The Variation of the Ratio of Positive to Negative Cosmic-Ray μ Mesons with Momentum and Altitude*†

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I. The mean lifetime of $\mu^{\pm}$ mesons in carbon and sulfur has been measured by the delayed coincidence technique. Data for decay times $>2\mu$sec were analyzed by the statistical method of Peierls and yield $\tau_s^+=2.09\pm0.05\mu$sec for the mean life of the μ^+ meson in sulfur and $\tau_c^-=1.92\pm0.04\mu$sec for the mean lifetime of the μ^- meson in carbon. τ_s^+ compares favorably with values found in material of low atomic number, but there appears to be a significant difference between these values and those obtained in high Z materials. Assuming a Z^4 dependent capture probability for μ^- mesons, τ_c^- is compatible with other lifetimes determined in materials of higher Z.

II. The integral time distributions of the delayed coincidences obtained above were extrapolated to zero delay time and allowance made for those μ^- mesons which are captured in carbon. These data were used to obtain the μ^+/μ^- ratio at a momentum of 325 ± 70 Mev/c at sea level: $\mu^+/\mu^-=1.06\pm0.03$. This ratio is compared with other experiments (which also provide good identification of the μ mesons) by plotting all ratios as a function of momentum at the top of the atmosphere (TOA). The best fit to the experimental points is given by the exponential expression

$$P=P_0 \exp[\mu^+/\mu^-/K],$$

where

$P<4$ Bev/c (TOA), $P_0\simeq0.165$ Bev/c (TOA), and $K\simeq0.38$.

The decrease of the μ^+/μ^- ratio with increasing altitude appears to be well established, although the exact values of P_0 and K are uncertain.

I. INTRODUCTION

FOR more than a decade it has been known that there is an excess of positive particles in the "hard component" at sea level. According to the best estimates[1–4] this excess amounts to $\sim$20 percent of the total penetrating radiation. Early studies gave no information regarding the distribution of the excess in the differential sea-level spectrum, but recent experiments[5–7] have shown that in the region 1→2 Bev/c the positive/negative ratio increases, reaches a broad maximum at 2→5 Bev/c and then falls off more slowly at high momenta.

Still less detailed information is available concerning the positive excess of μ mesons at altitudes above sea level. Most cloud-chamber data[8–10] show that the positive/negative ratio of the "hard component" increases with altitude, especially at low momenta. However, this increase can be related directly to the large increase of the proton intensity with altitude rather than any increase in the relative number of μ^+ mesons. Similarly, a magnetic lens experiment[11] showed that the ratio increased from sea level to 3.5 km, but did not change substantially from this latter value in going up to 7.6 km. To further complicate matters, delayed coincidence experiments[12–14] (in which μ mesons were identified by their characteristic decay) showed that the μ^+/μ^- ratio of 1.20 was fairly evenly distributed in the sea-level spectrum but dropped to $\sim$1 in going to 2.1 km and then increased with altitude.

Other experiments[15,16] in which the mesons were identified accurately by range-momenta criteria, showed no positive excess at 3.4 km.

Positive/negative ratios obtained from π mesons stopped in photographic emulsions[17–23] exposed at various altitudes and identified by their characteristic endings have generally exhibited ratios between 0.2 and 1.2.

It is the purpose of this paper to report an experimental determination of positive/negative ratio for low-energy μ mesons at sea level, and to compare this result with other experimental data in an attempt to resolve the major discrepancies which exist.

The delayed-coincidence technique was used in conjunction with absorbers of carbon and sulfur. A com-

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[8] Miller, Henderson, Potter, Todd, and Wotring, Phys. Rev. **79**, 459 (1950).
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[10] W. L. Whittmore and R. P. Shutt, Phys. Rev. **86**, 940 (1952).
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[13] O. Piccioni, Phys. Rev. **77**, 1 (1950).
[14] M. Conversi, Phys. Rev. **79**, 749 (1950).
[15] M. Correll, Phys. Rev. **72**, 1054 (1947).
[16] C. Franzinetti, Phil. Mag. **41**, 86 (1950).
[17] W. F. Fry, Phys. Rev. **83**, 594 (1951).
[18] I. Barbour, Phys. Rev. **78**, 319 (1950).
[19] Camerini, Muirhead, Powell, and Ritson, Nature **162**, 433 (1948).
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[21] Peyrou, Bousser, Fond, Juneau, Morellet, and Leprince-Ringuet, Nuovo cimento **6**, Supplement no. 3, 408 (1949).
[22] Barton, George, and Jason, Proc. Phys. Soc. (London) **A64**, 175 (1951).
[23] H. Yagoda, Phys. Rev. **85**, 891 (1952).

parison of the lifetimes obtained in the experiments with the characteristic values for μ mesons in these materials serves to identify the observed particles as μ mesons. Since essentially both positive and negative μ mesons decay in carbon, while only μ^+ mesons decay in sulfur[24] it is possible in principle to obtain the μ^+/μ^- ratio from the delayed coincidence rates, provided that the lifetimes of the μ mesons are known accurately for the two absorbers. This is a technique originated by Shamos *et al.*[12] and later used by Piccioni[13] and Conversi.[14]

II. APPARATUS

A. Counter Telescope

The counter telescope used in this experiment is shown in Fig. 1. Counter trays A and B, each consisting of five Geiger-Mueller counters, defined the incident beam. These two trays were connected to a special two-fold coincidence circuit. Tray C was connected in anti-coincidence, delayed-coincidence. The side trays labeled C' were connected in anticoincidence. In this arrangement, a meson was required to traverse trays A and B, come to rest in the absorber and then give rise to a decay electron in the interval 1.1 to 7.38μsec later.

The carbon absorber was in the form of graphite slabs (density 1.73 g/cm^3) and the sulphur was in the form of bricks (density 1.87 g/cm^3). The size of both absorbers was $4\frac{1}{2}$ in.$\times$14 in.$\times$36 in. A 10-cm thick lead filter cut out most of the incident electrons and photons. This filter was always in position.

The apparatus was operated at a height approximately 200 ft above sea level under a thin roof of glass and iron, the total thickness being $\sim$0.7 g/cm^2.

The brass Geiger-Mueller counters were filled with a self-quenching mixture of argon and ethyl acetate. The A and B tray counters were 1 in. in diameter and 23 in. long while the C tray counters were 1 in. in diameter and 28 in. long. The side counters were 2 in. in diameter and 36 in. long. The high voltage for each of the counters was individually adjusted by means of potentiometers across a regulated 300-volt supply which floated on a fixed high-tension supply. With the exception of the C' tray, all counters were operated at the upper end of their plateaus to minimize inherent lags.[25] The counter pulses were $\sim$75 volts in amplitude (at the cathode-follower output) with an over-all rise time to full height of $\sim$2μsec. The pulse amplitudes of all tubes were adjusted to the same height. The output of each counter was taken off through short leads to individual cathode followers and then mixed to provide tray outputs. Due to the individual coupling of each G-M counter, the dead time of a single counter could not block the entire tray.

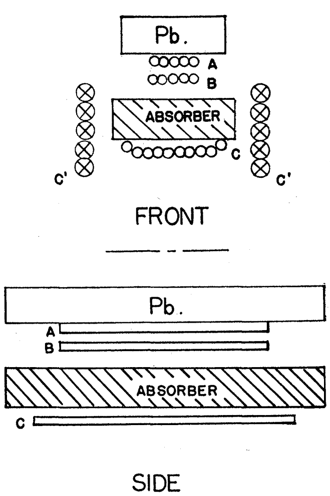

FIG. 1. Experimental arrangement. A, B, and C G-M counters are 1 inch in diameter. C' counters are 2 inches in diameter.

