobaric space. [See also K. M. Watson, Phys. Rev. **85**, 852 (1952).] Compare footnote 12.
[5] K. A. Brueckner, Phys. Rev. **86**, 106 (1952).
[6] G. S. Janes and W. L. Kraushaar, Phys. Rev. **90**, 341 (1953); and B. T. Feld, Phys. Rev. **90**, 342 (1953).
[7] G. Wentzel, Phys. Rev. **86**, 437 (1952).
[8] W. Pauli and S. M. Dancoff, Phys. Rev. **62**, 85 (1942).
[9] G. Wentzel (private communication).
[10] B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936); and N. Kemmer, Nature **140**, 192 (1937); Proc. Cambridge Phil. Soc. **34**, 354 (1938).

Here, $[1, T_\zeta]_{12}$ (with $T_\zeta=1, 0, -1$) are isobaric triplet functions and $[0, 0]_{12}$ is a singlet isobaric-spin function for the combined (two-nucleon) system.

The space- and ordinary-spin wave functions of the nucleoids in the final state we shall denote by $\dot{\varphi}$ and $\ddot{\varphi}$, and we put $\varphi^{s,a}=(\dot{\varphi}_1\ddot{\varphi}_2\pm\ddot{\varphi}_1\dot{\varphi}_2)/\sqrt{2}$. If one of the nucleoids is excited, we shall always denote by $\dot{\varphi}$ the state of the nucleon, and by $\ddot{\varphi}$ the state of the baryon. Such combinations as for instance $(\dot{\varphi}_1[\frac{1}{2}, -\frac{1}{2}]_1\ddot{\varphi}_2[\frac{3}{2}, \frac{1}{2}]_2 - \ddot{\varphi}_1[\frac{3}{2}, \frac{1}{2}]_1\dot{\varphi}_2[\frac{1}{2}, -\frac{1}{2}]_2)/\sqrt{2}$ we shall then denote by $[\dot{\varphi}_1\ddot{\varphi}_2[\frac{1}{2}, -\frac{1}{2}]_1[\frac{3}{2}, \frac{1}{2}]_2]^a$. If *both* nucleoids are excited to states of isobaric spin $\frac{3}{2}$, or if *neither one* is excited, we shall denote by $\dot{\varphi}$ the state of the particle with the largest charge. *In this case*, as is easily verified by studying the coefficients listed in the tables for $(1/2)\times(1/2)$ and for $(3/2)\times(3/2)$ in the Appendix, the isobaric-spin functions of odd total isobaric spin ($T=1$ or 3) are automatically symmetric in both particles, and the isobaric-spin functions of even total isobaric spin ($T=0$ or 2) are automatically antisymmetric, so that, in this case, for instance

$$[\dot{\varphi}_1\ddot{\varphi}_2[3, 0]_{12}]^a=\varphi^a[3, 0]_{12},$$

and

$$[\dot{\varphi}_1\ddot{\varphi}_2[1, 1]_{12}]^a=\varphi^a[1, 1]_{12},$$

but

$$[\dot{\varphi}_1\ddot{\varphi}_2[2, 1]_{12}]^a=\varphi^s[2, 1]_{12}.$$

(These relations are of course not true for the $[1, 1]_{12}$ and $[2, 1]_{12}$ found in the tables for $(1/2)\times(3/2)$.[11])

The spatial wave functions of pions we shall denote by $\gamma, \dot{\gamma}, \ddot{\gamma}, \cdots$. The final wave function of the complete system is denoted by Φ.

3. BARYON PRODUCTION

We consider the following processes: (The first particle mentioned is incident. The symbols between brackets will be used as labels on wave functions, cross sections etc., to distinguish these reactions.)

$p+p\rightarrow n\ +b^{++}$ ($pp2$), $\quad p+n\rightarrow n\ +b^{+}$ ($pn1$),
$p+p\rightarrow p\ +b^{+}$ ($pp1$), $\quad p+n\rightarrow p\ +b^{0}$ ($pn0$),
$p+p\rightarrow b^{++}+b^{0}$ ($pp20$), $\quad p+n\rightarrow b^{++}+b^{-}$ ($pn2-$),
$p+p\rightarrow b^{+}\ +b^{+}$ ($pp11$), $\quad p+n\rightarrow b^{+}\ +b^{0}$ ($pn10$).

Processes with initial states $n+n$ or $n+p$ are obtained from the above by substituting n, p, b^-, b^0, b^+, b^{++} for p, n, b^{++}, b^+, b^0, b^-, respectively. Because of the *mirror property*[12] of nuclear interactions, the cross sections for such processes are the same as for the ones listed above.

The normalized final wave functions for these processes are: (Compare the tables in the Appendix for vector addition of the isobaric spins of the final nucleoids; tables $(1/2)\times(3/2)$ and $(3/2)\times(3/2)$, respectively.)

$$\left.\begin{aligned}
\Phi_{pp2}&=[\dot{\varphi}_1\ddot{\varphi}_2[\tfrac{1}{2}, -\tfrac{1}{2}]_1[\tfrac{3}{2}, \tfrac{3}{2}]_2]^a\\
&=\tfrac{1}{2}[\dot{\varphi}_1\ddot{\varphi}_2[2, 1]_{12}]^a+\tfrac{1}{2}\sqrt{3}[\dot{\varphi}_1\ddot{\varphi}_2[1, 1]_{12}]^a;\\
\Phi_{pp1}&=\tfrac{1}{2}\sqrt{3}[\dot{\varphi}_1\ddot{\varphi}_2[2, 1]_{12}]^a-\tfrac{1}{2}[\dot{\varphi}_1\ddot{\varphi}_2[1, 1]_{12}]^a;\\
\Phi_{pp20}&=(\sqrt{\tfrac{1}{5}})\varphi^a[3, 1]_{12}+(\sqrt{\tfrac{1}{2}})\varphi^s[2, 1]_{12}\\
&\qquad+\sqrt{(3/10)}\cdot\varphi^a[1, 1]_{12};\\
\Phi_{pp11}&=(\sqrt{\tfrac{3}{5}})\varphi^a[3, 1]_{12}-(\sqrt{\tfrac{2}{5}})\varphi^a[1, 1]_{12};\\
\Phi_{pn1}&=(\sqrt{\tfrac{1}{2}})[\dot{\varphi}_1\ddot{\varphi}_2[2, 0]_{12}]^a\\
&\qquad+(\sqrt{\tfrac{1}{2}})[\dot{\varphi}_1\ddot{\varphi}_2[1, 0]_{12}]^a;\\
\Phi_{pn0}&=(\sqrt{\tfrac{1}{2}})[\dot{\varphi}_1\ddot{\varphi}_2[2, 0]_{12}]^a\\
&\qquad-(\sqrt{\tfrac{1}{2}})[\dot{\varphi}_1\ddot{\varphi}_2[1, 0]_{12}]^a;\\
\Phi_{pn2-}&=\sqrt{(1/20)}\cdot\varphi^a[3, 0]_{12}+\tfrac{1}{2}\varphi^s[2, 0]_{12}\\
&\qquad+\sqrt{(9/20)}\cdot\varphi^a[1, 0]_{12}+\tfrac{1}{2}\varphi^s[0, 0]_{12};\\
\Phi_{pn10}&=\sqrt{(9/20)}\cdot\varphi^a[3, 0]_{12}+\tfrac{1}{2}\varphi^s[2, 0]_{12}\\
&\qquad-\sqrt{(1/20)}\cdot\varphi^a[1, 0]_{12}-\tfrac{1}{2}\varphi^s[0, 0]_{12}.
\end{aligned}\right\}\quad(3.1)$$

