were made on the oscilloscope of a video type microwave spectrometer. Relative line frequencies were measured with frequency markers derived from a stabilized microwave oscillator. We plan to measure the magnetic moment of I^{131}. In view of the decreased magnitude of Q and the same spin as I^{129} it will be interesting to see if the magnetic moment decreases.[3]

The I^{131} was obtained from the isotope production division at Oak Ridge National Laboratory. We wish to acknowledge the help of O. R. Gilliam in the early phases of the work.

Note added in proof:—In later work the $K=1$, $F=9/2\rightarrow 9/2$ and the $K=2$, $F=7/2\rightarrow 7/2$, $7/2\rightarrow 5/2$, $7/2\rightarrow 9/2$ lines were seen at frequencies in good agreement with those calculated from the parameters of Table I.

PHYSICAL REVIEW VOLUME 92, NUMBER 5 DECEMBER 1, 1953

Photoproduction of π-Meson Pairs*

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(Received August 25, 1953)

A perturbation study of the photoproduction of π-meson pairs by gamma rays incident on protons is made for both pseudoscalar and pseudovector coupling. Expressions are derived for the possible cross sections assuming the nucleon is infinitely heavy. The effect of first-order nucleon recoil on the (π^+, π^-) cross section, assuming pseudoscalar coupling, is considered. Curves illustrating the results are given. The possibility of obtaining information on the type of interaction operative between mesons and nucleons from a study of the pair production cross sections is discussed.

INTRODUCTION

Although the pseudoscalar nature of the π meson has been more or less definitely established,[1] the question of the coupling between the meson and nucleon fields remains unanswered. It is well known[2] that the pseudoscalar and pseudovector interactions give identical results to lowest order in the coupling constant provided $f=(2M/\mu)g$; where f and g are the pseudoscalar and pseudovector coupling constants respectively, and M and μ are the nucleon and meson masses. Assuming weak coupling theory to be approximately valid we must, therefore, study processes which do not proceed in lowest order of the coupling constant in order to gain insight into the type of interaction operative between mesons and nucleons.

Kaplon[3] has investigated the cross section for the production of π mesons in nucleon-nucleon collisions. His calculations indicate that the differential cross sections for the production process differ when the two types of coupling are used. However, aside from the assumption of weak coupling theory, the interaction between the two final state nucleons is neglected and consequently one cannot draw quantitative conclusions from the calculation.

This latter difficulty is partly obviated when we consider the photoproduction of π-meson pairs by gamma rays incident on protons.[4] It has been shown[5] that in the low-energy region the (π^+, π^-) and (π^0, π^0) cross sections for pseudoscalar coupling are considerably larger than those for the pseudovector interaction. On the other hand, the (π^+, π^0) cross section has approximately the same magnitude when either type of coupling is used. In this paper we shall investigate the possibility of obtaining information on the meson-nucleon interaction from a study of the photoproduction of meson pairs.

(π^+, π^-) PAIR PRODUCTION

A. Pseudoscalar Coupling

Assuming pseudoscalar coupling, the interaction energy between the meson, nucleon, and electromagnetic field is written as[6]

$$H=\int H_1 d\mathbf{r}+\int H_2 d\mathbf{r}+\int H_3 d\mathbf{r},$$

where

$$\begin{aligned} H_1&=if\bar{\psi}\gamma_5\tau_\alpha\phi_\alpha\psi,\\ H_2&=-eA_\nu\left(\phi_1\frac{\partial\phi_2}{\partial x_\nu}-\phi_2\frac{\partial\phi_1}{\partial x_\nu}\right),\\ H_3&=-ie\bar{\psi}\gamma_\nu A_\nu\tau_p\psi,\end{aligned} \quad (1)$$

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission, and by the Office of Scientific Research, Air Research and Development Command.

[1] R. E. Marshak, *Meson Phyics* (McGraw-Hill Book Company, Inc., New York, 1952), pp. 1–201.

[2] F. J. Dyson, Phys. Rev. **73**, 929 (1948); L. L. Foldy, Phys. Rev. **84**, 168 (1951); G. Wentzel, Phys. Rev. **86**, 802 (1952); S. D. Drell and E. M. Henley, Phys. Rev. **88**, 1053 (1952).

[3] M. F. Kaplon, Ph.D. thesis, University of Rochester, 1951 (unpublished). Details of the calculation are given by R. E. Marshak, reference 1, p. 47.

[4] The possibility of a strong attractive interaction between the emerging π mesons has been pointed out by K. A. Brueckner and K. M. Watson, Phys. Rev. **87**, 621 (1952).

