Semiclassical Methods in Meson Processes*

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(Received July 29, 1953)

Simple calculations analogous to the electrodynamical Weizsäcker-Williams method have been carried out for pseudoscalar mesons with pseudoscalar coupling. By integrating a fast nucleon's virtual meson spectrum with the appropriate meson scattering cross section, a nucleon-nucleon cross section is obtained. Results are given for neutron-proton bremsstrahlung and nucleon-nucleon meson production. When comparison is possible, results obtained by this method agree approximately with more exact calculations.

THE analogue to the Weizsäcker-Williams[1] method in meson theory was introduced by Heitler and Peng[2] and by Wang.[3] These authors calculated nucleon-nucleon meson production by scattering the fast nucleon's beam of virtual mesons on the other nucleon. From field theory they find that the number of charged virtual pseudoscalar mesons within an energy interval, $d\omega$, accompanying a moving nucleon of energy, γM, is ($c=\hbar=\mu=1$)

$$N(\omega)d\omega=\frac{g^2\omega z^2}{4\pi\gamma^2M^2}\left\{\frac{K_1^2-K_0^2}{1+\gamma^2\omega^{-2}}+K_0^2-K_1^2+2\frac{K_0K_1}{z}\right\}d\omega. \quad (1)$$

Here the argument of the K's[4] is $z=(1+\omega^2\gamma^{-2})^{\frac{1}{2}}/M$ and g is the $PSPS$ coupling constant (not in Heaviside units). In (1) it is assumed that (a) $\gamma\gg 1$, (b) corrections resulting from nonclassical effects will not give rise to appreciably different final results, and (c) weak-coupling theory may be used. Integration of (1) with the appropriate meson-induced reaction cross section, $\sigma_m(\omega)$,

$$\sigma=\int_{\omega_{\min}}^{\omega_{\max}}N(\omega)\sigma_m(\omega)d\omega, \quad (2)$$

gives the nucleon's total cross section.

To calculate $n-p$ bremsstrahlung the $PSPS$ radiative capture cross section is needed. By Feynman-Dyson techniques one finds for $p(\pi^-,\gamma)n$ and $n(\pi^+,\gamma)p$, respectively[5]

$$\frac{d\sigma_-}{d\omega'}=\frac{\pi g^2e^2}{M(\omega^2-1)}\left\{\frac{2M\omega+1}{2M\omega'}+\frac{1}{2\omega'^2}+\frac{2}{[2M(\omega-\omega')+1]^2}-\frac{2\omega}{\omega'[2M(\omega-\omega')+1]}\right\}, \quad (3)$$

$$\frac{d\sigma_+}{d\omega'}=\frac{2\pi g^2e^2}{(\omega^2-1)(2M\omega+1)^2}\left\{\omega'(2M\omega+1)+M+\frac{4M\omega'^2}{[2M(\omega-\omega')+1]^2}-\frac{4M\omega\omega'}{2M(\omega-\omega')+1}\right\}, \quad (4)$$

where ω and ω' are the meson and γ-ray energies. The limits in the integration over ω, Eq. (2), are determined by $\cos\theta=\pm1$ in the Compton law for radiative capture, although $\omega_{\max}=\gamma M/2$ is used when it is a stronger limit.[2] Using $M=20/3$, (1) may be approximated as

$$N(\omega)=\frac{9g^2}{1600\pi\omega}\times\frac{3\omega^2}{\gamma^2}\qquad\text{for }\omega\leqslant\gamma, \quad (5a)$$

$$N(\omega)=\frac{9g^2}{1600\pi\omega}\left\{21.5-0.6\left(6.6-\frac{\omega}{\gamma}\right)^2\right\}\qquad\text{for }\omega>\gamma. \quad (5b)$$

After integrating (3) and (4) with (5) over ω and then over ω', one adds the results to obtain the total cross section for $n-p$ bremsstrahlung resulting from π^+ and π^- exchange,[6]

$$\sigma_++\sigma_-=0.0054e^2g^4\gamma^{-1}[\ln(0.57\gamma)-0.53\gamma^{-1}\ln(13\gamma)], \quad (6)$$

where $1/\gamma^2$ terms are neglected. Presumably a relativistic third-order field theoretic calculation of the charge exchange contribution would have a leading term of the same order as (6).

Although nucleon-nucleon meson production by the

* This work was performed while the author held a U. S. Atomic Energy Commission Postdoctoral Fellowship at Cornell University.

[1] C. F. v. Weizsäcker, Z. Physik **88**, 612 (1934); E. J. Williams, Phys. Rev. **45**, 729 (1934).

[2] W. Heitler and H. W. Peng, Proc. Roy. Irish Acad. **A49**, 101 (1943); see also W. Heitler and P. Walsh, Revs. Modern Phys. **17**, 252 (1945).

[3] F. S. Wang, Z. Physik **115**, 431 (1940).

[4] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Macmillan Company, New York, 1948), p. 78.

[5] Nonrelativistic calculations have been published by Aidzu, Fujimoto, Fukuda, Hayakawa, Takayanagi, Takeda, and Yamaguchi, Progr. Theoret. Phys. (Japan) **5**, 931 (1950); S. Ogawa and E. Yamada, Progr. Theoret. Phys. (Japan) **5**, 977 (1950); R. E. Marshak and A. S. Wightman, Phys. Rev. **76**, 114 (1949). It is somewhat disturbing that these three calculations, while retaining terms of order μ/M, disagree among themselves regarding these terms.