The differential cross sections for these reactions are proportional to the absolute squares of the matrix elements M of the scattering matrix (say R_N) between the initial states (2.1) and the final states (3.1). (The label N may denote the number of excited nucleoids.) Because of the assumed invariance under rotations in isobaric space, we have[11]

$$\left.\begin{aligned}
&([\dot{\varphi}_1\ddot{\varphi}_2[1, 1]_{12}]^a, R_{\rm I}\psi^a[1, 1]_{12})\\
&\qquad=([\dot{\varphi}_1\ddot{\varphi}_2[1, 0]_{12}]^a, R_{\rm I}\psi^a[1, 0]_{12})\equiv A;\\
&(\varphi^a[1, 1]_{12}, R_{\rm II}\psi^a[1, 1]_{12})\\
&\qquad=(\varphi^a[1, 0]_{12}, R_{\rm II}\psi^a[1, 0]_{12})\equiv B;\\
&\quad(\varphi^s[0, 0]_{12}, R_{\rm II}\psi^s[0, 0]_{12})\equiv C;
\end{aligned}\right\}\quad(3.2)$$

while matrix elements between different T values vanish. Thence

$$\left.\begin{aligned}
M_{pp2}&=\tfrac{1}{2}\sqrt{3}A,\\
M_{pp1}&=-\tfrac{1}{2}A,\\
M_{pp20}&=\sqrt{(3/10)}\cdot B,\\
M_{pp11}&=-\sqrt{(2/5)}\cdot B,\\
M_{pn1}&=\tfrac{1}{2}A;\\
M_{pn0}&=-\tfrac{1}{2}A;\\
M_{pn2-}&=\sqrt{(9/40)}\cdot B+(\sqrt{\tfrac{1}{8}})C;\\
M_{pn10}&=-\sqrt{(1/40)}\cdot B-(\sqrt{\tfrac{1}{8}})C.
\end{aligned}\right\}\quad(3.3)$$

The differential cross section for scattering into the solid angles $d\dot{\omega}$ and $d\ddot{\omega}$ is then given by $d\sigma=\mathcal{C}|M|^2d\dot{\omega}d\ddot{\omega}$, where $\mathcal{C}$ is independent of the charge state of the particles. The matrix elements A, B, and C depend on energies and directions of the final wave functions $\dot{\varphi}$ and $\ddot{\varphi}$ and contain delta functions to take care of con-

[11] Obviously, the $[1, 1]_{12}$ used for one-baryon processes and defined by the Table $(1/2)\times(3/2)$ of the Appendix, and the $[1, 1]_{12}$ from the Table $(3/2)\times(3/2)$ for two-baryon processes, are not identical and are even orthogonal to each other. Similarly for $[1, 0]_{12}$, etc. In reading such equations as Eqs. (3.1), (3.2), etc., this should always be kept in mind. One might distinguish such different isobaric-spin functions with identical values of T and T_ζ by primes or other symbols. We have not taken the trouble of doing so.

[12] We avoid the term "charge symmetry," as various authors have used it for Kemmer's symmetrical meson theory and as Watson and Brueckner have occasionally referred to their postulate of invariance under rotations in isobaric-spin space as the "hypothesis of charge symmetry." Compare footnote 4.

servation laws. The total cross sections are given by

$$\sigma=\int\int d\dot{\omega}d\ddot{\omega}\mathcal{C}|M|^2$$

except (3.4)

$$\sigma_{pp11}=\frac{1}{2}\int\int d\dot{\omega}d\ddot{\omega}\mathcal{C}|M|^2,$$

where the factor $\frac{1}{2}$ is due to the indistinguishability of the two final single-charged baryons in the reaction $p+p\rightarrow b^++b^+$.

The differential cross sections $d\sigma_{pn2-}$ and $d\sigma_{pn10}$ contain terms proportional to Re $\{B^*C\}$. These terms do not contribute to the total cross section. This is most easily seen by calculating the differential cross section for the process, in which the final space- and ordinary-spin states of the two baryons are interchanged.[3] This is equivalent to a change of sign of φ^a, thence of B and of Re $\{B^*C\}$, so that contributions of Re $\{B^*C\}$ to the differential cross sections for interchanged directions of the final baryons cancel each other and $\int\int d\dot{\omega}d\ddot{\omega}\mathcal{C}\,\mathrm{Re}\{B^*C\}=0$.

Putting $\int\int\mathcal{C}|A|^2d\dot{\omega}d\ddot{\omega}=360\,a^2$, $\int\int\mathcal{C}|B|^2d\dot{\omega}d\ddot{\omega}=360\,b^2$, $\int\int\mathcal{C}|C|^2d\dot{\omega}d\ddot{\omega}=360\,c^2$, we thus find for the total cross sections

$$\left.\begin{array}{lll}\sigma_{pp2}=270\,a^2, & \sigma_{pp1}=90\,a^2, & \sigma_{pn1}=90\,a^2;\\ \sigma_{pp20}=108\,b^2, & \sigma_{pp11}=72\,b^2, & \sigma_{pn0}=90\,a^2;\\ \sigma_{pn2-}=81\,b^2+45\,c^2, & \sigma_{pn10}=9\,b^2+45\,c^2. & \end{array}\right\}\quad(3.5)$$