[5] R. D. Lawson and S. D. Drell, Phys. Rev. **90**, 326 (1953).

[6] Throughout this work the system of units in which $\hbar=c=1$ will be used. The fine structure constant in our notation is $e^2/4\pi$ and the meson-nucleon coupling constant is $f^2/4\pi$.

and ψ is the nucleon field strength; $\bar\psi=\psi^*\gamma_4$; ϕ_α is the meson field strength, $\alpha=3$ describing neutral mesons; τ_α is the isotopic spin operator with $\tau_p=\frac{1}{2}(1-\tau_3)$; A_ν ($\nu=1-4$) refers to the electromagnetic field; and the gammas are the usual Dirac matrices with $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$.

The Feynman diagrams to order ef^2 for the process $\gamma+p\rightarrow p'+\pi^++\pi^-$ are shown in Fig. 1. The effective S matrix for the process is calculated by the usual methods[7] and yields

$$S=\frac{-ief^2\epsilon_\nu\delta(p_1+K-p_2-k_+-k_-)}{[(2\pi)^7 2E_\gamma\omega(k_+)\omega(k_-)]^{\frac{1}{2}}}\left\{\frac{k_{+\nu}k_{-\lambda}\bar{u}_f\gamma_\lambda u_i}{(k_+K)(k_-^2+2p_2k_-)}+\frac{k_{-\nu}k_{+\lambda}\bar{u}_f\gamma_\lambda u_i}{(k_-K)(k_+^2-2p_1k_+)}-\frac{k_{-\lambda}\bar{u}_f\gamma_\lambda(2p_{1\nu}+\gamma_\alpha\gamma_\nu K_\alpha)u_i}{2(p_1K)(k_-^2+2p_2k_-)}-\frac{k_{+\lambda}\bar{u}_f(2p_{2\nu}-\gamma_\nu\gamma_\alpha K_\alpha)\gamma_\lambda u_i}{2(p_2K)(k_+^2-2p_1k_+)}\right\}, \quad (2)$$

where k_+, k_-, and K refer to the energy-momentum four vectors of the π^+, π^- and gamma ray respectively; u_i and u_f are the Dirac spinors for the initial and final nucleons whose energy-momentum four-vectors are p_1 and p_2; $\bar{u}_f=u_f^*\gamma_4$; $\boldsymbol{\varepsilon}$ is a unit vector in the direction of polarization of the gamma ray; and all products are understood to be four-vector products. The order of the terms in Eq. (2) is the same as the order of the Feynman diagrams in Fig. 1.

In the extreme nonrelativistic limit for the nucleon, $M\rightarrow\infty$, only the first two Feynman diagrams are important. In this approximation the differential cross section as a function of $\omega(k_-)$ is

$$\frac{d\sigma}{d\omega(k_-)}=\frac{8}{137}\left(\frac{g^2}{4\pi}\right)^2\left(\frac{2M}{\mu}\right)^2\frac{k_-k_+}{\mu^2E_\gamma^3}\{A+B\}, \quad (3)$$

where

$$A=\frac{\omega(k_+)}{k_+}\left(\ln\frac{\omega(k_+)+k_+}{\omega(k_+)-k_+}\right)-2,$$
$$B=\frac{\omega(k_-)}{k_-}\left(\ln\frac{\omega(k_-)+k_-}{\omega(k_-)-k_-}\right)-2; \quad (4)$$

and for convenience we have set

$$f=(2M/\mu)g, \quad (5)$$

where g is the coupling constant in the derivative coupling theory.

To find the total cross section we need only integrate Eq. (3) over the energy range of $\omega(k_-)$. For 400-Mev incident gamma rays in the laboratory coordinate system, the energy E_γ in the center-of-mass coordinate system is 294 Mev. Thus on the basis of a strict no recoil approximation the energy range of the emitted mesons is from 140 Mev (their rest energy) to an upper limit of 154 Mev. Carrying out the integration of Eq. (3) numerically we find

$$\sigma=2.3\times10^{-2}\left(\frac{g^2}{4\pi}\right)^2 \text{ millibarns.}$$

In view of the small energy spread it is necessary to look more closely into the kinematics of the process since any kinetic energy carried by the nucleon could easily affect this energy interval by an order of magnitude and hence materially change the cross section.[8] The kinetic energy of the initial proton in the center-of-mass coordinate system is easily found to be $\sim$46 Mev. To calculate the energy carried off by the recoil nucleon at the upper end of the spectrum we use the laws of conservation of energy and momentum,

