and the first five terms retained [this corresponds to retaining powers through y^6 in the expansion of $P(y)$], Eq. (3) becomes

$$(d^2F/dz^2)+[(3/16z^2)+(\lambda_1/z)+(\lambda_2/z^{\frac{1}{2}})+\lambda_0]F=0, \quad (5)$$

where

$$\lambda_0=\{a^2+(a^2\epsilon/2c_2)[-(3c_4/2c_2)+(103c_3^2/72c_2^2)]+(3/2c_2)[-(5c_6/3c_2)+(5c_3c_5/2c_2^2)+(109c_4^2/96c_2^2)-(1723c_3^2c_4/576c_2^3)+(12061c_3^4/13824c_2^4)]\},$$

$$\lambda_1=(1/2c_2^{\frac{1}{2}})\{a^2\epsilon-(1/8)[(3c_4/c_2)-(7c_3^2/4c_2^2)]\},$$

$$\lambda_2=(2^{\frac{1}{2}}/c_2^{\frac{3}{2}})\{-(a^2\epsilon c_3/3c_2)+(3/10)[-(2c_5/c_2)+(8c_3c_4/3c_2^2)-(28c_3^3/27c_2^3)]\}.$$

The independent solutions $F_{1,2}$ of (5) are expressible in terms of the parabolic cylinder functions:[3]

$$F_1=z^{\frac{1}{2}}D_n(\xi), \quad F_2=z^{\frac{1}{2}}D_{-n-1}(i\xi), \quad (6)$$

where

$$n=-(1/2)+(i/\lambda_0^{\frac{1}{2}})[\lambda_1-(\lambda_2^2/4\lambda_0)],$$

$$\xi=2\lambda_0^{\frac{1}{2}}[z^{\frac{1}{2}}+(\lambda_2/2\lambda_0)]\exp(-i\pi/4).$$

Consequently, the functions

$$\psi_1=P^{-\frac{1}{2}}z^{\frac{1}{2}}D_n(\xi), \qquad \psi_2=P^{-\frac{1}{2}}z^{\frac{1}{2}}D_{-n-1}(i\xi) \quad (7)$$

are very good approximations to the solutions of the wave equation near $y=0$. These solutions have the advantage of being one-valued. For bound states, the parabolic cylinder function is expressible in terms of a Hermite polynomial.

The solution (7) has numerous applications. For example, a solution valid near the minimum of a potential trough yields an accurate evaluation of the energies of the low-lying levels of the potential. Also, a solution valid near the maximum of a potential barrier determines accurately the transmission coefficient of particles having energies nearly equal to the height of the barrier. The simple asymptotic form of the single-valued parabolic cylinder function readily yields the "connection" between the asymptotic solutions of (1).

* Supported in part by the U. S. Office of Naval Research.

[1] S. Goldstein, Proc. London Math. Soc. **33**, 246 (1932); R. E. Langer, Trans. Am. Math. Soc. **36**, 90 (1934); E. Guth and C. J. Mullin, Phys. Rev. **59**, 575 (1941); S. C. Miller and R. H. Good, Phys. Rev. **91**, 174 (1953).

[2] The terminology and the transformations used here are similar to those employed by I. Imai [Phys. Rev. **74**, 113 (1948) and Phys. Rev. **80**, 1112 (1950)] in obtaining an accurate solution to the wave equation near a linear turning point.

[3] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, London, 1940), fourth edition, p. 347.

The Long-Wavelength Edge of Photographic Sensitivity and of the Electronic Absorption of Solids*

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IN an extensive investigation of the sensitivity spectrum of various types of photographic emulsions, it was noticed that, for the simplest unsensitized emulsions, the plot of the logarithm of sensitivity *versus* frequency approaches a straight line, as shown in Fig. 1.[1] For pure silver bromide emulsions near room temperature, the slope of this line was found to be very near to $1/kT$. If this behavior is more than a mere coincidence, it should be expected to occur over a reasonable temperature range. In the wavelength range of low sensitivity, the absorption coefficient of the halide is expected to be proportional to the sensitivity. A set of measurements carried out by F. Moser, of these laboratories, of the absorption spectrum of silver bromide crystals in the region of the absorption edge showed that the relation $d\log\epsilon/d\nu=-1/kT$ is closely approached for absorption coefficients ϵ of the order of 10^{-1} cm^{-1} for temperatures ranging at least from 200°K to 620°K (Fig. 2). The absorption spectrum behaves in this range as if, by

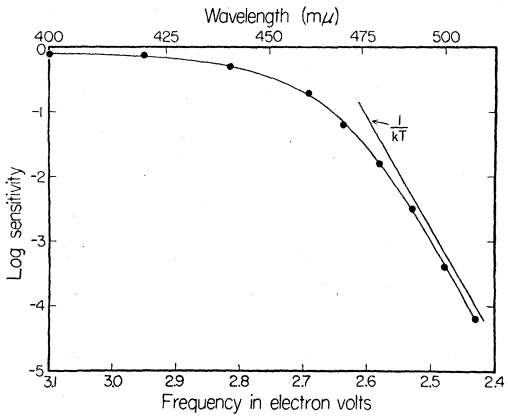

FIG. 1. Spectral sensitivity of an unfinished pure AgBr emulsion at 22°C.