4. BARYON DECAY

We now consider the decay of the baryon produced,

$$\begin{array}{lll} b^{++}\rightarrow p+\pi^+ & (2+) & \\ b^+\nearrow n+\pi^+ & (1+) & \\ b^+\searrow p+\pi^0 & (10) & \\ b^0\nearrow n+\pi^0 & (00) & \\ b^0\searrow p+\pi^- & (0-) & \\ b^-\rightarrow n+\pi^- & (--). & \end{array}$$

In order to find the branching ratios for the b^+ and the b^0 decay, we write down the initial and final wave functions. We may as well leave out the space- and ordinary-spin functions ψ_1, φ_1, and γ_3, which all processes have in common. Then (compare the tables $(1/2)\times(1)$ in the Appendix),

$$\left.\begin{array}{l}\Psi_1=[\tfrac{3}{2},\tfrac{1}{2}]_1;\\ \Phi_{1+}=[\tfrac{1}{2},-\tfrac{1}{2}]_1[1,1]_3\\ \qquad=(\sqrt{\tfrac{1}{3}})[\tfrac{3}{2},\tfrac{1}{2}]_{13}-(\sqrt{\tfrac{2}{3}})[\tfrac{1}{2},\tfrac{1}{2}]_{13};\\ \Phi_{10}=[\tfrac{1}{2},\tfrac{1}{2}]_1[1,0]_3=(\sqrt{\tfrac{2}{3}})[\tfrac{3}{2},\tfrac{1}{2}]_{13}+(\sqrt{\tfrac{1}{3}})[\tfrac{1}{2},\tfrac{1}{2}]_{13}.\\ \Psi_0=[\tfrac{3}{2},-\tfrac{1}{2}]_1;\\ \Phi_{00}=[\tfrac{1}{2},-\tfrac{1}{2}]_1[1,0]_3\\ \qquad=(\sqrt{\tfrac{2}{3}})[\tfrac{3}{2},-\tfrac{1}{2}]_{13}-(\sqrt{\tfrac{1}{3}})[\tfrac{1}{2},-\tfrac{1}{2}]_{13};\\ \Phi_{0-}=[\tfrac{1}{2},\tfrac{1}{2}]_1[1,-1]_3\\ \qquad=(\sqrt{\tfrac{1}{3}})[\tfrac{3}{2},-\tfrac{1}{2}]_{13}+(\sqrt{\tfrac{2}{3}})[\tfrac{1}{2},-\tfrac{1}{2}]_{13}.\end{array}\right\}\quad(4.1)$$

The only matrix elements different from zero are

$$\langle[\tfrac{3}{2},\tfrac{1}{2}]_{13}|R|[\tfrac{3}{2},\tfrac{1}{2}]_1\rangle=\langle[\tfrac{3}{2},-\tfrac{1}{2}]_{13}|R|[\tfrac{3}{2},-\tfrac{1}{2}]_1\rangle,$$

so that we find the branching ratios

$$\sigma_{1+}:\sigma_{10}:\sigma_{00}:\sigma_{0-}=\tfrac{1}{3}:\tfrac{2}{3}:\tfrac{2}{3}:\tfrac{1}{3}.\quad(4.2)$$

The mass of the baryon may be just large enough to make two-pion decay ($b^{++}\rightarrow p+\pi^++\pi^0$, etc.) energetically possible. Here, we ignore this possibility as probably an improbable process. If, however, a baryon would decay into a nucleon and two pions, the tracks of these particles would lie very close together because of the small amount of kinetic energy available in the center-of-mass system.

5. CREATION AND DECAY OF HIGHER-EXCITED NUCLEOIDS

The next excited state of the nucleon is supposed to have a (5/2) isobaric spin.[8] If we denote a particle in this state by $\bar{b}$, and a nucleon by N, the reaction $N+N\rightarrow N+\bar{b}$ is forbidden by conservation of isobaric spin. Also, the decay $\bar{b}\rightarrow N+\pi$ is forbidden. These spin (5/2) nucleoids can however be produced according to $N+N\rightarrow b+\bar{b}$ or $N+N\rightarrow\bar{b}+\bar{b}$. We shall consider the former process only. The calculations run analogous to the ones given above. This time one needs the formulas for vector addition of spins 3/2 and 5/2. (See the tables in the Appendix.) We list the reactions to be considered and the matrix elements for the corresponding differential cross sections, in terms of a matrix element D defined similar to A in Eq. (3.2) but for the occurrence of a different operator R for the present reactions:

$$\left.\begin{array}{ll}p+p\rightarrow\bar{b}^{+++}+b^-, & M_{p3}=-(\sqrt{\tfrac{1}{2}})\cdot D;\\ p+p\rightarrow\bar{b}^{++}+b^0, & M_{p2}=+\sqrt{(3/10)}\cdot D;\\ p+p\rightarrow\bar{b}^{+}+b^+, & M_{p1}=-\sqrt{(3/20)}\cdot D;\\ p+p\rightarrow\bar{b}^{0}+b^{++}, & M_{p0}=+\sqrt{(1/20)}\cdot D;\\ p+n\rightarrow\bar{b}^{++}+b^-, & M_{n2}=-\sqrt{(1/10)}\cdot D;\\ p+n\rightarrow\bar{b}^{+}+b^0, & M_{n1}=+\sqrt{(3/20)}\cdot D;\\ p+n\rightarrow\bar{b}^{0}+b^+, & M_{n0}=-\sqrt{(3/20)}\cdot D;\\ p+n\rightarrow\bar{b}^{-}+b^{++}, & M_{n-}=+\sqrt{(1/10)}\cdot D.\end{array}\right\}\quad(5.1)$$

The baryon b disintegrates as discussed in chapter 4. The spin (5/2) particle $\bar{b}$ can either decay stepwise by $\bar{b}\rightarrow b+(1, 2,$ or 3 pions$)$, $b\rightarrow N+\pi$, or directly by $\bar{b}\rightarrow N+(2, 3, 4,$ or 5 pions$)$. (See Sec. 6 for discussion of energy available.) Either way for conservation of isobaric spin at least two decay pions appear.