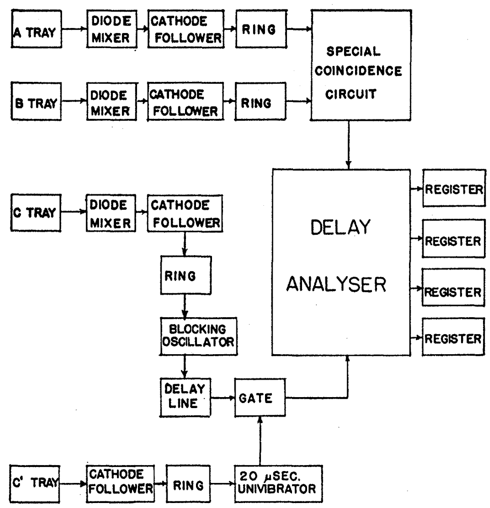

FIG. 2. Block diagram of the electronic apparatus. The time delay between C and the earliest signal from either A or B is recorded in four adjacent time channels provided there is an AB coincidence within 1μsec and that there is no signal from C'. C channel is dead for $\sim$12μsec following each signal.

B. Circuits

An ove-rall block diagram of the apparatus is shown in Fig. 2.

The timing circuit generally followed a design reported by Sands.[26] However, the inputs were ringing circuits of a type proposed by Elmore.[27] The special twofold coincidence circuit differs in detail from that proposed by Sands, but is designed to accomplish essentially the same purpose, namely, to reduce the effect of spontaneous counter lags by starting the timing cycle from the earliest signal obtained in a twofold coincidence.

The channel widths in the delay discriminator were derived from fixed delay lines driven by blocking oscillators whose bias was controlled by a highly regulated power supply. All bias voltages were monitored daily.

[24] H. K. Ticho, Phys. Rev. **74**, 1337 (1948).

[25] A. R. Laufer, Rev. Sci. Instr. **21**, 244 (1950).

[26] M. Sands, Massachusetts Institute of Technology Technical Report 28, 1949 (unpublished).

[27] R. Cool (private communication).

TABLE I(a). Differential data.

Channel	No. 1	No. 2	No. 3	No. 4
Width (Δt)	1.07μsec	1.80μsec	2.00μsec	1.41μsec
Edges (from $t=0$)	1.10–2.17μsec	2.17–3.97μsec	3.97–5.97μsec	5.97–7.38μsec
N_c (1408.9 hr)	12 816±113	10 281±101	4865±69	2022±45
N_s (1609.7 hr)	11 380±107	7862±88	4068±64	1805±42
N_b (822.2 hr)	2392±49	1676±41	1042±32	595±24

TABLE I(b). Differential data reduced to equal channel widths ($\Delta t=1.10\mu$sec).

Channel	No. 1	No. 2	No. 3	No. 4
Calculated "center"	1.58μsec	2.95μsec	4.83μsec	6.57μsec
$\frac{dN_{c-b}}{dt}$ (hr^{-1})	6.19±0.10	3.21±0.06	1.20±0.06	0.56±0.03
$\frac{dN_{s-b}}{dt}$ (hr^{-1})	4.16±0.09	1.74±0.03	0.69±0.03	0.31±0.03
$\frac{dN_{c-s}}{dt}$ (hr^{-1})		1.47±0.06	0.52±0.04	0.25±0.04

Due to these precautions, the shift in the channel widths over the period of observation was $<0.02\mu$sec. Average values for the time widths of the channels reported in Table I(a) were used in all calculations. This average was computed from weekly calibrations made with a delay line controlled double pulse generator developed in this laboratory[28–30] and used in previous delayed coincidence experiments.

All of the critical power supplies were either of the degenerative feedback type or were controlled by *VR* tubes.

III. EXPERIMENTAL PROCEDURE

The apparatus was operated for a total of 3840 hours. Of this time, 3018 hours were foreground and 822 hours were background (no absorber). The results are summarized in Table I.

Individual tray counting rates and the twofold and threefold prompt coincidence rates were monitored daily. In order to minimize any effects due to fluctuations in the incident intensity, runs with each of the two absorbers were alternately sandwiched in time with background runs. The measured background was subtracted so that all final results are based upon the true delayed coincidence rate for each absorber. Thus, delayed counts due to accidental coincidences and spontaneous counter lags are cancelled out. Mesons stopping in the support material or Geiger-Mueller counter walls are likewise eliminated from the results.

The differential time distributions plotted in Fig. 3 were obtained by reducing the data from each channel to correspond to equal widths ($\Delta t=1.1\mu$sec) and calculating the weighted "center" of each reduced channel. This procedure [see Table I(b)] was used only in plotting Fig. 3. *All other information was obtained from the raw data listed in* Table I(a).

IV. MEAN LIFETIMES OF μ MESONS

A. General

The mean lifetimes reported here were calculated from the data in Table I(a) by the statistical method of Peierls.[31] This procedure allows the mean lifetime and standard statistical error to be computed from an ob-

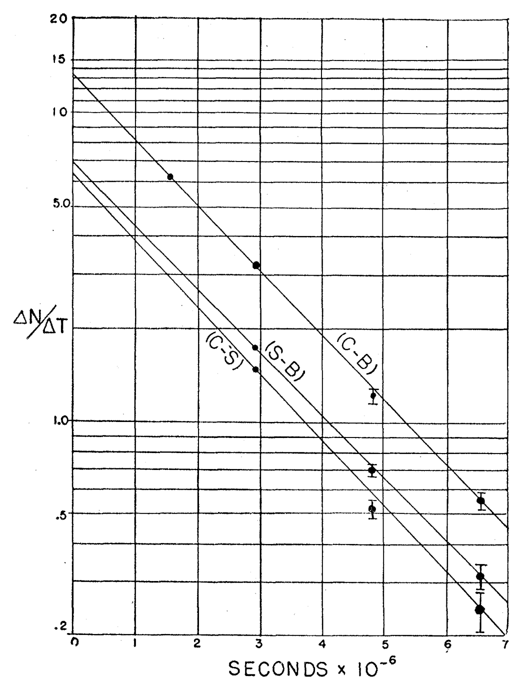

FIG. 3. Differential time distribution of delayed coincidence counts. All channels reduced to equal widths ($\Delta t=1.1\mu$sec).

[28] M. H. Shamos and M. G. Levy, Phys. Rev. **73**, 1396 (1948).
[29] M. H. Shamos and A. Russek, Phys. Rev. **74**, 1546 (1948).
[30] J. L. Zar, Phys. Rev. **83**, 761 (1951).
[31] R. Peierls. Proc. Roy. Soc. (London) **A149**, 473 (1935).

served set of individual delays. Since the various delay channel widths were in all cases greater than 0.3τ (τ = mean lifetime), Peierls' series expression could not be used and his more general integral form was required. The resulting calculations were more accurate since no approximations were involved. As a further consequence of using the general expression, it was not necessary to reduce all channels to the same width, thus eliminating a source of error in the handling of the data.

In computing the lifetimes involving the sulfur data, the results for delay times earlier than 2.17μsec were omitted, since there is in this region an appreciable counting rate due to the μ^- mesons decaying with a shortened mean life. The contribution of μ^- mesons in the succeeding channels is completely negligible[32] provided that $\tau_s^- \lesssim 0.54\mu$sec. We find

Lifetime of μ^+ mesons in sulfur: ($S-B$, less No. 1 channel) $\quad \tau_s^+ = 2.09+0.05\mu$sec.