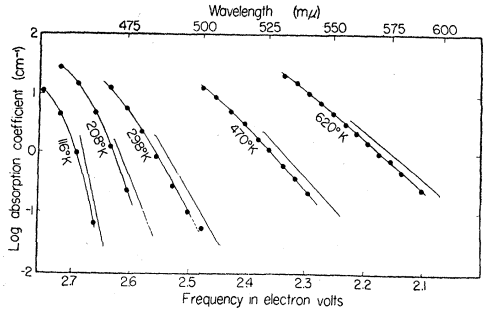

FIG. 2. Temperature dependence of absorption of an AgBr crystal. The straight lines are the $1/kT$ slopes at the corresponding temperatures.

some mechanism not known in detail, the thermal energy were contributing directly to the transitions responsible for the absorption edge. An analysis of data from various sources on materials, such as AgCl,[2] Ge,[3] TiO_2,[4] and CdS,[5] indicates a somewhat similar behavior, with slopes within a factor of 2 of $1/kT$, in regions of sufficiently high temperatures and low absorption constants.

Attempts to provide a more precise formulation and a definite theoretical basis for this apparent regularity have not been successful. More detailed data and some theoretical considerations will be published elsewhere.

* Communication No. 1620 from the Kodak Research Laboratories.

[1] See also the data of J. Eggert and M. Biltz, Trans. Faraday Soc. **34**, 892 (1938); J. Eggert and F. G. Kleinschrod, Z. wiss. Phot. **39**, 155 (1940), and of M. Biltz, J. Opt. Soc. Am. **39**, 994 (1949).

[2] Measurements in these laboratories and recent data by M. A. Gilleo, Phys. Rev. **91**, 534 (1953).

[3] M. Becker and H. Y. Fan, Phys. Rev. **76**, 1530 (1940).

[4] D. C. Cronemeyer, Phys. Rev. **87**, 876 (1952).

[5] C. C. Klick, Phys. Rev. **89**, 274 (1953).

Nuclear Spin of Bi^{210}

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THE hyperfine structure of λ3067, the resonance line of Bi I, has been investigated for ordinary bismuth and for Bi^{210} with a large grating spectrograph. Solutions of the samples were evaporated on the ends of $\frac{1}{16}$-in. diameter silver electrodes and were then excited in a dc arc. A cylindrical lens was mounted in front of the photographic plate to shorten the astigmatic line image, giving a strong exposure in the third order of the grating (0.61A/mm) with 10^{-7} gram of bismuth. Under these conditions

the Bi^{209} line appeared as a well-resolved doublet having a separation given by the splitting of the upper state, 0.828 cm^{-1}. The close components in each member of the doublet due to the much smaller ground-state splitting could not be resolved but produced a noticeable widening.

The Bi^{210} samples were obtained as the RaE (5-day half-life) in equilibrium with 13 millicuries of RaD. The Bi^{210} was isolated by the ion exchange method of Raby and Hyde,[1] which was found to give a 98-percent yield with negligible loss of Pb. A number of separations were made at intervals of several weeks. Considerable difficulty was experienced at first with Bi^{209} contamination, so that an efficient separation process was essential. The amount of RaE isolated in each separation was determined by standard β-counting procedures and was usually about 1.5×10^{-7} gram. For Bi^{210}, λ3067 was found to be a single sharp line. An upper limit to any hfs which might possibly be present is roughly 0.1 the width of the Bi^{209} line. Since such a small splitting would lead to an unreasonably small g_I value, it seems much more probable that no splitting is present and $I=0$. Hence the 127th neutron is almost certainly in a $g_{9/2}$ state.

The isotope shift was measured by superimposing an exposure of Cs λ4593 in the second order of the grating which served as a fiducial mark. The Bi^{210} line was shifted to shorter wavelengths by 0.12 cm^{-1}. Further exposures with an improved light source are planned in order to obtain more accurate and extensive shift measurements.

[1] B. A. Raby and E. K. Hyde, University of California Radiation Laboratory Report AECD-3524, 1953 (unpublished).