Of all these possibilities we shall discuss here only the ones of the form $\bar{b}\rightarrow N+2\pi$. The final wave function must be symmetric in the pions. We obtain the isobaric spin functions of the pion system from the $(1)\times(1)$ table in the Appendix, and we use the $(1/2)\times(2)$ table to find the part of the isobaric spin function of the whole $(N+2\pi)$ system belonging to isobaric spin (5/2). In this way only one single unknown matrix element,

$$\begin{aligned}E&=\langle[(5/2),\tfrac{1}{2}]_{134}\varphi_1\gamma^s|R|[(5/2),\tfrac{1}{2}]_1\psi_1\rangle\\ &=\langle[(5/2),\tfrac{3}{2}]_{134}\varphi_1\gamma^s|R|[(5/2),\tfrac{3}{2}]_1\psi_1\rangle\\ &=\langle[(5/2),(5/2)]_{134}\varphi_1\gamma^s|R|[(5/2),(5/2)]_1\psi_1\rangle\end{aligned}$$

$$(\text{with }\gamma^s=\{\dot{\gamma}_3\ddot{\gamma}_4+\ddot{\gamma}_3\dot{\gamma}_4\}/\sqrt{2}),$$

TABLE I. Total cross sections σ for pion production by decay of baryons produced by incident protons. The brackets group together particles from a single decay. N is the number of charged particles after the collision. The constants a, b, c, d depend on unknown matrix elements.

Reaction	N	Cross section
$p+p\rightarrow n+(p+\pi^+)$	2	$\sigma(p;p+) = 270\,a^2$
$\rightarrow p+(n+\pi^+)$	2	$\sigma(p;n+) = 30\,a^2$
$\rightarrow p+(p+\pi^0)$	2	$\sigma(p;p0) = 60\,a^2$
$p+p\rightarrow(n+\pi^+)+(n+\pi^+)$	2	$\sigma(p;n+n+) = 8\,b^2$
$\rightarrow(p+\pi^0)+(n+\pi^+)$	2	$\sigma(p;p0n+) = 32\,b^2$
$\rightarrow(p+\pi^+)+(n+\pi^0)$	2	$\sigma(p;p+n0) = 72\,b^2$
$\rightarrow(p+\pi^+)+(p+\pi^-)$	4	$\sigma(p;p+p-) = 36\,b^2$
$\rightarrow(p+\pi^0)+(p+\pi^0)$	2	$\sigma(p;p0p0) = 32\,b^2$
$p+p\rightarrow(n+\pi^++\pi^0)+(n+\pi^+)$	2	$\sigma(p;n+0n+) = 2\,d^2$
$\rightarrow(n+\pi^++\pi^+)+(n+\pi^0)$	2	$\sigma(p;n++n0) = 4\,d^2$
$\rightarrow(n+\pi^++\pi^-)+(p+\pi^+)$	4	$\sigma(p;n+-p+) = d^2$
$\rightarrow(n+\pi^++\pi^+)+(p+\pi^-)$	4	$\sigma(p;n++p-) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^-)+(n+\pi^+)$	4	$\sigma(p;p+-n+) = d^2$
$\rightarrow(p+\pi^++\pi^+)+(n+\pi^-)$	4	$\sigma(p;p++n-) = 50\,d^2$
$\rightarrow(n+\pi^0+\pi^0)+(p+\pi^+)$	2	$\sigma(p;n00p+) = 2\,d^2$
$\rightarrow(n+\pi^++\pi^0)+(p+\pi^0)$	2	$\sigma(p;n+0p0) = 4\,d^2$
$\rightarrow(p+\pi^0+\pi^0)+(n+\pi^+)$	2	$\sigma(p;p00n+) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^0)+(n+\pi^0)$	2	$\sigma(p;p+0n0) = 16\,d^2$
$\rightarrow(p+\pi^0+\pi^-)+(p+\pi^+)$	4	$\sigma(p;p0-p+) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^-)+(p+\pi^0)$	4	$\sigma(p;p+-p0) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^0)+(p+\pi^-)$	4	$\sigma(p;p+0p-) = 8\,d^2$
$\rightarrow(p+\pi^0+\pi^0)+(p+\pi^0)$	2	$\sigma(p;p00p0) = 4\,d^2$
$p+n\rightarrow n+(n+\pi^+)$	1	$\sigma(n;n+) = 30\,a^2$
$\rightarrow n+(p+\pi^0)$	1	$\sigma(n;p0) = 60\,a^2$
$\rightarrow p+(n+\pi^0)$	1	$\sigma(n;n0) = 60\,a^2$
$\rightarrow p+(p+\pi^-)$	3	$\sigma(n;p-) = 30\,a^2$
$p+n\rightarrow(n+\pi^+)+(n+\pi^0)$	1	$\sigma(n;n+n0) = 2\,b^2+10\,c^2$
$\rightarrow(n+\pi^-)+(p+\pi^+)$	3	$\sigma(n;n-p+) = 81\,b^2+45\,c^2$
$\rightarrow(n+\pi^+)+(p+\pi^-)$	3	$\sigma(n;n+p-) = b^2+5\,c^2$
$\rightarrow(n+\pi^0)+(p+\pi^0)$	1	$\sigma(n;n0p0) = 4\,b^2+20\,c^2$
$\rightarrow(p+\pi^-)+(p+\pi^0)$	3	$\sigma(n;p-p0) = 2\,b^2+10\,c^2$
$p+n\rightarrow(n+\pi^0+\pi^0)+(n+\pi^+)$	1	$\sigma(n;n00n+) = 2\,d^2$
$\rightarrow(n+\pi^++\pi^0)+(n+\pi^0)$	1	$\sigma(n;n+0n0) = 4\,d^2$
$\rightarrow(n+\pi^++\pi^-)+(n+\pi^+)$	3	$\sigma(n;n+-n+) = d^2$
$\rightarrow(n+\pi^++\pi^+)+(n+\pi^-)$	3	$\sigma(n;n++n-) = 2\,d^2$
$\rightarrow(n+\pi^0+\pi^-)+(p+\pi^+)$	3	$\sigma(n;n0-p+) = 8\,d^2$
$\rightarrow(n+\pi^++\pi^-)+(p+\pi^0)$	3	$\sigma(n;n+-p0) = 2\,d^2$
$\rightarrow(n+\pi^++\pi^0)+(p+\pi^-)$	3	$\sigma(n;n+0p-) = 2\,d^2$
$\rightarrow(p+\pi^0+\pi^-)+(n+\pi^+)$	3	$\sigma(n;p0-n+) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^-)+(n+\pi^0)$	3	$\sigma(n;p+-n0) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^0)+(n+\pi^-)$	3	$\sigma(n;p+0n-) = 8\,d^2$
$\rightarrow(n+\pi^0+\pi^0)+(p+\pi^0)$	1	$\sigma(n;n00p0) = 4\,d^2$
$\rightarrow(p+\pi^0+\pi^0)+(n+\pi^0)$	1	$\sigma(n;p00n0) = 4\,d^2$
$\rightarrow(p+\pi^-+\pi^-)+(p+\pi^+)$	5	$\sigma(n;p--p+) = 2\,d^2$
$\rightarrow(p+\pi^++\pi^-)+(p+\pi^-)$	5	$\sigma(n;p+-p-) = d^2$
$\rightarrow(p+\pi^0+\pi^-)+(p+\pi^0)$	3	$\sigma(n;p0-p0) = 4\,d^2$
$\rightarrow(p+\pi^0+\pi^0)+(p+\pi^-)$	3	$\sigma(n;p00p-) = 2\,d^2$