Lifetime of μ^- mesons in carbon: $\quad \tau_c^- = 1.92\pm0.04\mu$sec.

Composite lifetime of the natural mixture of both μ^+ and μ^- mesons in carbon under ~11 cm of Pb at sea level ($C-B$): $\quad \tau_c^\pm = 2.06\pm0.03\mu$sec.

The errors associated with the above lifetimes are the standard statistical errors computed by Peierls' method.

B. Lifetime of the μ^+ Meson

Ticho[24] has used magnetic separation in conjunction with the delayed coincidence technique and found that the average μ^+ meson lifetime in absorbers from oxygen to sulfur is $\tau^+ = 2.11\pm0.1\mu$sec. Valley[33] using a similar method and an aluminum absorber found $\tau^+ = 2.06 \pm0.08\mu$sec. Alvarez *et al.*[34] observed the decay of the μ^+ mesons derived from artificially generated π^+ mesons and found $\tau^+ = 2.09\pm0.03\mu$sec for the μ^+ lifetime in stilbene (carbon). This is in good agreement with Steinberger and Bishop,[35] who used a similar technique.

The μ^+ meson lifetime determined in the present experiment compares favorably with values found in materials of low atomic number, but disagrees with a measurement recently reported by Bell and Hincks,[36] who used an iron absorber (in which μ^- mesons do not decay) and obtained: $\tau^+ = 2.22\pm0.02\mu$sec. The results of all pertinent experiments[24,33–41] are summarized in Table II.

TABLE II. Mean lifetime of the μ^+ meson at rest.

Author	Z	μsec
Valley (1952)[a]	13	2.06±0.08
Alvarez *et al.* (1950)[b]	6	2.09±0.03
Present experiment (1952)	16	2.09±0.05
Steinberger and Bishop (1950)[c]	6	2.10±0.10
Ticho (1948)[d]	8–16	2.11±0.10
Nereson and Rossi (1943)[e]	26, 29, 82	2.15±0.10
Maze *et al.* (1945)[f]	26	2.2 ±0.2
Bartman, Harrison, and Reynolds (1949)[g]	82	2.2 ±0.2
Bell and Hincks (1951)[h]	26	2.22±0.02
Rossi and Nereson (1942)[i]	82	2.3 ±0.2
Conversi and Piccioni (1946)[j]	26	2.33±0.15

[a] See reference 33. [b] See reference 34. [c] See reference 35. [d] See reference 24. [e] See reference 37. [f] See reference 38. [g] See reference 39. [h] See reference 36. [i] See reference 40. [j] See reference 41.

It is difficult to draw firm conclusions in this respect at the present time, but the data might be considered consistent with the view that τ^+ increases with Z. The various lifetime determinations appear to fall into two groups, according to the Z of the absorber.[42] The weighted means from Table II become

$$\tau^+ = 2.09\pm0.03\ \mu\text{sec} \quad \text{for} \quad Z \leq 16;$$
$$\tau^+ = 2.22\pm0.02\ \mu\text{sec} \quad \text{for} \quad Z \geq 26.$$

This trend takes no account of possible systematic errors in the lifetime determinations reported by the various authors, and hence the indicated effect may not be real.

C. Comparison of μ^- Lifetimes

Wheeler and Budini[43] have shown from theoretical considerations that the capture probability for μ^- mesons should vary as Z^4 for $Z<29$. Thus, the stopped μ^- meson falls into a K orbit and either is captured by the nucleus or decays spontaneously.[44] The mean lifetime for a μ^- meson depends, therefore, upon the competition between the relative probabilities for capture and for decay, and should (in elements of high Z) differ markedly from the free-space lifetime. This is not the case for μ^+ mesons, which (because of electrostatic repulsion) cannot approach sufficiently close to the nucleus for capture to take place.

The predicted Z dependence of μ^- meson lifetime has

[32] Ticho's μ^- meson lifetimes include a 4 percent correction for assumed magnetic lens inefficiency. The lifetime in sulfur required by the present data as well as the lifetime in aluminum recently obtained by Valley are more compatible with Ticho's uncorrected lifetimes. This leads to the suggestion that Ticho's correction may have been unnecessary and that $\tau_s^- = 0.66\pm0.05\mu$sec is more nearly the true lifetime.

[33] G. E. Valley, quoted by B. Rossi, *High Energy Particles* (Prentice-Hall Publishing Company, New York, 1952), p. 68.

[34] Alvarez Longacre, Orgen, and Thomas, Phys. Rev. **77**, 752 (1950).

[35] J. Steinberger and A. S. Bishop, Phys. Rev. **78**, 39 (1950).

[36] W. E. Bell and E. P. Hincks, Phys. Rev. **84**, 1243 (1951).

[37] N. Nereson and B. Rossi, Phys. Rev. **64**, 199 (1943); recalculated by B. Rossi, reference 33, p. 160.

[38] Maze, Chaminade, and Freon, J. phys. **4**, 202 (1945).

[39] Bartman, Harrison, and Reynolds, Princeton University Technical Report No. 2, Chap. X, 5, 1949 (unpublished).

[40] B. Rossi and N. Nereson, Phys. Rev. **62**, 417 (1942).

[41] M. Conversi and O. Piccioni, Phys. Rev. **70**, 859 (1946).

[42] The standard statistical errors assigned to these weighted values were obtained by assuming that the ith experiment contributed N_i events to the final result and then using $\delta\tau_{Av}/\tau_{Av} = 1/(\Sigma_i N_i)^{\frac{1}{2}}$, where $N_i = (\tau_i/\delta\tau_i)^2$.

[43] J. A. Wheeler, Revs. Modern Phys. **21**, 133 (1949); P. Budini, Nuovo cimento **8**, 901 (1951).

[44] H. K. Ticho and M. Schein, Phys. Rev. **72**, 248 (1947).

TABLE III. Mean lifetime of the μ^- meson at rest.

Author	Z	$Z_{(\text{eff})}$	τ^- μsec	$\Lambda_{\text{cap}}{}^{a}\times 10^6$ sec^{-1}
Hincks and Bell (1952)	3	3	2.15 ± 0.09^{b}	$<0.035^{b}$
	4	3.93	2.05 ± 0.06^{b}	0.037 ± 0.015^{b}
	6	5.78	1.98 ± 0.08^{b}	0.055 ± 0.021^{b}
Present experiment (1952)	6	5.78	1.92 ± 0.04	0.043 ± 0.010
Ticho (1948)	8	7.56	1.89 ± 0.15	0.051 ± 0.043
Ticho (1948)	10	9.25	1.28 ± 0.12	0.304 ± 0.073
Ticho (1948)	12	10.83	0.96 ± 0.06	0.562 ± 0.065
Valley (1949)	12	10.83	1.1 ± 0.2	0.431 ± 0.166
Valley (1949)	13	11.58	0.81 ± 0.06	0.76 ± 0.09
Ticho (1948)	13	11.58	0.75 ± 0.07	0.853 ± 0.124
			0.82 ± 0.05^{c}	0.74 ± 0.09
Cathey (1952)	14	12.33	0.60 ± 0.09	1.19 ± 0.25
Ticho (1948)	16	13.7	0.54 ± 0.12	1.377 ± 0.413
			0.66 ± 0.05^{c}	1.04 ± 0.13
Conforto and Sard (1952)	20	16.1	0.81 ± 0.24^{d}	0.77 ± 0.35
Keuffel *et al.* (1952)	26	19.5	0.163 ± 0.027^{e}	5.66 ± 0.94^{e}
	29	20.6	0.116 ± 0.009^{e}	8.15 ± 0.63^{e}

[a] Under the assumption that $\tau^+=2.09\pm 0.02$ unless otherwise indicated.
[b] Calculated from the unseparated delayed-coincidence data under the assumption: $\mu^+/\mu^-=1.22$ and $\tau^+=2.22\pm 0.02\mu$sec.
[c] Less the correction for 4 percent magnetic lens inefficiency included by Ticho (see footnote 32).
[d] Computed graphically from the data given by these authors. They attribute the large lifetime as compared to that obtained from a Z^4 law to magnetic lens inefficiency, although data taken with other absorbers in the same apparatus do not confirm this view.
[e] Obtained by detecting the capture neutrons and gammas.

been confirmed experimentally by Ticho,[24] and by Valley[33] and by Cathey[45] for $8<Z<16$, and by Keuffel *et al.*[46] for copper and iron. The latter group has recently shown that an anomaly, which is attributed to the influence of the shell structure of the nucleus,[47] occurs for large Z.