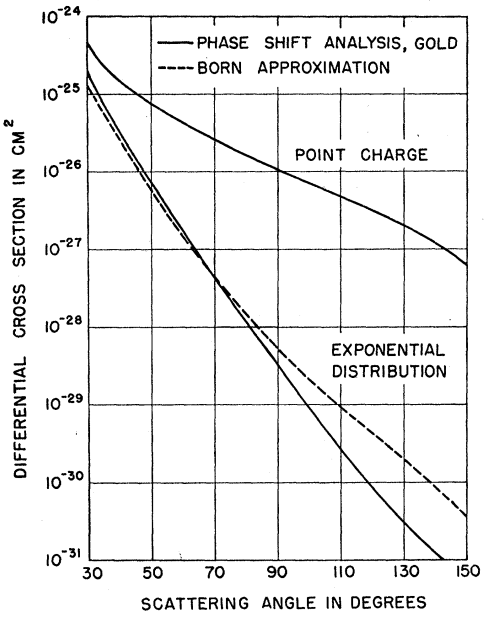

FIG. 2. Scattering from an exponential charge distribution, $ka=0.91$.

Phase-Shift Calculation of High-Energy Electron Scattering by Nuclei*

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EXPERIMENTS on the elastic scattering of high-energy electrons ($E\sim125$ Mev) by several elements have been carried out by Hofstadter, Fechter, and McIntyre.[1] The results have been analyzed by them[1] and by Schiff[2] using the Born approximation to estimate the effects of the finite nuclear size.

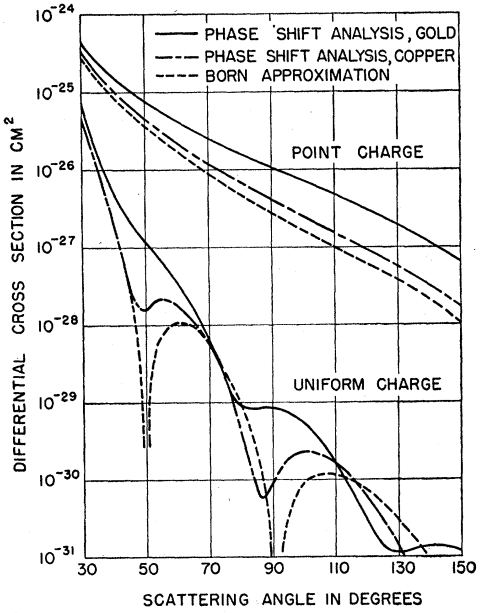

FIG. 1. Scattering from a uniform charge distribution, $kR=5.4$.

On the basis of such an analysis, the results can be fitted well by assuming an exponential distribution of the nuclear charge. Other charge distributions, such as the uniform distribution, are discriminated against because in Born approximation they predict diffraction maxima and minima in the angular distribution. In view of the fact that the Born approximation is not accurate for the heavier elements considered, we have carried out a numerical phase-shift calculation of the process. Our object in the present note is to examine the accuracy of the Born approximation, rather than to attempt to fit the experimental results. Our results are in disagreement with those of Parzen.[3]

The details of the calculation will be published later. Generally the calculation follows the methods given by Parzen[3] and Acheson,[4] with minor modifications. The phase shift relative to the Coulomb phase shift is calculated for the first nine partial waves; the last phase shift used is less than 0.003° in all cases. All the phase shifts are negative and tend monotonically to zero, in disagreement with Elton's assertion[5] that the phase shifts should tend to zero from positive values. We have found it necessary to increase the accuracy of the point scattering amplitudes given by Feshbach,[6] since at large angles this amplitude is largely canceled out by the nuclear size modifications. We estimate that the calculated cross sections given below are accurate to about 5 percent out to 110°; the major errors are contained in the first two phase shifts, which are known to only 0.01°.

The following charge distributions have been used for gold ($Z=79$):

(a) Uniform; $\rho(r)=\rho_0$, $r<R$,
$=0$, $r>R$; $kR=5.4$;

(b) Exponential; $\rho(r)=\rho_0 e^{-r/a}$; $ka=0.91$.

The cross sections given by (a) and (b) are found to decrease in approximately the same ratio between 30° and 90°, and this ratio is roughly that given by Hofstadter's experiment.

For purposes of comparison, we carried through the calculation for copper ($Z=29$) for the uniform charge distribution, with $kR=5.4$. The Born approximation to this cross section is the same, apart from a constant factor $(Z_{\text{copper}}k_{\text{gold}}/Z_{\text{gold}}k_{\text{copper}})^2$, as the Born approximation to (a).

In Fig. 1 are shown the cross sections given by the two uniform charge distributions. If we assume a nuclear radius $r_0A^{\frac{1}{3}}$, with