

and the first five terms retained [this corresponds to retaining powers through  $y^6$  in the expansion of  $P(y)$ ], Eq. (3) becomes

$$(d^2F/dz^2) + [(3/16z^2) + (\lambda_1/z) + (\lambda_2/z^3) + \lambda_0]F = 0, \quad (5)$$

where

$$\begin{aligned} \lambda_0 &= \{a^2 + (a^2\epsilon/2c_2)[- (3c_4/2c_2) + (103c_3^2/72c_2^2)] \\ &\quad + (3/2c_2)[- (5c_6/3c_2) + (5c_3c_6/2c_2^2) + (109c_4^2/96c_2^2) \\ &\quad - (1723c_3^2c_4/576c_2^3) + (12061c_3^4/13824c_2^4)]\}, \\ \lambda_1 &= (1/2c_2^3)\{a^2\epsilon - (1/8)[(3c_4/c_2) - (7c_3^2/4c_2^2)]\}, \\ \lambda_2 &= (2^3/c_2^3)\{- (a^2\epsilon c_3/3c_2) + (3/10)[- (2c_6/c_2) \\ &\quad + (8c_3c_4/3c_2^2) - (28c_3^3/27c_2^3)]\}. \end{aligned}$$

The independent solutions  $F_{1,2}$  of (5) are expressible in terms of the parabolic cylinder functions:<sup>3</sup>

$$F_1 = z^{\frac{1}{2}} D_n(\xi), \quad F_2 = z^{\frac{1}{2}} D_{-n-1}(i\xi), \quad (6)$$

where

$$n = -(1/2) + (i/\lambda_0^{\frac{1}{2}})[\lambda_1 - (\lambda_2^2/4\lambda_0)],$$

$$\xi = 2\lambda_0^{\frac{1}{2}}[z^{\frac{1}{2}} + (\lambda_2/2\lambda_0)] \exp(-i\pi/4).$$

Consequently, the functions

$$\begin{aligned} \psi_1 &= P^{-\frac{1}{2}} z^{\frac{1}{2}} D_n(\xi), \\ \psi_2 &= P^{-\frac{1}{2}} z^{\frac{1}{2}} D_{-n-1}(i\xi) \end{aligned} \quad (7)$$

are very good approximations to the solutions of the wave equation near  $y=0$ . These solutions have the advantage of being one-valued. For bound states, the parabolic cylinder function is expressible in terms of a Hermite polynomial.

The solution (7) has numerous applications. For example, a solution valid near the minimum of a potential trough yields an accurate evaluation of the energies of the low-lying levels of the potential. Also, a solution valid near the maximum of a potential barrier determines accurately the transmission coefficient of particles having energies nearly equal to the height of the barrier. The simple asymptotic form of the single-valued parabolic cylinder function readily yields the "connection" between the asymptotic solutions of (1).

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<sup>1</sup> S. Goldstein, Proc. London Math. Soc. **33**, 246 (1932); R. E. Langer, Trans. Am. Math. Soc. **36**, 90 (1934); E. Guth and C. J. Mullin, Phys. Rev. **59**, 575 (1941); S. C. Miller and R. H. Good, Phys. Rev. **91**, 174 (1953).

<sup>2</sup> The terminology and the transformations used here are similar to those employed by I. Imai [Phys. Rev. **74**, 113 (1948) and Phys. Rev. **80**, 1112 (1950)] in obtaining an accurate solution to the wave equation near a linear turning point.

<sup>3</sup> E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, London, 1940), fourth edition, p. 347.