At the time of the present experiment no reliable evidence existed concerning the capture of μ^- mesons in materials of $Z<8$. Conversi *et al.*,[48] who were the first to observe the Z dependence of the μ^- meson capture process, found that all μ^- mesons decay in carbon. Similar results have been obtained by Kissinger and Cooper,[49] and Valley.[50] All of the references cited above have large statistical errors. Nereson,[51] however, found that in carbon only 76 ± 17.6 percent of the μ^- mesons decay. In their experiments on the range of decay electrons in carbon, Shamos and Russek[29] observed that their results did not tend toward unit yield (i.e., one decay electron per stopped meson) at zero thickness of absorber. The general trend observed in both of these experiments is consistent with Wheeler's theory, which predicts that $\sim$90 percent of the μ^- mesons decay in carbon.

Recently, Hincks and Bell[52] have reported a series of measurements somewhat similar in principle to the present one in absorbers of lithium, beryllium, and carbon. Their $\tau_c{}^-$ compares favorably with the value reported in the present paper. However, capture probabilities computed from their $\tau_{\text{Fe}}{}^+$, $\tau_{\text{Be}}{}^-$, and $\tau_{\text{Li}}{}^-$ are not in agreement with other experiments[24,33] extrapolated to low Z by means of Wheeler's theory.[53]

The various experimental values of the lifetime of the μ^- meson as a function of Z are tabulated in Table III.

Under the hypothesis that the shortened μ^- meson lifetime is due solely to the competition between capture and decay processes,[44,54] the fraction of μ^- mesons which undergo spontaneous decay is given by[55]

$$f=\tau^-/\tau^+.$$

From the values of μ^+ and μ^- lifetimes determined in the present paper, it is seen that 92 ± 3 percent of the μ^- mesons decay in carbon. This confirms previous estimates[29,51] as to the magnitude of this effect.

Using a similar hypothesis, the capture probability is given by

$$\Lambda_{\text{cap}}=1/\tau^- - 1/\tau^+, \quad \text{where} \quad \tau^+=2.09\pm 0.02\mu\text{sec}.$$

The results of this calculation are plotted in Fig. 4 using the effective Z computed by Wheeler.[42] The slope of the resulting plot closely corresponds to the Z^4 effective law. However, one should expect to find local fluctuations in μ^- meson capture probabilities due to nuclear "shell structure" effects. This problem has been examined theoretically by Tiomno and Wheeler[56] for O^{16} and by Kennedy[47] for Ca^{40} and Pb^{208}. These authors find

$$\Lambda_{O^{16}}=2.29\times 10^{101}g^2\ \text{sec}^{-1},$$
$$\Lambda_{Ca^{40}}=2.7\times 10^{103}g^2\ \text{sec}^{-1},$$
$$\Lambda_{Pb^{208}}=1.56\times 10^{104}g^2\ \text{sec}^{-1},$$
$$\Lambda_{Pb}=1.6\times 10^{104}g^2\ \text{sec}^{-1}.$$

Kennedy has compared his calculated capture probability for Pb with the experimentally derived value obtained by Keuffel *et al.*[46] and finds $g\simeq 3\times 10^{-49}$ erg cm^3 with a probable error of 25 percent. Substituting this value in the expression for O^{16} and Ca^{40}, one obtains

$$\Lambda_{O^{16}}=2.06\times 10^4\ \text{sec}^{-1},$$
$$\Lambda_{Ca^{40}}=2.43\times 10^6\ \text{sec}^{-1}.$$

These agree, within the experimental error, with the

[45] Le Conte Cathey, Phys. Rev. **87**, 169 (1952).
[46] Keuffel, Harrison, Godfrey, and Reynolds, Phys. Rev. **87**, 942 (1952).
[47] J. M. Kennedy, Phys. Rev. **87**, 953 (1952).
[48] Conversi, Pancini, and Piccioni, Phys. Rev. **71**, 209 (1947).
[49] C. W. Kissinger and D. Cooper, Phys. Rev. **74**, 349 (1948).
[50] G. E. Valley, Phys. Rev. **73**, 1251 (1948).
[51] N. Nereson, Phys. Rev. **73**, 569 (1948).
[52] E. P. Hincks and W. E. Bell, Phys. Rev. **88**, 168 (1952); **88**, 1424 (1952).

[53] In calculating their lifetimes, Hincks and Bell *assume* that the μ^+/μ^- ratio is 1.24 at 500 Mev/c at sea level in order to find the fraction of decays in each absorber which are due to μ^+ mesons. (As is shown by Fig. 6 of the present paper, $\mu^+/\mu^-\simeq 1.1$ at 500 Mev/c at sea level.) Furthermore, they choose 2.22μsec as the μ^+ meson lifetime, and this may not be the correct lifetime of the μ^+ in materials of low Z (see Table II). Both effects combine to give τ^- lifetimes which are of the proper order of magnitude, but result in capture probabilities which are too large.
[54] A. M. Conforto and R. D. Sard, Phys. Rev. **86**, 465 (1952).
[55] Several authors [see reference 24 and P. Budini, Nuovo cimento **9**, 445 (1952); N. Dallaporta, Nuovo cimento **9**, 450 (1952)] have suggested that the nucleus may emit an energetic charged particle directly after capture of a μ^- meson. In this case the fraction (f) of μ^- mesons which decay would not be equal to τ^-/τ^+.
[56] J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 144 (1949).

capture probability of μ^- mesons in oxygen[24] but not with that of μ^- mesons in calcium[54] (see Table III).

Recently, "shell structure" effects have also been found for the capture of π^- mesons in oxygen,[57] although the much higher energy transfers which occur in this case should greatly weaken the selection rules.[58]

V. μ^+/μ^- RATIO

Since only positive mesons decay in sulfur for delay times $>2\mu$sec, and both positive and negative mesons decay in carbon (see Sec IV), the μ^+/μ^- ratio may be obtained by extrapolating the time distributions of the decay electrons in the two absorbers to zero delay time. The lifetimes of the μ mesons affect the calculated μ^+/μ^- ratio in two ways: first, the fraction of μ^- mesons which decay in carbon is given by the ratio τ_c^-/τ_s^+, and secondly, the value of the delayed coincidence counting rates, extrapolated to zero delay is dependent upon both τ_c^- and τ_s^+. By utilizing the lifetimes and delayed coincidence counting rates obtained in the present experiment we have computed the μ^+/μ^- ratio for mesons of 325 ± 70 Mev/c momentum at sea level,[59]

