excited electron states. An estimate of these terms shows that they contribute less than $\frac{1}{2}$ percent to H_{12}. In the region where H_{12} obtains most of its value, the difference between Hartree and Hartree-Fock is not significant.

H_3 is an operator which does not weight regions near the nucleus and is sensitive to the amount of electron correlation. We have therefore evaluated H_3 using the Hylleraas six-term function.

The results for a hydrogenic function with $Z=1.6875$, the Hylleraas function, and Hartree functions are given in Table I. An evaluation of H_3 with Hartree functions yields 0.59 ($\alpha^2 Ry$).

It will be noticed that our results for columns 1 and 3 disagree with those of Bethe.[2] This discrepancy seems to arise from an error of a factor of 2 in his H_3, and a numerical error in his evaluation of H_1.

TABLE I. Relativistic corrections with various wave functions, in α^2 *Ry*.

	Hydrogenic	Hylleraas	Present calculation
$-H_1$	20.27		26.23
$-H_2$	−19.22		−22.60
$-H_3$	0.60		0.71
Sum	+1.65	4.22	+4.34
Ion	−4.00	−4.00	−4.00
Total ($\alpha^2 Ry$)	−2.35	0.22	0.34
Total (cm^{-1})	$-13.7\ \text{cm}^{-1}$	$1.28\ \text{cm}^{-1}$	$2.0\pm0.5\ \text{cm}^{-1}$

The hydrogenic values for H_1 and H_2 may be brought into substantial agreement with the Hartree values by using $Z=1.8$. This is reasonable in view of the weighting of the region near the nucleus by H_{12}.

Combining the interelectron magnetic interaction,[5] the mass polarization term,[2] the relativistic term, the Breit operator term,[3] and the contribution of the Lamb shift as calculated by Hakansson,[6] we obtain

$$(-16.4-5.2+2.0+0.15+1.2)\ \text{cm}^{-1}=-18.3\pm1\ \text{cm}^{-1}.$$

Using the most recent value of the electrostatic nonrelativistic eigenvalue:[1] $I=198290.9\ \text{cm}^{-1}$, and adding the above, we get $I_{\text{corrected}}=198272.7\ \text{cm}^{-1}$, $I_{\text{observed}}=198313\pm5\ \text{cm}^{-1}$. This leaves a discrepancy of 40 cm⁻¹. With the Hylleraas "eighth approximation"[1] the discrepancy is 10 cm⁻¹.

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† National Science Foundation Predoctoral Fellow.

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Mobility of Impurity Ions in Germanium and Silicon

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LI and Cu have been shown to diffuse rapidly into Ge and Si and to behave as donor and acceptor elements, respectively, in these semiconductors.[1,2] It is therefore of interest to investigate the mobilities of these elements in order to secure information on their state of ionization and to provide, also, independent and more accurate measures of their diffusivities.

By diffusing hemispherical n-type regions (20 mils in diam) of Li into p-type Ge and Si, in the form of bars 0.7 in.×0.10 in.×0.05 in., and applying dc fields of 1–10 volts per cm, we have determined Li^+ mobilities in the range 150–851°C. The dc fields were obtained by passing large currents through the specimen, the current serving at the same time to heat up the specimen to the desired temperature. The electric field was measured between two tungsten probes fixed 1.0 cm apart which could be brought into contact with the specimens. Temperatures were calculated to ±5°C from the resistivities, employing data of Morin.[3] Cooling was provided by steel rods which contacted the specimens through thin mica separators. The drift times varied from 5 minutes to 3 hours and the drift distances, which were measured by means of a micrometer microscope, ranged from 3 to 25 mils. The latter were determined by measuring the distance between the centers of the circular pn junctions before and after subjection to the field. The pn junctions were made visible by means of the deposition of barium titanate on the boundaries under reverse voltages of about 40 volts.[4]

The results for Li are shown in Fig. 1. The diffusivities, D, were calculated from the Einstein relation,

$$D=\mu kT/q,$$

where μ is the ion mobility in cm² per volt sec and q is the ionic charge. The curves in Fig. 1 are given by

$$D=25\times10^{-4}\exp(-11\,800/RT),$$

$$D=19\times10^{-4}\exp(-14\,700/RT),$$

for Li in Ge and Si, respectively. These equations agree within experimental error with those previously published. Therefore Li follows the Einstein relation in this concentration range and migrates as a singly-charged positive ion.