## The Long-Wavelength Edge of Photographic Sensitivity and of the Electronic Absorption of Solids\*

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IN an extensive investigation of the sensitivity spectrum of various types of photographic emulsions, it was noticed that, for the simplest unsensitized emulsions, the plot of the logarithm of sensitivity versus frequency approaches a straight line, as shown in Fig. 1.<sup>1</sup> For pure silver bromide emulsions near room temperature, the slope of this line was found to be very near to  $1/kT$ . If this behavior is more than a mere coincidence, it should be expected to occur over a reasonable temperature range. In the wavelength range of low sensitivity, the absorption coefficient of the halide is expected to be proportional to the sensitivity. A set of measurements carried out by F. Moser, of these laboratories, of the absorption spectrum of silver bromide crystals in the region of the absorption edge showed that the relation  $d \log \epsilon / d\nu = -1/kT$  is closely approached for absorption coefficients  $\epsilon$  of the order of  $10^{-1} \text{ cm}^{-1}$  for temperatures ranging at least from 200°K to 620°K (Fig. 2). The absorption spectrum behaves in this range as if, by

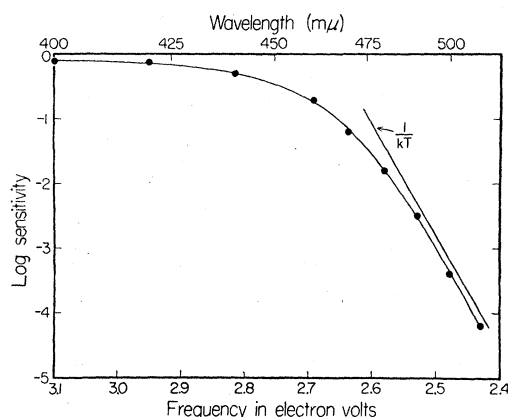


FIG. 1. Spectral sensitivity of an unfinished pure AgBr emulsion at 22°C.

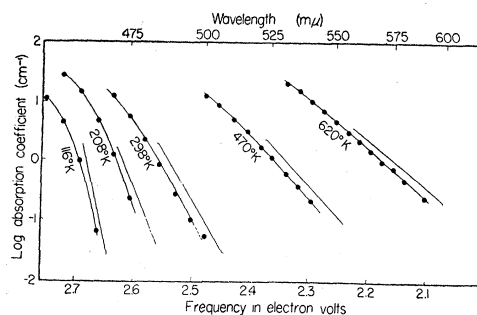


FIG. 2. Temperature dependence of absorption of an AgBr crystal. The straight lines are the  $1/kT$  slopes at the corresponding temperatures.

some mechanism not known in detail, the thermal energy were contributing directly to the transitions responsible for the absorption edge. An analysis of data from various sources on materials, such as  $\text{AgCl}$ ,<sup>2</sup>  $\text{Ge}$ ,<sup>3</sup>  $\text{TiO}_2$ ,<sup>4</sup> and  $\text{CdS}$ ,<sup>5</sup> indicates a somewhat similar behavior, with slopes within a factor of 2 of  $1/kT$ , in regions of sufficiently high temperatures and low absorption constants.

Attempts to provide a more precise formulation and a definite theoretical basis for this apparent regularity have not been successful. More detailed data and some theoretical considerations will be published elsewhere.

\* Communication No. 1620 from the Kodak Research Laboratories.

<sup>1</sup> See also the data of J. Eggert and M. Biltz, Trans. Faraday Soc. **34**, 892 (1938); J. Eggert and F. G. Kleinschrod, Z. wiss. Phot. **39**, 155 (1940), and of M. Biltz, J. Opt. Soc. Am. **39**, 994 (1949).

<sup>2</sup> Measurements in these laboratories and recent data by M. A. Gilleo, Phys. Rev. **91**, 534 (1953).

<sup>3</sup> M. Becker and H. Y. Fan, Phys. Rev. **76**, 1530 (1940).

<sup>4</sup> D. C. Cronmeyer, Phys. Rev. **87**, 876 (1952).

<sup>5</sup> C. C. Klick, Phys. Rev. **89**, 274 (1953).

## Nuclear Spin of $\text{Bi}^{210}$

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THE hyperfine structure of  $\lambda 3067$ , the resonance line of  $\text{Bi I}$ , has been investigated for ordinary bismuth and for  $\text{Bi}^{210}$  with a large grating spectrograph. Solutions of the samples were evaporated on the ends of  $\frac{1}{8}$ -in. diameter silver electrodes and were then excited in a dc arc. A cylindrical lens was mounted in front of the photographic plate to shorten the astigmatic line image, giving a strong exposure in the third order of the grating (0.61A/mm) with  $10^{-7}$  gram of bismuth. Under these conditions