$$E_\gamma+p_1^2/2M=\omega(k_+)+\omega(k_-)+(\mathbf{k}_-+\mathbf{k}_+)^2/2M, \quad (6)$$

and assume that the upper end of the π^- spectrum is defined by setting $\omega(k_+)=\mu$ in Eq. (6). Under this assumption the kinetic energy of the recoil nucleon is 9 Mev. Thus, according to this reasoning, the energy spread of the emitted mesons would be from $\omega(k_-)=140$ Mev to $\omega(k_-)=154+46-9=191$ Mev. One would certainly expect the numerical value of the cross section to be closer to that observed if in some way the energy range of the emitted mesons were made close to the experimentally observed range. For this reason, in the no-recoil formula, Eq. (3), let us set $\omega(k_-)+\omega(k_+)=294+46-9=331$ Mev. In this case the energy limits are at least almost correct.[9] The total cross section is then

$$\sigma_{ps}(\pi^+,\pi^-)=1.15(g^2/4\pi)^2 \text{ millibarns.} \quad (7)$$

One might doubt the validity of Eq. (7) because of our method of treating energy conservation. We there-

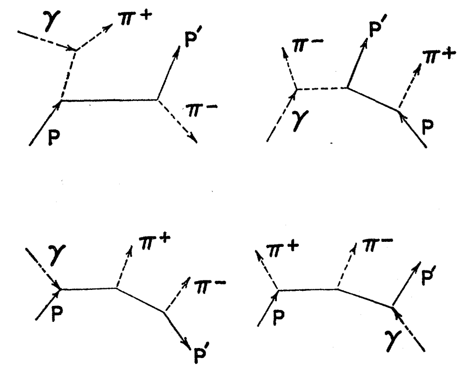

FIG. 1. Feynman diagrams for the process $\gamma+p\rightarrow p'+\pi^++\pi^-$ assuming pseudoscalar coupling.

[7] F. J. Dyson, Phys. Rev. **75**, 486 (1949); R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949); G. C. Wick, Phys. Rev. **80**, 268 (1950).

[8] Since we are producing meson pairs, the volume element in phase space is extremely sensitive to the available energy.

[9] In Using Eq. (6) to calculate the recoil energy of the proton, if the angle between the momentum vectors of the π^+ and π^- mesons is taken to be 180°, one finds that at the upper end of the differential cross section $\omega(k_+)$ is slightly greater than μ. However, our original assumption gives results valid to about three percent.

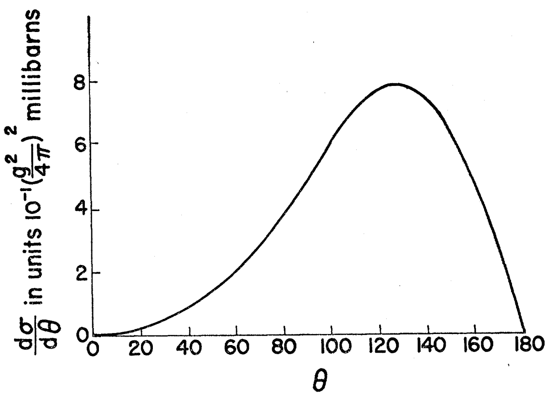

FIG. 2. Differential cross section for the photoproduction of (π^+, π^-) pairs as a function of θ, the angle between the two mesons in the center-of-mass coordinate system. The curve is plotted for 400-Mev incident gamma rays in the laboratory coordinate system. Pseudoscalar coupling is assumed. First-order nucleon recoil effects are considered.

fore calculate the cross section taking into account first-order nucleon recoil effects. To do this it is necessary to evaluate S^*S to order M^{-1}. This result should then be combined with the approximate expression for the density of final states obtained by making a binomial expansion of the denominator of the expression

$$\rho = \frac{k_- k_+ \omega(k_-)\omega(k_+) d\Omega_- d\Omega_+}{1+[\omega(k_+)/Mk_+](k_+ + k_- \cos\theta)}; \tag{8}$$

where ρ is the density of final states, k_- and k_+ refer to the magnitude of $\mathbf{k}_-$ and $\mathbf{k}_+$ respectively, $d\Omega_-$ and $d\Omega_+$ are the elements of solid angle in k_- and k_+ phase space, and θ is the angle between $\mathbf{k}_-$ and $\mathbf{k}_+$ in the center-of-mass coordinate system.

However, the binomial expansion of Eq. (8) does not converge near the upper end of the π^- spectrum since there $k_+ \to 0$. On the other hand, no terms of the form $1/k_+$ occur in the expansion of S^*S. Consequently, if we convert S^*S to a cross section by using the exact nonrelativistic expression for the density of final states, Eq. (8), the results will be valid to order $(\mu/M)^2$. The results for 400-Mev gamma rays are shown in Figs. 2 and 3.