[6] J. Ashkin and R. E. Marshak, Phys. Rev. **76**, 58 (1949), find that charge exchange gives the major contribution at low energies.

method of virtual quanta has been treated[2] using various meson theories, it is of interest to calculate the *PSPS* result. Using the meson-scattering cross sections of Ashkin *et al.*[7] and proceeding exactly as above, one obtains for charged mesons from $n-p$ collisions:

$$\sigma=0.0034g^6\gamma^{-1}(1-0.94\gamma^{-1}), \tag{7}$$

and for π^+ from $p-p$ (or π^- from $n-n$) collisions:

$$\sigma=0.0068g^6\gamma^{-1}[\ln(0.31\gamma)+4.3\gamma^{-1}]. \tag{8}$$

Comparison with the third-order field-theoretic calculation of Morette[8] shows substantial agreement in the leading term. Similar agreement between virtual quanta methods and rigorous calculations in the scalar theory has been found by Strick and ter Haar.[9]

[7] Ashkin, Simon, and Marshak, Progr. Theoret. Phys. (Japan) **5**, 634 (1950).
[8] C. Morette, Phys. Rev. **76**, 1432 (1949).
[9] E. Strick and D. ter Haar, Phys. Rev. **76**, 304 (1949).

Because of the assumptions entailed in (1), the above results are of theoretical interest only. In practice one should integrate perhaps a phenomenological $N(\omega)$ with experimental meson-scattering cross sections. This might enable one to correlate meson-induced with nucleon-induced reactions, in a manner similar to Guth and Mullin's[10] correlation of experiments on the disintegration of Be^9 by γ rays and by electrons. A comparison of a high energy $n-p$ bremsstrahlung calculation using nuclear force phenomenology[11] with the method of virtual quanta might enable one to express $N(\omega)$ in terms of nuclear force parameters. Finally it might be pointed out that the method of virtual quanta would lend itself more readily to heavier particles with weaker coupling.

[10] E. Guth and C. J. Mullin, Phys. Rev. **76**, 234 (1949).
[11] J. Ashkin and R. E. Marshak, Phys. Rev. **76**, 58 (1949) and T. Muto, Phys. Rev. **59**, 837 (1941) have done low-energy $n-p$ bremsstrahlung calculations in which the nucleon-nucleon interaction is handled by nuclear force phenomenology rather than by weak-coupling meson theory.

PHYSICAL REVIEW VOLUME 92, NUMBER 5 DECEMBER 1, 1953

The Theory of Quantized Fields. IV

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(Received August 6, 1953)

The principal development in this paper is the extension of the eigenvalue-eigenvector concept to complete sets of anticommuting operators. With the aid of this formalism we construct a transformation function for the Dirac field, as perturbed by an external source. This transformation function is enlarged to describe phase transformations and, when applied to the isolated Dirac field, yields the charge and energy-momentum eigenvalues and eigenfunctions. The transformation function describing the system in the presence of the source is then used as a generating function to construct the matrices of all ordered products of the field operators, for the isolated Dirac field. The matrices in the occupation number representation are exhibited with a classification that effectively employs a time-reversed description for negative frequency modes. The last section supplements III by constructing the matrices of all ordered products of the potential vector, for the isolated electromagnetic field.

INTRODUCTION

THIS paper and its sequel are continuations of III[1] in their concern with a single externally perturbed field. We shall discuss the Dirac field as perturbed by a second prescribed Dirac field, which appears as an external source, or by a prescribed Bose-Einstein field, as exemplified by a given electromagnetic field. The Lagrange function of this system is

$$\begin{aligned}\mathfrak{L}=-\tfrac{1}{4}[\bar\psi,\gamma_\mu(-i\partial_\mu-eA_\mu)\psi+m\psi]\\ -\tfrac{1}{4}[(i\partial_\mu-eA_\mu)\bar\psi\gamma_\mu+m\bar\psi,\psi]\\ +\tfrac{1}{2}[\bar\psi,\eta]+\tfrac{1}{2}[\bar\eta,\psi].\end{aligned} \tag{1}$$

The resulting field equations are

$$\begin{aligned}\gamma_\mu(-i\partial_\mu-eA_\mu)\psi+m\psi=\eta,\\ (i\partial_\mu-eA_\mu)\bar\psi\gamma_\mu+m\bar\psi=\bar\eta,\end{aligned} \tag{2}$$

[1] J. Schwinger, Phys. Rev. **91**, 728 (1953).

and the generators of infinitesimal changes in ψ or $\bar\psi$ on a surface σ are given by

$$G(\psi)=i\int d\sigma_\mu\bar\psi\gamma_\mu\delta\psi=i\int d\sigma\bar\psi\gamma_{(0)}\delta\psi \tag{3}$$

and

$$G(\bar\psi)=-i\int d\sigma_\mu\delta\bar\psi\gamma_\mu\psi=-i\int d\sigma\delta\bar\psi\gamma_{(0)}\psi. \tag{4}$$

It was shown in III that the vacuum state of a closed system, Ψ_0 can be characterized as the right eigenvector, with zero eigenvalues, of the positive frequency parts of the field components, and that $\Psi_0^\dagger$ is the left eigenvector, with zero eigenvalues, of the negative frequency parts of the field components. The inference that the totality of eigenvectors of these types would be of particular utility led us, in discussing a Bose-Einstein system, to introduce eigenvectors and eigenvalues for complete sets