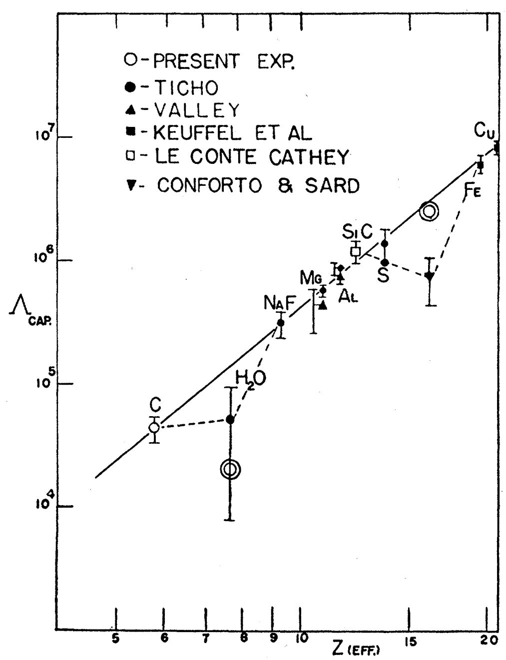

FIG. 4. Capture probabilities *vs* Z_{eff} of Wheeler theory. Solid line represents Z_{eff} law. Dotted line shows experimental deviations near closed shell nuclei. Double circles represent calculations from shell model of nucleus for charge-exchange type of interaction with a coupling constant of $g\sim3\times10^{-49}$ erg cm^3. The lower point at sulfur results from neglecting a correction originally given by Ticho (see reference 32).

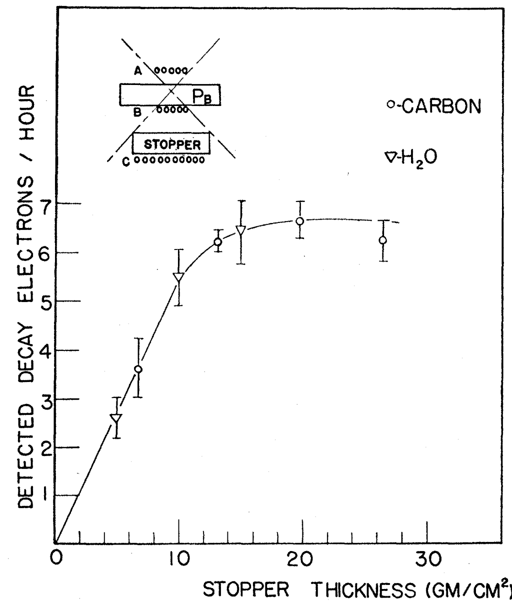

FIG. 5. Delayed coincidence counting rate as a function of absorber thickness. The curve saturates at $\sim$16 g/cm^2 indicating that the top portion of the absorbers in the main experiment are not useful in producing detectable decay electrons. Thus, the effective thickness is determined by the range of the decay electrons in each absorber.

obtaining

$$\mu^+/\mu^-=1.06\pm0.03. \quad (1)$$

The error quoted above is the pure statistical error alone and does not include any estimate of the systematic error, which will be discussed in the next section.

VI. SYSTEMATIC ERROR

A possible source of systematic error in the present experiment lies in the implicit assumption that the relative efficiencies of the two absorbers for stopping μ mesons and allowing their decay electrons to be detected is the same. A 10 percent change in the ratio of the relative efficiencies (E) causes a 16 percent shift in the μ^+/μ^- ratio and a 7 percent change in τ_c^-.

This ratio (E) has been calculated for the geometry of the present experiment by a semi-empirical method. It can be shown that (E) depends only on the product of the ratio of the number of mesons stopped in each absorber ($N_c/N_s=0.89\pm0.02$), and the ratio of the ranges of the decay electrons. In each case the thickness of absorber which was effective for stopping the detected μ mesons (see Fig. 5) was determined by the maximum range of the decay electrons. These ranges were obtained from the theory recently proposed by Wilson[60] and include corrections for radiation losses and multiple scattering. Under the assumption that the

[57] Camac, McGuire, Platt, and Schulte, Phys. Rev. **88**, 134 (1952).

[58] A. M. Messiah and R. E. Marshak, Phys. Rev. **88**, 678 (1952).

[59] The momentum of the observed μ mesons is determined by the 10-cm thick lead filter, the thickness of the roof and counter walls (1 g/cm^2), and the *effective* thickness of the absorber (see Sec. VI). The filter is increased by the cos^2 intensity distribution of the incident mesons and by multiple Coulomb scattering in the lead [H. Koenig, Phys. Rev. **69**, 590 (1946)].

[60] R. R. Wilson, Phys. Rev. **84**, 100 (1951).

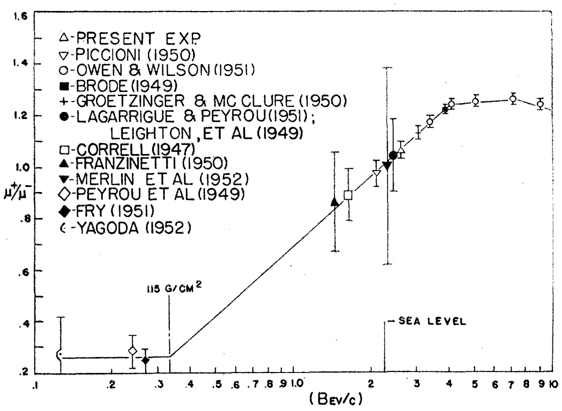

FIG. 6. μ^+/μ^- ratios *vs* momenta at the top of the atmosphere (TOA) for the selected experiments. Those points labeled (π) are obtained from π mesons stopped in photographic emulsions exposed above 60 000 ft.

stopped mesons are evenly distributed throughout each absorber[61] one finds that

$$E = 1.01 \pm 0.10.$$

The estimated uncertainty in the computed ratio (E) is due in part to the uncertainty in the theoretical expressions for the average electron ranges, which are only good to about 5 percent. Such phenomena as backscattering, etc., were neglected. The geometrical correction for edge effects (which is included) is <2 percent.

An experimental value for the ratio of relative efficiencies of carbon and sulfur has been obtained by Piccioni[13] for the delayed coincidences from μ^+ mesons observed in hard showers. When his data are corrected for spurious showers actually produced by knock-on electrons accompanying incident μ mesons, the ratio of the efficiencies becomes

$$(E) = \frac{\mu^+ \text{ delayed coincidences in carbon}}{\mu^+ \text{ delayed coincidences in sulfur}} = 1.08 \pm 0.10.$$

Another approach to this question can be obtained from the sulfur data alone in the present experiment. The data for decay times between 1.1 and 2.17μsec can be corrected for μ^+ mesons in this channel by extrapolating from the following channels. This procedure yields only one point on the decay curve for μ^- mesons in sulfur, but has the advantage that no questions of relative efficiencies are involved. If one assumes that $\tau_s^- = 0.66 \pm 0.05\mu$sec (see reference 32), then one obtains

$$\mu^+/\mu^- = 0.94 \pm 0.20.$$

When this is compared to Eq. (1) it implies

$$E = 0.96 \pm 0.15.$$

These three estimates indicate that (E) is approximately unity, but do not permit an accurate evaluation of this quantity. It is for this reason that no corrections for this effect are included in the μ^+/μ^- ratio [Eq. (1)]. While a more accurate evaluation of the present data awaits a better determination of (E), it will be shown in the following section that the value obtained under the assumption that $E=1$ is in good agreement with the results obtained by other methods.