is introduced. Integrating the differential cross sections over angles, we must add a factor $\frac{1}{2}$ whenever the two pions are in the same charge state. Thus we find for the disintegration probabilities Δ, in terms of one unknown constant $e^2=\frac{1}{10}\int\int\int \mathfrak{C}'|E|^2 d\omega_1 d\omega_3 d\omega_4$,

$$\left.\begin{aligned} \mathfrak{b}^{+++}&\rightarrow p+\pi^++\pi^+, & \Delta_{3++}&=5e^2;\\ \mathfrak{b}^{++}&\rightarrow n+\pi^++\pi^+, & \Delta_{2++}&=e^2;\\ \mathfrak{b}^{++}&\rightarrow p+\pi^++\pi^0, & \Delta_{2+0}&=4e^2;\\ \mathfrak{b}^{+}&\rightarrow n+\pi^++\pi^0, & \Delta_{1+0}&=2e^2;\\ \mathfrak{b}^{+}&\rightarrow p+\pi^0+\pi^0, & \Delta_{100}&=2e^2;\\ \mathfrak{b}^{+}&\rightarrow p+\pi^++\pi^-, & \Delta_{1+-}&=e^2. \end{aligned}\right\} \tag{5.2}$$

Decay probabilities for $\mathfrak{b}^{--}$, $\mathfrak{b}^{-}$, and $\mathfrak{b}^{0}$ are equal to the above by the mirror property.[12]

6. PION PRODUCTION FROM BARYON DECAY. Discussion

We calculate total cross sections for pion production by protons from decay of produced baryons and spin-(5/2) nucleoids by combining our results (3.5), (4.2), (5.1), and (5.2). We put $\int\int \mathfrak{C}'' e^2|D|^2 d\omega_1 d\omega_2 = 20\,d^2$. The results are shown in Table I. In the listings of the reactions, the brackets group together the particles from each single decay. The number of charged particles arising from the collision (N) is also shown. Because of the short life time of the baryon, all tracks seem to come from the same point.

The threshold[13] for the one-baryon processes is about 600 Mev for the incident proton in the laboratory system; the threshold for the two-baryon processes of chapter 3 is about 1285 Mev in the laboratory system. Just above the 600-Mev threshold, processes $(p;p+)$ and $(p;n+)$ at the top of Table I can be distinguished by the energy distributions of the protons, as the first nucleon is (with the baryon) about at rest in the center-of-mass system, and thus at the threshold has almost a fixed energy of nearly 140 Mev in the laboratory system. The second nucleon arising from the decay of the baryon has its energy in the laboratory system spread between approximately 43 and 290 Mev. Similarly, the processes $(n;p0)$ and $(n;n0)$ are distinct. Near the 1285-Mev threshold,[13] the processes $(p;p0n+)$ and $(p;p+n0)$ could be distinguished only by taking also the directions of the tracks of the proton and the pion into account; similarly, the processes $(n;n-p+)$ and $(n;n+p-)$. About differential cross sections for such processes, compare also the remarks at the end of Sec. 3 on the unknown quantity $\mathrm{Re}\{B^*C\}$ not appearing in Table I.

The nucleoid state of spin (5/2) according to strong coupling theory[8] would lie about $2\frac{2}{3}$ times as high above the nucleon ground state as the baryon state. This would place the mass of the spin-(5/2) nucleoid at around 3300 m_ec^2 and the threshold[13] for the processes (5.1) at over 2.6 Gev for the incident proton in the laboratory system.[14] The next higher threshold,[13] for the $p+p\rightarrow\mathfrak{b}+\mathfrak{b}$ reaction, lies then a little below 4.2 Gev.

Only the two-pion decay of the spin-(5/2) particle was taken into account. There is enough energy for up to five pions from decay of $\mathfrak{b}$. These possibilities increase the number of unknown matrix elements to be taken into account; we have ignored them hoping for the best. Thus, in the energy region 0.6–1.3 Gev only a^2 does not vanish in Table I; for 1.3–2.6 Gev protons also b^2 and c^2 become important; from 2.6 Gev up to

[13] The word "threshold" as used in this paper should not be taken too literally. As these "thresholds" have been calculated neglecting proper motion of the target nucleon, the actual thresholds may lie considerably lower for heavy target nuclei.

[14] 1 Gev$=10^9$ ev. See for instance Phys. Today 4, No. 11, 31 (1951); and F. J. Belinfante, Am. J. Phys. 21, 474 (1953).

4.2 Gev many more constants enter the theory, of which we mentioned only d^2; and above 4.2 Gev the number of processes becomes too large for a brief survey.[13]

Besides the processes discussed by us, there are of course the competing direct single and multiple pion productions by nucleon-nucleon interaction discussed by others,[1–3] which include the processes in which the baryon disintegrates before it escapes from the pion cloud created *with* it; and there are also the processes involving more than one target nucleon in the nucleus. There is, however, at least a possibility that the baryon reactions discussed in this paper may be predominant for energies above the 600-Mev threshold[13] for baryon production.

SUMMARY

In this paper we have proposed the hypothesis of predominance of baryon production over direct pion production as a possible simplification of the theory. For the validity of this hypothesis we cannot give at this time any binding *a priori* arguments. Experimental verification of our hypothesis by a comparison of the various total (or better differential) cross sections as functions of the energy in the 0.6–4.2 Gev region should become possible in the near future as machines producing protons over 600 Mev will go into full operation.

APPENDIX ON VECTOR ADDITION OF SPINS

In the following tables, signs have been chosen in such a way that the matrix elements of the total isobaric spin **T** are

$$\left.\begin{aligned}\langle T', T_\zeta' | \mathbf{T}_\xi \pm i\mathbf{T}_\eta | T, T_\zeta\rangle &\\ = \delta(T', T)\cdot\delta(T_\zeta', T_\zeta \pm 1)&\\ \cdot\sqrt{\{(T\mp T_\zeta)(T\pm T_\zeta+1)\}},&\\ \langle T', T_\zeta' | \mathbf{T}_\zeta | T, T_\zeta\rangle = \delta(T', T)\cdot\delta(T_\zeta', T_\zeta)\cdot T_\zeta,&\end{aligned}\right\} \quad (A.1)$$

where $\delta(m, n)$ are Kronecker symbols.