The above techniques, as well as techniques using radioactive copper, Cu^{64}, have been employed to determine the drift mobility of copper ions in Ge. Radioautographs taken of Cu^{64} particles before and after diffusion for 2–3 minutes at 800–850°C in a field of about 1 volt per cm showed a drift of about 7–15 mils toward the negative electrode. However, the excess copper was not removed in these experiments so that some doubt exists whether the result was a true measure of the impurity ions in the solid. In another experiment done at 800°C, the excess copper was removed. A drift of 4 mils toward the negative electrode was observed for a field of 1 volt/cm in 45 sec. The diffusion constant calculated from

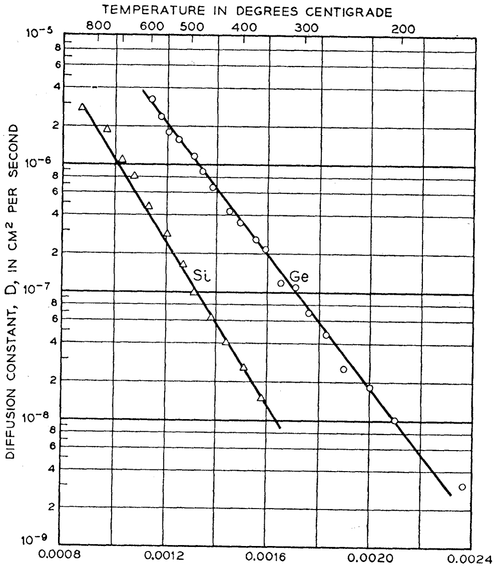

FIG. 1. Plot of the diffusivity of Li in Si and Ge against reciprocal of absolute temperature.

this mobility measurement and assuming that Cu moves as a singly-charged positive ion is 2.2×10^{-5} cm^2/sec (±25 percent). This result is of the same order as that previously published.[2] Subsequent results obtained in the same manner leave no doubt that copper migrates as a positive ion at these temperatures, but more work will be required to determine definitely the magnitude of its charge.

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Variations in the Decay of Phosphorescence with Frequency of Applied Electric Field*†

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DESTRIAU[1] has reported on the momentary illumination and extinguishing effects of an electric field applied to a phosphor which has a long phosphorescence decay time. The effects of varying the frequency of the applied field on the wavelength of maximum phosphorescence intensity from an ultraviolet-excited ZnS/CdS:Cu phosphor have been examined by the author. This letter discusses the qualitative features of these variations.

The samples were suspensions of the phosphor in a commercial thermo-setting plastic. The liquid suspension was poured on a polished metal plate and pressed to a thickness of about 0.005 in. with a slide of conducting glass. The thickness was controlled by Teflon spacers. Hardening was accomplished by heating to 80°C for about one hour. The samples were irradiated by near ultraviolet light for twenty seconds, and at specified times after the ultraviolet was turned off the field was applied. The emitted light was passed through a monochromator which was adjusted to transmit only the wavelength of maximum intensity, to a refrigerated photomultiplier. The current from the photomultiplier was fed through a bridge-type electrometer to a very sensitive galvanometer.

With an emf of 500 volts applied across a typical sample of RMA phosphor No. P-7 (*Y*),[2] the curves obtained at 28°C are shown in Fig. 1. Curve 1 is the ordinary phosphorescence decay curve with no field applied. Curves 2, 3, and 4 are, respectively, those for fields of 80, 800, and 5000 cps applied at 8 and 72 sec, and removed at 40 and 104 sec after the ultraviolet was turned off.