VII. COMPARISON WITH OTHER EXPERIMENTS

A. General

An attempt has been made to resolve the ambiguities concerning the variation of the μ^+/μ^- ratio as a function of meson momentum and as a function of the altitude at which the ratio is determined. It is possible to correlate both variations in a consistent way by studying the μ^+/μ^- ratio as a function only of meson momentum (P) referred to the top of the atmosphere (TOA).[62] Experiments *selected on the basis that they provide good identification of the μ mesons* have been compared in this manner; Fig. 6 shows the result of this compilation. The "selected" experiments can be fitted quite well by an expression of the form

$$P = P_0 \exp\left[\frac{\mu^+/\mu^-}{K}\right], \qquad (2)$$

where $P<4$ Bev/c (TOA), $P_0 = 0.165$ Bev/c (TOA), and $K = 0.38$.

The fact that the experimental data seem to be only a function of momenta (TOA) implies that there is no appreciable meson production in the lower atmosphere as has been suggested by recent experiments.[10,63] However, it will be necessary to obtain data at different altitudes which overlap in momentum (TOA) before this can be confirmed. Furthermore, although *the decrease of the μ^+/μ^- ratio with increasing altitude appears to be well established*, the exact values of both P_0 and K in Eq. (2) are uncertain in the region below 2 Bev/c (TOA) because of the large statistical errors in the data obtained at altitudes above sea level.

[61] Both the main experiment and the auxiliary experiment (Fig. 5) are performed in an approximately "flat" region of the sea-level momentum spectrum. This means that the stopped mesons are to a first approximation distributed evenly throughout the absorber. The indication of a dip at 26 g/cm² may be caused by a characteristic anomaly in the spectrum at 400 Mev/c [see A. Rogozinski and A. G. Voisin, Compt. rend. **230**, 2092 (1950); L. Eisen, Masters thesis, New York University, 1952 (unpublished); E. W. Kellerman and K. Westerman, Proc. Phys. Soc. (London) **A62**, 356 (1949)].

[62] In the discussion that follows, all momenta labeled (TOA) are given at the top of the atmosphere. We use the NACA standard atmosphere [see W. G. Brombacher, National Advisory Committee for Aeronautics Report NACA-538, 1935 (unpublished)] which approximates the yearly average of altitude as a function of pressure for latitude 40°N in the United States. All range (altitude in g/cm²) to momentum conversions are made by means of the Princeton tables [E. P. Gross, *Range-Energy-Ionization Curves* (Princeton University, Princeton, 1947)]. These are based mainly upon the theoretical work of G. C. Wick [Nuovo cimento **1**, 310 (1943).], which includes polarization effects.

[63] W. L. Kraushaar, Phys. Rev. **76**, 1056 (1949).

Nevertheless, it is instructive to sketch in the altitude variation of the μ^+/μ^- ratio by extrapolating Eq. (2) to high altitudes where it can be compared to the π^+/π^- ratio obtained in the production region. In this region the π^+/π^- ratio appears to have a constant value of ~ 0.26 (see Table IV).[17,21,23,64] Since μ mesons are the daughters of π mesons, then this ratio could be considered to be that of low-energy μ mesons at production. Therefore, if one assumes that the momentum dependence of the μ^+/μ^- ratio [as given by Eq. (2)] is the same as that of mesons at production, the curve may be extrapolated to $\mu^+/\mu^- = 0.26$ to obtain a value for the average atmospheric depth of the production layer. This value is ~ 115 g/cm^2, in agreement with most conventional estimates.[26]

From available cloud-chamber data,[8–10] the altitude dependence of the μ^+/μ^- ratio deduced above allows the proton component at various altitudes to be computed. Such spectra show that a much larger fraction of protons exists at low momenta than most previous estimates infer.[65]

B. Discussion of the Selected Experiments

Owen and Wilson[5] using an air-gap magnetic spectrograph have investigated the μ^+/μ^- ratio for six momenta at sea level. These ratios were carefully corrected for included protons by an auxiliary absorption experiment[66] performed with the same apparatus. The apparatus provided good discrimination against electrons by shower production in a 2-cm lead plate. The results were not subject to multiple scattering corrections because no iron was in the path of the mesons. This series of measurements provides the most reliable determination to date of the μ^+/μ^- ratio at sea level in the momentum region above 3.25 Bev/c (TOA).

A magnetic lens (iron) coincidence telescope in which the counters were vertically out-of-line was developed by Brode.[67] This design was free of the large corrections which had characterized previous "magnetic lens" experiments. Brode's apparatus counted both positive and negative particles simultaneously, thus reducing the systematic errors associated with variations in the incident intensity. Geometrical effects were eliminated by alternately reversing the magnetic field and inverting the telescope during the course of the measurements, so that each channel in turn counted positive and then negative mesons. The effect of side showers was eliminated by using counters in adjacent channels as anticoincidence protection. Protons did not contribute to the measured ratio, because protons of the energy required to penetrate 60 cm of iron do not exist in appreciable numbers at sea level.[66]

[64] C. F. Powell, *Colston Papers* (Interscience Publishers, Inc., New York, 1949), p. 83.
[65] G. Puppi and N. Dallaporta, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952), Chap. VI, p. 360.
[66] M. G. Mylroi and J. G. Wilson, Proc. Phys. Soc. (London) **A64**, 404 (1951).
[67] R. B. Brode, Nuovo cimento **6**, Supplement No. 3 465 (1949).

TABLE IV. π^+/π^- ratios for π mesons above 60 000 feet.

Author	Method	Altitude	π^+/π^-
Yagoda[a] (1952)	π mesons (produced in air by primaries) stop in emulsion (Ilford *G*-5); corrected for starless π mesons.	$\sim$14 g/cm^2	0.69 $\pm$0.04[b]
Yagoda[a] (1952)	π mesons (produced in gelatin (H, C, N, O) of emulsion by primaries) stop in emulsion (Ilford *G*-5); corrected for starless π mesons.	$\sim$14 g/cm^2	0.276 $\pm$0.151 ($0 < E < 6$ Mev)
Powell[c] (1949)	π mesons (produced in gelatin (H, C, O, N) and AgBr of emulsion stop in emulsion; uncorrected for geometry; corrected for starless π mesons.	$\sim$27 g/cm^2	0.023 $\pm$0.023[d]
Peyrou *et al.*[e] (1949)	π mesons (produced in air by primaries) stop in emulsion; corrected for starless π mesons.	$\sim$58 g/cm^2	0.28 $\pm$0.064
Fry[f] (1951)	π mesons (produced in air by primaries) stop in emulsion; (Kodak *NTB*-3); corrected for starless π mesons.	$\sim$73 g/cm^2	0.25 $\pm$0.045

[a] See reference 23.
[b] Since $\pi^+ \rightarrow \mu^+$ decays are more easily recognized than one prong σ stars, π^+ mesons will have a higher detection efficiency than π^- mesons if $\sim$100 percent scanning is not employed in the search for characteristic endings. Yagoda estimates that the over-all detection efficiency is $<$80 percent in the thick emulsions employed in determining this ratio. Note that this ratio ($\pi^+/\pi^- = 0.69$) can be compared with the ratio for mesons produced internally in the gelatin (H, C, O, N) of the emulsion. Those internally produced mesons for which an identical method of scanning was employed, exhibit a ratio of 0.73, while those which were traced from their parent stars give $\pi^+/\pi^- = 0.28$. This latter group was presumably scanned with almost 100 percent efficiency. However, J. Y. Mei and E. Pickup [Can. J. Phys. **30**, 430 (1952)] assume that all of the π mesons are generated locally (including those which are apparently externally incident). The change in the ratio with meson energy is then attributed to a Coulomb effect.
[c] See reference 64.
[d] This ratio would be higher for stars produced in the gelatin of the emulsion alone, and would be increased still further if the π mesons were corrected for those which escape detection because of geometry. This latter correction becomes important for thin emulsions such as were employed here.
[e] See reference 21.
[f] See reference 17.