By $(T)_i\times(T')_j$ we shall indicate a listing of the coefficients in the expansion of isobaric-spin functions, for a system ij composed of component systems i and j of isobaric spin T and T', respectively, in terms of products of isobaric-spin functions of the single systems i and j separately. We save space by omitting the trivial coefficients 1 in expressing the two states $T_{ij}=\pm(T_\zeta)_{ij}=T_i+T_j$ in terms of the products of states with $(T_\zeta)_i=\pm T_i$ and $(T_\zeta)_j=\pm T_j$. (For instance $[1, 1]_{ij}=[\frac{1}{2}, \frac{1}{2}]_i[\frac{1}{2}, \frac{1}{2}]_j$ and $[1, -1]_{ij}=[\frac{1}{2}, -\frac{1}{2}]_i[\frac{1}{2}, -\frac{1}{2}]_j$ for the $(1/2)\times(1/2)$ case.) We have also left out the trivial addition of an isobaric spin 0 of system i to the isobaric spin T of a system j, in which case the functions of the combined system ij are simply $[0, 0]_i$ times those of system j.

For every given pair of isobaric spin values T and T' for the component systems, we group together the coefficients for fixed total T_ζ. As each such table forms an orthogonal matrix, the tables may then also be used for expressing the old products of separate-system isobaric-spin functions in terms of the new combined-system isobaric-spin functions of different possible values of T_{ij} (as listed by $(\cdots)_{ij}\&(\cdots)_{ij}\&\cdots$ in the table heading), with coefficients taken from the columns of the matrices.

$$(1/2)_i\times(1/2)_j=(1)_{ij}\,\&(0)_{ij}:$$

$T_\zeta=0$	$[\frac{1}{2}, \frac{1}{2}]_i[\frac{1}{2}, -\frac{1}{2}]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[\frac{1}{2}, \frac{1}{2}]_j$
$[1, 0]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[0, 0]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

(Interpretation:

$$[1, 0]_{ij}=(\sqrt{\tfrac{1}{2}})[\tfrac{1}{2}, \tfrac{1}{2}]_i[\tfrac{1}{2}, -\tfrac{1}{2}]_j+(\sqrt{\tfrac{1}{2}})[\tfrac{1}{2}, -\tfrac{1}{2}]_i[\tfrac{1}{2}, \tfrac{1}{2}]_j;$$

$[\frac{1}{2}, -\frac{1}{2}]_i[\frac{1}{2}, \frac{1}{2}]_j=(\sqrt{\frac{1}{2}})[1, 0]_{ij}-(\sqrt{\frac{1}{2}})[0, 0]_{ij}$; etc.)

$$(1/2)_i\times(1)_j=(3/2)_{ij}\,\&(1/2)_{ij}:$$

$T_\zeta=\frac{1}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[1, 0]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[1, 1]_j$
$[\frac{3}{2}, \frac{1}{2}]_{ij}$	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$[\frac{1}{2}, \frac{1}{2}]_{ij}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$

$T_\zeta=-\frac{1}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[1, -1]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[1, 0]_j$
$[\frac{3}{2}, -\frac{1}{2}]_{ij}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
$[\frac{1}{2}, -\frac{1}{2}]_{ij}$	$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$

$$(1/2)_i\times(3/2)_j=(2)_{ij}\,\&(1)_{ij}:$$

$T_\zeta=1$	$[\frac{1}{2}, -\frac{1}{2}]_i[\frac{3}{2}, \frac{3}{2}]_j$	$[\frac{1}{2}, \frac{1}{2}]_i[\frac{3}{2}, \frac{1}{2}]_j$
$[2, 1]_{ij}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$
$[1, 1]_{ij}$	$\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$

$T_\zeta=0$	$[\frac{1}{2}, -\frac{1}{2}]_i[\frac{3}{2}, \frac{1}{2}]_j$	$[\frac{1}{2}, \frac{1}{2}]_i[\frac{3}{2}, -\frac{1}{2}]_j$
$[2, 0]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[1, 0]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$T_\zeta=-1$	$[\frac{1}{2}, -\frac{1}{2}]_i[\frac{3}{2}, -\frac{1}{2}]_j$	$[\frac{1}{2}, \frac{1}{2}]_i[\frac{3}{2}, -\frac{3}{2}]_j$
$[2, -1]_{ij}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$
$[1, -1]_{ij}$	$\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$

$$(1/2)_i\times(2)_j=(5/2)_{ij}\,\&(3/2)_{ij}:$$

$T_\zeta=\frac{3}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[2, 1]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[2, 2]_j$
$[(5/2), \frac{3}{2}]_{ij}$	$\sqrt{\frac{4}{5}}$	$\sqrt{\frac{1}{5}}$
$[\frac{3}{2}, \frac{3}{2}]_{ij}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{4}{5}}$

$T_\zeta=\frac{1}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[2, 0]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[2, 1]_j$
$[(5/2), \frac{1}{2}]_{ij}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$
$[\frac{3}{2}, \frac{1}{2}]_{ij}$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$

$T_\zeta = -\frac{1}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[2, -1]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[2, 0]_j$
$[(5/2), -\frac{1}{2}]_{ij}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$
$[\frac{3}{2}, -\frac{1}{2}]_{ij}$	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$

$T_\zeta = -\frac{3}{2}$	$[\frac{1}{2}, \frac{1}{2}]_i[2, -2]_j$	$[\frac{1}{2}, -\frac{1}{2}]_i[2, -1]_j$
$[(5/2), -\frac{3}{2}]_{ij}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$
$[\frac{3}{2}, -\frac{3}{2}]_{ij}$	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$

$$(1)_i \times (1)_j = (2)_{ij} \,\&\, (1)_{ij} \,\&\, (0)_{ij}:$$

$T_\zeta = 1$	$[1, 0]_i[1, 1]_j$	$[1, 1]_i[1, 0]_j$
$[2, 1]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[1, 1]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$T_\zeta = 0$	$[1, -1]_i[1, 1]_j$	$[1, 0]_i[1, 0]_j$	$[1, 1]_i[1, -1]_j$
$[2, 0]_{ij}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{6}}$
$[1, 0]_{ij}$	$\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$
$[0, 0]_{ij}$	$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$