The initial momentary illumination is greatest for the 80-cps field, decreasing with increasing frequency—see *A*. Also the length of time for the decay to return to its normal scheme after the field is removed is greater for the lower-frequency fields, as can be seen in time intervals *B–C* and just after *D*. However, the magnitudes of the extinguishing effect after the initial increase, the increase at the first removal of the field, *B*, the extinguishing effect at *C*, and the final increase at *D* are considerably greater for the higher-frequency fields. Finally, the actual form of the decay varies appreciably with the field frequency when the field is on—shown in *C–D*.

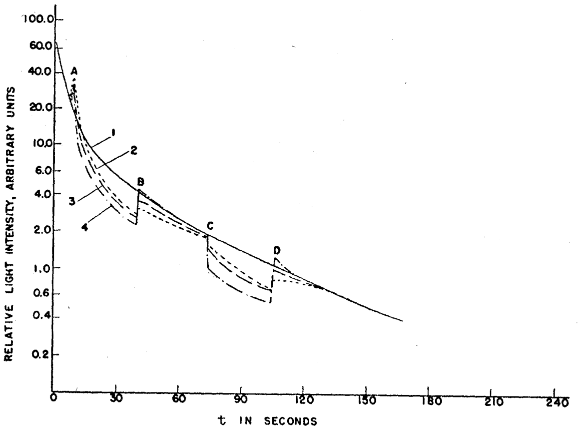

FIG. 1. Light output of ZnS/CdS:Cu phosphor *vs* time after the ultraviolet was turned off. 1. Normal phosphorescence decay curve. 2, 3, 4: Applied field frequency: 80, 800, and 5000 cps, respectively. A, C: Field on; B, D: Field off.

All of these phenomena show strong dependence on field strength, temperature, the particular wavelength examined, and the relative amounts of ZnS and Cds. Destriau shows curves of a ZnS/Cds phosphor subjected to a 50-cps field in which the momentary increase at field removal times comparable to *B* and *D* gives an intensity considerably greater than that from the normal decay at the given time. For the phosphor discussed in this letter the intensity remained lower than the normal decay intensity for all the 80-cps runs and exceeded the normal intensity only for the 5000-cps field.

Work is now in progress to obtain more quantitative information on these effects as functions of temperature, field strength, and phosphor composition. Such studies may increase our understanding of trapping levels and luminescence centers in these phosphors.

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Solution of the Wave Equation Near an Extremum of the Potential*

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IN previous work reported in the literature,[1] solutions of the wave equation near an extremum of the potential have been obtained by expanding the potential in powers of the distance from the potential extremum y and retaining only the constant and quadratic terms. In certain problems it is desirable to have a more accurate solution which will be valid over a more extended range of the variable y. In this paper a solution is obtained which is consistent with retaining all powers through y^6 in the expansion of the potential in the vicinity of the extremum. The present work is confined to one dimension.

The wave equation is written[2]

$$(d^2\psi/dy^2)+a^2[\epsilon+P(y)]\psi=0, \tag{1}$$

with $y=0$ taken as the location of the extremum of the potential, and $P(0)=0$. The transformation

$$\psi=P^{-\frac{1}{4}}F,\quad z=\int_0^y P^{\frac{1}{2}}dy \tag{2}$$

yields the transformed wave equation

$$(d^2F/dz^2)+[a^2+(a^2\epsilon/P)-Q]F=0, \tag{3}$$

where

$$Q=-P^{-\frac{3}{4}}(d^2P^{-\frac{1}{4}}/dy^2)=P^{-\frac{1}{4}}(d^2P^{\frac{1}{4}}/dz^2).$$

Near $y=0$, $P(y)$ can be developed in a series of ascending powers of y:

$$P(y)=c_2y^2+c_3y^3+\cdots. \tag{4}$$

If z and $[(a^2\epsilon/P)-Q]$ are expanded in ascending powers of y