Brode's published data contain a further correction based on the counting rates determined by reversing the lower half of the magnet with respect to the upper half; these rates were ascribed to field-insensitive particles and were subtracted from the average values of the normal rates before the μ^+/μ^- ratio was computed. Owen and Wilson[5] have pointed out that this represents a severe overcorrection since the probability that scattering will simultaneously reverse the particle trajectory in the upper and lower halves is negligible. We have therefore recalculated the μ^+/μ^- ratio from Brode's data, neglecting the reversed half-magnet correction.

Although this was a coincidence measurement, the momentum resolution of the apparatus was such that the bulk of the μ mesons which contribute to the measured ratio had momenta between 3.25 and 5.75 Bev/c (TOA). Since the momentum interval is not defined precisely, an average momentum is used in plotting this point. This experiment has low statistical error and good identification.

Groetzinger and McClure,[68] using a telescope similar in principle to that developed by Brode, measured the

[68] G. Groetzinger and G. W. McClure, Phys. Rev. **77**, 777 (1950).

μ^+/μ^- ratio at $\sim$0.8 Bev/c at sea level and at 4 km. They determined the μ^+/μ^- ratio at two zenith angles in both the east and west azimuths. Using their raw data we have averaged[69] over both angles and azimuths and computed a single μ^+/μ^- ratio for each altitude. Their sea-level measurement contains no protons because the 40 cm of iron in the lens effectively attenuates those protons of $\sim$0.8 Bev/c momenta which were present in the incident beam. Unfortunately this is not true at 4 km where protons constitute $\sim$28 percent of the total incident beam at this momentum or $\sim$18 percent of the positive particle beam capable of causing a coincidence. As a result this determination at 4 km indicates 20 percent more positive particles than do the selected experiments (see following section).

Leighton *et al.*[70] have observed 75 $\mu\rightarrow\beta$ decays in a cloud chamber with a magnetic field at sea level. Of these events, 12 occurred in an absorber (a mixture of paraffin and copper) whose atomic number (Z) could not be stated precisely, and one occurred in brass. (In the discussion that follows these 13 events are not considered.) The remaining decays occurred in carbon and Bakelite (54 events) or in glass and argon (8 events). Lagarrigue and Peyrou[71] have observed 150 $\mu\rightarrow\beta$ decays under 20 cm of lead at sea level by means of a cloud chamber and magnetic field. All of their decays occurred either in carbon or glass; but the majority took place in carbon.

The data of the two experiments were combined under the assumption that all of the decays occurred in carbon. Thus, allowing for those μ^- mesons which would be captured in "carbon," we compute the μ^+/μ^- ratio to be

$$\mu^+/\mu^-=1.04\pm0.14 \quad \text{at} \quad \sim2.5 \text{ Bev}/c \text{ (TOA)}.$$

This ratio is based on excellent identification, but only indicates the trend at low momenta at sea level because of the assumption involved and the poor statistical accuracy.

A cloud chamber in a magnetic field with an absorber placed below the chamber was used by Correll[15] to determine the μ^+/μ^- ratio for mesons of 125 to 250 Mev/c momenta at 3.5 km. Electrons were eliminated by shower production in three 1-cm lead plates (two were inside the chamber); protons were eliminated by range-momenta criteria. In this momentum region confusion with mesons is extremely unlikely. Although this measurement has a statistical precision of only 11 percent, the identification is good.

Franzinetti[12] exposed vertical photographic plates in a 30-kilogauss magnetic field at 11 000 ft and identified the particles which stopped in the plates both by their mass (momentum and residual range) and by their characteristic endings. Since he was primarily interested in a mass spectrum of the cosmic radiation at these altitudes, he included in his μ^+/μ^- ratio eight μ^+ mesons resulting from $\pi^+\rightarrow\mu^+$ decay in the plates. In using his results we have omitted these eight locally-produced μ^+ mesons. A histogram of the entrance angles of the stopped μ mesons shows that they were deflected by the magnetic field before entering the plates; the angles of incidence are consistent with the hypothesis that these mesons were not locally produced. Recently, Merlin *et al.*[72] have essentially repeated Franzinetti's experiment at sea level. The statistical errors of both experiments were large, but the identification was exceptionally good.

Piccioni[13] has performed a delayed coincidence experiment at 3.5 km under 19.5 in. of lead. Carbon and sulfur absorbers were used in a manner similar to that employed in the present experiment. Most locally produced μ^+ mesons were eliminated by not counting those events in which more than a single counter was discharged in the first tray of his telescope. (This precaution is necessary at mountain altitudes.) Since his experimental arrangement and the present one are similar, the same considerations employed in reducing the present data apply.

We have recomputed[73] from Piccioni's raw data the μ^+/μ^- ratio for those delayed coincidences unaccompanied by hard shower events and find that

$$\mu^+/\mu^-=0.97\pm0.05 \quad \text{at} \quad 2.09 \text{ Bev}/c \text{ (TOA)}.$$

Because of the possibility that locally generated hard showers can contribute extra μ^+ mesons to the data, this ratio must be considered only as an upper limit.

There are two other experiments which tend to point up the various effects mentioned above, although they do not do this directly. In one of these experiments, Ticho[74] has measured the composite lifetime of the natural mixture of cosmic-ray mesons in aluminum both at sea level and at 3.5 km. Since the μ^- mesons decay with a considerably shortened lifetime in aluminum, a decrease in composite lifetime with altitude is consistent with a decrease in the μ^+/μ^- ratio with altitude, although the precision of this experiment does not permit definite conclusions to be drawn. In the second, Brode has reported a continuation of his sea-level magnetic lens experiments at 3.5 km altitude.[75] Although the reported ratios are diluted by an overcorrection (see the discussion above) they show that the ratio decreases with altitude, and that the altitude variation can be represented as a momentum variation, since the addi-

[69] The ratios determined at the two zenith angles were not statistically different.

[70] Leighton, Anderson, and Seriff, Phys. Rev. **75**, 1432 (1949).

[71] A. Lagarrigue and C. Peyrou, Compt. rend. **233**, 478 (1951).

[72] Merlin, Vitale, and Goldschmidt-Clermont, Nuovo cimento **9**, 422 (1952).

[73] Piccioni assumed that the mean lifetime of μ^+ mesons in sulfur as well as the composite lifetime of μ mesons in carbon was 2.2μsec. The carbon/sulfur ratio thus obtained corresponds to $\mu^+/\mu^-=1.2$. This is actually the ratio for delay times greater than 1.3μsec and only corresponds to the zero-time extrapolated value provided that the lifetimes are the same.

[74] H. K. Ticho, Phys. Rev. **71**, 463 (1947).

[75] R. B. Brode, Phys. Rev. **78**, 92 (1950).

tion of 45 cm of lead[76] at altitude results in the same ratio obtained at sea level.

C. Other Experiments

The experiments which were not selected for this compilation fall into two categories; first, those which cover the same momentum region as the "selected" group, but which have poor statistical precision, and secondly, those in which the μ mesons are not differentiated completely from other particles in the incident cosmic radiation. In the latter group are included most of the cloud-chamber and magnetic lens experiments performed at altitude. This follows from the large increase in the proton component (relative to the meson component) with altitude, and is considered in detail in the following discussion.

Caro *et al.*[7] using an air-gap magnetic spectrograph have investigated the variation of the μ^+/μ^- ratio as a function of momentum at sea level. The statistical precision of this experiment is not as good as that of Owen and Wilson.[5] Furthermore, no correction was made for included protons. Nevertheless, the results generally indicate the same trend as the "selected experiments" at momenta below 4 Bev/c (TOA).