$T_\zeta = -1$	$[1, -1]_i[1, 0]_j$	$[1, 0]_i[1, -1]_j$
$[2, -1]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[1, -1]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$$(1)_i \times (3/2)_j = (5/2)_{ij} \,\&\, (3/2)_{ij} \,\&\, (1/2)_{ij}:$$

$T_\zeta = \frac{3}{2}$	$[1, 1]_i[\frac{3}{2}, \frac{1}{2}]_j$	$[1, 0]_i[\frac{3}{2}, \frac{3}{2}]_j$
$[(5/2), \frac{3}{2}]_i$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{2}{5}}$
$[\frac{3}{2}, \frac{3}{2}]_i$	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{3}{5}}$

$T_\zeta = \frac{1}{2}$	$[1, 1]_i[\frac{3}{2}, -\frac{1}{2}]_j$	$[1, 0]_i[\frac{3}{2}, \frac{1}{2}]_j$	$[1, -1]_i[\frac{3}{2}, \frac{3}{2}]_j$
$[(5/2), \frac{1}{2}]_{ij}$	$\sqrt{(3/10)}$	$\sqrt{\frac{3}{5}}$	$\sqrt{(1/10)}$
$[\frac{3}{2}, \frac{1}{2}]_{ij}$	$\sqrt{(8/15)}$	$-\sqrt{(1/15)}$	$-\sqrt{\frac{2}{5}}$
$[\frac{1}{2}, \frac{1}{2}]_{ij}$	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{2}}$

$T_\zeta = -\frac{1}{2}$	$[1, 1]_i[\frac{3}{2}, -\frac{3}{2}]_j$	$[1, 0]_i[\frac{3}{2}, -\frac{1}{2}]_j$	$[1, -1]_i[\frac{3}{2}, \frac{1}{2}]_j$
$[(5/2), -\frac{1}{2}]_{ij}$	$\sqrt{(1/10)}$	$\sqrt{\frac{3}{5}}$	$\sqrt{(3/10)}$
$[\frac{3}{2}, -\frac{1}{2}]_{ij}$	$\sqrt{\frac{2}{5}}$	$\sqrt{(1/15)}$	$-\sqrt{(8/15)}$
$[\frac{1}{2}, -\frac{1}{2}]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{6}}$

$T_\zeta = -\frac{3}{2}$	$[1, 0]_i[\frac{3}{2}, -\frac{3}{2}]_j$	$[1, -1]_i[\frac{3}{2}, -\frac{1}{2}]_j$
$[(5/2), -\frac{3}{2}]_{ij}$	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{3}{5}}$
$[\frac{3}{2}, -\frac{3}{2}]_{ij}$	$\sqrt{\frac{3}{5}}$	$-\sqrt{\frac{2}{5}}$

$(3/2)_i \times (3/2)_j = (3)_{ij} \& (2)_{ij} \& (1)_{ij} \& (0)_{ij}$:

$T_\zeta=2$	$[\frac{3}{2},\frac{3}{2}]_i[\frac{3}{2},\frac{1}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[\frac{3}{2},\frac{3}{2}]_j$
$[3,2]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[2,2]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$T_\zeta=1$	$[\frac{3}{2},\frac{3}{2}]_i[\frac{3}{2},-\frac{1}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[\frac{3}{2},\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[\frac{3}{2},\frac{3}{2}]_j$
$[3,1]_{ij}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{5}}$
$[2,1]_{ij}$	$\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$
$[1,1]_{ij}$	$\sqrt{(3/10)}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{(3/10)}$

$T_\zeta=0$	$[\frac{3}{2},\frac{3}{2}]_i[\frac{3}{2},-\frac{3}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[\frac{3}{2},-\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[\frac{3}{2},\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[\frac{3}{2},\frac{3}{2}]_j$
$[3,0]_{ij}$	$\sqrt{(1/20)}$	$\sqrt{(9/20)}$	$\sqrt{(9/20)}$	$\sqrt{(1/20)}$
$[2,0]_{ij}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$[1,0]_{ij}$	$\sqrt{(9/20)}$	$-\sqrt{(1/20)}$	$-\sqrt{(1/20)}$	$\sqrt{(9/20)}$
$[0,0]_{ij}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

$T_\zeta=-1$	$[\frac{3}{2},\frac{1}{2}]_i[\frac{3}{2},-\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[\frac{3}{2},-\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[\frac{3}{2},\frac{1}{2}]_j$
$[3,-1]_{ij}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{3}{5}}$	$\sqrt{\frac{1}{5}}$
$[2,-1]_{ij}$	$\sqrt{\frac{1}{2}}$	0	$-\sqrt{\frac{1}{2}}$
$[1,-1]_{ij}$	$\sqrt{(3/10)}$	$-\sqrt{\frac{2}{5}}$	$\sqrt{(3/10)}$

$T_\zeta=-2$	$[\frac{3}{2},-\frac{1}{2}]_i[\frac{3}{2},-\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[\frac{3}{2},-\frac{1}{2}]_j$
$[3,-2]_{ij}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$
$[2,-2]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$

$(3/2)_i \times (5/2)_j = (4)_{ij} \& (3)_{ij} \& (2)_{ij} \& (1)_{ij}$:

$T_\zeta=3$	$[\frac{3}{2},\frac{3}{2}]_i[(5/2),\frac{3}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),(5/2)]_j$
$[4,3]_{ij}$	$\sqrt{\frac{5}{8}}$	$\sqrt{\frac{3}{8}}$
$[3,3]_{ij}$	$\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{5}{8}}$

$T_\zeta=2$	$[\frac{3}{2},\frac{3}{2}]_i[(5/2),\frac{1}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),(5/2)]_j$
$[4,2]_{ij}$	$\sqrt{(5/14)}$	$\sqrt{(15/28)}$	$\sqrt{(3/28)}$
$[3,2]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{(1/12)}$	$-\sqrt{(5/12)}$
$[2,2]_{ij}$	$\sqrt{(1/7)}$	$-\sqrt{(8/21)}$	$\sqrt{(10/21)}$

$T_\zeta=1$	$[\frac{3}{2},\frac{3}{2}]_i[(5/2),-\frac{1}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[(5/2),(5/2)]_j$
$[4,1]_{ij}$	$\sqrt{(5/28)}$	$\sqrt{(15/28)}$	$\sqrt{(15/56)}$	$\sqrt{(1/56)}$
$[3,1]_{ij}$	$\sqrt{(9/20)}$	$\sqrt{(1/60)}$	$-\sqrt{(49/120)}$	$-\sqrt{\frac{1}{8}}$
$[2,1]_{ij}$	$\sqrt{(9/28)}$	$-\sqrt{(25/84)}$	$\sqrt{(1/42)}$	$\sqrt{(5/14)}$
$[1,1]_{ij}$	$\sqrt{(1/20)}$	$-\sqrt{(3/20)}$	$\sqrt{(3/10)}$	$-\sqrt{\frac{1}{2}}$

$T_\zeta=0$	$[\frac{3}{2},\frac{3}{2}]_i[(5/2),-\frac{3}{2}]_j$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),-\frac{1}{2}]_i$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[(5/2),\frac{3}{2}]_j$
$[4,0]_{ij}$	$\sqrt{(1/14)}$	$\sqrt{(3/7)}$	$\sqrt{(3/7)}$	$\sqrt{(1/14)}$
$[3,0]_{ij}$	$\sqrt{(3/10)}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{\frac{1}{5}}$	$-\sqrt{(3/10)}$
$[2,0]_{ij}$	$\sqrt{(3/7)}$	$-\sqrt{(1/14)}$	$-\sqrt{(1/14)}$	$\sqrt{(3/7)}$
$[1,0]_{ij}$	$\sqrt{\frac{1}{5}}$	$-\sqrt{(3/10)}$	$\sqrt{(3/10)}$	$-\sqrt{\frac{1}{5}}$

$T_\zeta=-1$	$[\frac{3}{2},\frac{3}{2}]_i[(5/2),-(5/2)]_j$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),-\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-\frac{1}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[(5/2),\frac{1}{2}]_j$
$[4,-1]_{ij}$	$\sqrt{(1/56)}$	$\sqrt{(15/56)}$	$\sqrt{(15/28)}$	$\sqrt{(5/28)}$
$[3,-1]_{ij}$	$\sqrt{\frac{1}{8}}$	$\sqrt{(49/120)}$	$-\sqrt{(1/60)}$	$-\sqrt{(9/20)}$
$[2,-1]_{ij}$	$\sqrt{(5/14)}$	$\sqrt{(1/42)}$	$-\sqrt{(25/84)}$	$\sqrt{(9/28)}$
$[1,-1]_{ij}$	$\sqrt{\frac{1}{2}}$	$-\sqrt{(3/10)}$	$\sqrt{(3/20)}$	$-\sqrt{(1/20)}$

$T_\zeta=-2$	$[\frac{3}{2},\frac{1}{2}]_i[(5/2),-(5/2)]_j$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-\frac{3}{2}]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[(5/2),-\frac{1}{2}]_j$
$[4,-2]_{ij}$	$\sqrt{(3/28)}$	$\sqrt{(15/28)}$	$\sqrt{(5/14)}$
$[3,-2]_{ij}$	$\sqrt{(5/12)}$	$\sqrt{(1/12)}$	$-\sqrt{\frac{1}{2}}$
$[2,-2]_{ij}$	$\sqrt{(10/21)}$	$-\sqrt{(8/21)}$	$\sqrt{(1/7)}$

$T_\zeta=-3$	$[\frac{3}{2},-\frac{1}{2}]_i[(5/2),-(5/2)]_j$	$[\frac{3}{2},-\frac{3}{2}]_i[(5/2),-\frac{3}{2}]_j$
$[4,-3]_{ij}$	$\sqrt{\frac{3}{8}}$	$\sqrt{\frac{5}{8}}$
$[3,-3]_{ij}$	$\sqrt{\frac{5}{8}}$	$-\sqrt{\frac{3}{8}}$

Addition of more than two vectors may be performed in steps. For instance, for a system of one nucleon and two pions,

$$(1/2)_1\times(1)_3\times(1)_4=(1/2)_1\times\{(2)_{34}\&(1)_{34}\&(0)_{34}\}$$
$$=\{(5/2)_{134}\&(3/2)_{134}\}\&\{(3/2)'_{134}\&(1/2)'_{134}\}\&(1/2)_{134},$$

where for instance $(3/2)'_{134}$ differs from $(3/2)_{134}$ by the fact that the isobaric spins of the two pions are parallel in $(3/2)$, but form a total pion isobaric spin 1 in $(3/2)'$.

PHYSICAL REVIEW VOLUME 92, NUMBER 1 OCTOBER 1, 1953

Scattering of Mesons by Complex Nuclei

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(Received March 25, 1953)

The result of the previous calculation on the scattering of mesons by a single nucleon has been extended to the case of multiple scattering by nucleons in nuclear matter. Methods familiar in the dispersion theory in physical optics are used, and the index of refraction of meson wave has been formally calculated. The absorption and diffraction scattering cross sections are computed and relation with the results of the recent experimental observation is discussed.

I. INTRODUCTION

The interactions of high-energy particles with complex nuclei have been discussed theoretically by several authors in the recent years. Fernbach, Serber, and Taylor[1] considered nuclear matter as a continuous optical medium characterized by an index of refraction and an absorption coefficient with respect to the incoming neutron waves. Recently a detailed quantum-mechanical theory of the interaction of high-energy π mesons with nuclear matter based on the optical model has been given by Watson.[2] He introduced phenomenological interactions for the scattering and absorption of π mesons by individual nucleon into a many-body Schrödinger equation and showed that the solution of the equation has the structure of a multiply scatter wave.

In this paper an attempt is made to discuss the interaction of π mesons with complex nuclei in a manner similar to that of Fernbach, Serber, and Taylor for high-energy neutrons. While the index of refraction there was obtained by simply ascribing a uniform potential well for the nuclear matter, in our case we pay a special attention on the calculation of the index of refraction[3] by the methods familiar in physical optics. We consider a nuclear matter of a large extent and assume that the mesons with which we are concerned are of sufficiently low energy that their wavelengths are large compared with the average internucleon distance.

For the scattering of a meson wave by the individual nucleon we make use of the results of the previous paper on the subject.[4] The fact that the nucleons within a

[1] Fernbach, Serber, and Taylor, Phys. Rev. **75**, 1352 (1949).
[2] K. M. Watson, Phys. Rev. **89**, 575 (1953).
[3] See M. Lax, Revs. Modern Phys. **23**, 287 (1951), for the extensive literature on the subject.
[4] W. W. Wada, Phys. Rev. **88**, 1032 (1952). This will be referred to as I. Also D. Feldman, Phys. Rev. **88**, 890 (1952).