An air-gap magnetic spectrometer has been used by Bassi *et al.*[6] to investigate three points in the sea-level momentum spectrum below 4 Bev/c (TOA). They do not correct for included protons although the anticoincidence method of event selection which they used is very sensitive to protons. Their ratios (corrected by the sea-level proton spectrum of Mylroi and Wilson[66]), agree with the "selected" experiments, but the statistical precision is much poorer than that obtained in the experiment of Owen and Wilson.[5]

Bassi *et al.*[77] have performed a measurement at 2 km using the same apparatus as that used in the above experiment. The method of selection in this case was changed to a coincidence system in an attempt to eliminate protons. The two points which were obtained at this altitude contain ~20 percent more "positive mesons" than do the "selected" experiments. Their lowest momentum point agrees with the ratio obtained at the same momentum (TOA) by Groetzinger and McClure,[68] which experiment, according to the discussion in Sec. VII (B) was contaminated by ~18 percent protons.

An experiment which has been widely reported with other sea-level results is that of Nereson.[51] However, his experiment was performed in the basement of a building (~18 in. of concrete)[78] *at an altitude of 5000 ft.* An out-of-line magnetic lens telescope was used to observe the positive/negative ratio for those particles which stopped in $\frac{1}{2}$ in. of lead after traversing 28 in. of iron. Since this ratio is 10 percent higher than that obtained in plain coincidence at the same time, then either μ^+/μ^- decreases with increasing momentum or the *protons must have made a considerable contribution to the measured ratios.*

Background runs in which the iron lens was not magnetized were reported but were not used in calculating the ratios (both coincidence and anticoincidence). Thus, no corrections for multiple scattering are included. If the appropriate backgrounds are subtracted one finds

Coincidence: plus/minus $= 1.38 \pm 0.03$ at 3.1 Bev/c (TOA).

Anticoincidence: plus/minus $= 1.48 \pm 0.03$ at 2.9 Bev/c (TOA).

These coincidence values computed above compare favorably with the altitude experiments of Groetzinger and McClure[68] (4 km) and Bassi *et al.*[77] (2 km), but do not agree with the "selected" experiments. It is apparent that these experiments (magnetic lenses in out-of-line coincidence) were contaminated by protons.

Barbour[18] has reported a series of stratosphere experiments with vertical photographic plates in the field of a permanent magnet. The identification of the particles observed in this series of experiments should presumably be as good as the results obtained in Franzinetti's experiment;[12] and these ratios should be comparable to the ones in Table IV; however, the magnet in this case was suspended in such a manner that 69 kg of iron was available for the local production of mesons directly above the plates. Consequently, the observed π^+/π^- and μ^+/μ^- ratios bear little relation to the ratios existing in the free atmosphere at those altitudes. In fact, Barbour was able to find a production mean-free-path for π mesons from the separate experiments of this series performed at different altitudes.

Conversi[16] has described an iron-core magnetic lens experiment which he and Nappo performed at sea level in Rome. The counter system used in this experiment defined those particles which, after deflection in 28 cm of iron, stopped in 7 cm of lead. No correction was made for included protons, and sufficient details are not available to enable one to evaluate other systematic errors which might be involved.

A series of measurements similar in principle to that of the present experiment has also been reported by Conversi.[16] These measurements, which cover a wide range of momenta at sea level and several altitudes, were not absolute ratios but were normalized to the magnetic lens experiment reported above. No correction was made for those μ^- mesons which decay in carbon, and all data were extrapolated to zero delay time with an assumed lifetime of 2.2μsec. The ratios were renormalized to the "selected experiments" at one point, and in general (except the 9-km point) exhibit the same momentum variation as the "selected" experiments, within the large statistical errors (>20 percent).

[76] The lead equivalent of the atmosphere between 3.6 km and sea level.

[77] Bassi, Filosofo, Manduchi, and Prinzi, Nuovo cimento 8, 469 (1951).

[78] V. H. Regener (private communication).

Nonnemaker and Street[79] have used a cloud chamber at sea level to study the momenta of singly occurring particles which traversed 140 g/cm^2 and stopped in an additional 88 g/cm^2 below the chamber. This apparatus is somewhat similar to that of Correll's,[11] but no "shower plates" were provided in the chamber. The momentum cutoff is that of μ mesons; however single electrons could not be distinguished from μ^- mesons over most of the region of momentum considered.

VIII. COMPARISON WITH THEORY

Interpretation of the well-known 20 percent excess of positive mesons observed at sea level is generally based[80] on the model of a primary radiation consisting exclusively of protons which produce charged mesons upon colliding with air nuclei. Charge conservation then leads to the generation of more positive than negative mesons. That the simultaneous production in the primary collision of charged nucleons may upset the charge balance is not usually considered. Thus, it is not surprising that this simple picture is not entirely compatible with the results reported in the present paper. In fact, no theory of meson production is successful in accounting for the variation of the μ^+/μ^- ratio at momenta below 4 Bev/c (TOA).

One theory, based on plural production of mesons[81] has been proposed recently by Caldirola *et al.*[82] The theoretical variation of the μ^+/μ^- ratio at sea level does not fit the experimental points too well below 4 Bev/c (TOA). This theory also predicts that the μ^+/μ^- ratio should increase with altitude, which is distinctly at variance with the present results.

Other models, based upon multiple-production theories[83,84] give distributions of the ratio which vary with the inverse square root [85–87] of the meson energy. The poor agreement between the experimental results and these theories probably lies in the fact that they are applicable only at sufficiently high energies.

Puppi and Dallaporta[88] have attempted to construct a phenomenological theory to describe the momentum dependence of the ratio at sea level. They assume that the positive excess of mesons generated by the primaries is either uniformly distributed, or a slowly varying function of meson momentum. According to their picture, the initial excess is diluted by mesons produced throughout the atmosphere by secondary nucleons. These secondaries consist of approximately equal numbers of protons and neutrons, and, as a consequence the second generation of mesons shows no excess. By using Sands'[26] data for the production of slow mesons as a function of altitude and choosing 55 g/cm^2 as the mean free path for inelastic collisions of the nucleonic component, they obtain a rough fit to the experimental points at sea level. They conclude that better agreement would be obtained if they considered the small excess of neutrons in collisions after the first; this would diminish slightly the computed values at low energies and bring them more into agreement with the experimental ratios. Despite the good agreement at sea level, this theory must be treated with reserve since the dilution parameter would require the positive/negative ratio to increase with altitude or at least remain relatively constant.

The preponderance of negative mesons in the region below 2.25 Bev/c (TOA) can be attributed to the action of the incident neutrons contained in the heavy particle primaries.[89] (These neutrons constitute $\sim$25 percent of the *incident nucleons* and are predominantly of low energy.) It is difficult to obtain more than a qualitative explanation of the observed momentum dependence of the μ^+/μ^- ratio since little is known about the relevant cross sections and modes of interaction, even at those values of momenta which are accessible to present day accelerators.[90]

IX. ACKNOWLEDGMENTS

It is a pleasure to thank Professor L. Spruch and Mr. A. Fafarman for many helpful discussions.

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[87] Uri Haber-Schaim, Phys. Rev. **84**, 1199 (1951).
[88] See reference 65, p. 344.
[89] B. Peters, *Progress in Cosmic Ray Physics* (Interscience Publishers, Inc., New York, 1952), Vol. I, Chap. IV.
[90] E. M. Henley, Phys. Rev. **85**, 204 (1952); H. W. Wilson and W. H. Barkas, Phys. Rev. **89**, 758 (1953).