The total cross section obtained in this way is

$$\sigma_{ps}(\pi^+, \pi^-) = 1.16(g^2/4\pi)^2 \text{ millibarns}, \tag{9}$$

which certainly compares favorably with our "pseudo no-recoil" result. One might argue that this agreement is fortuitous. However, if the first order recoil calculation for 600-Mev incident gamma rays in the laboratory coordinate system is carried out one obtains a cross section, $\sigma = 16.1(g^2/4\pi)^2$ millibarns, whereas a calculation similar to that used in deriving Eq. (7) yields $\sigma = 15.9\ (g^2/4\pi)^2$ millibarns. From these results it follows that for incident gamma-ray energies up to 600 Mev an excellent approximation to the total cross section is obtained by using Eq. (3), providing we set $\omega(k_+)+\omega(k_-)$ equal to the actual energy available to the two mesons when recoil is considered.

One may understand the shift of the differential cross section to higher values of θ (see Fig. 2) by noting that as θ increases, the recoil energy of the proton decreases. Thus the volume element in phase space is not symmetric about $\theta = 90°$, but tends to have a maximum value for $\theta > 90°$.

The plot of the cross section as a function of the energy of the emitted meson, Fig. 3, shows that when recoil is considered the π^- tends to come out with higher energy than the π^+. It appears difficult to attach much physical significance to this result. The origin of part of the shift is evident from the form of the energy denominators in the S matrix, Eq. (2). In the low energy limit, the denominator of the term corresponding to the emission of the π^+ in a P state (the $k_{+\nu}$ term) is larger than the denominator for the term in which the π^- is emitted in a P state. Thus the matrix elements favor the π^- coming off in a P state. Since it is more probable for mesons with $l \neq 0$ to be emitted with greater energy, it follows that these two terms shift the differential cross section to higher π^- energies. The remainder of the shift comes from the interference of the first and second terms of Eq. (2) with the third and fourth.

A useful expression which gives the form of the differential cross section as a function of the energy of one of the emitted mesons, without recourse to tedious numerical integrations, is obtained by evaluating S^*S to order M^{-1} and combining this with the zero order density of final states [i.e., $M \to \infty$ in Eq. (8)]. The differential cross section in the center-of-mass coordinate system as a function of the energy of the π^- meson is

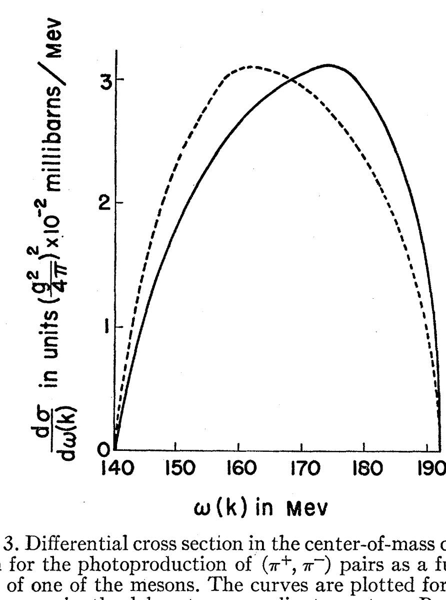

FIG. 3. Differential cross section in the center-of-mass coordinate system for the photoproduction of (π^+, π^-) pairs as a function of energy of one of the mesons. The curves are plotted for 400-Mev gamma rays in the laboratory coordinate system. Pseudoscalar coupling is assumed and first-order nucleon recoil effects are considered. The dotted (solid) curve refers to the cross section as a function of the energy of the $\pi^+(\pi^-)$ meson.

then given by

$$\frac{d\sigma}{d\omega(k_-)}=\frac{8}{137}\left(\frac{2M}{\mu}\right)^2\left(\frac{g^2}{4\pi}\right)^2\frac{k_-k_+}{\mu^2E_\gamma^3}\left\{\left(1-\frac{\omega(k_-)}{M}\right)A+\left(1+\frac{\omega(k_+)}{M}\right)B-\frac{k_-[E_\gamma-2\omega(k_-)]}{2M\omega(k_-)}C+\frac{k_+[E_\gamma-2\omega(k_+)]}{2M\omega(k_+)}D+\frac{k_-k_+[\omega(k_+)-\omega(k_-)]}{8M\omega(k_+)\omega(k_-)}CD\right\}, \quad (10)$$

where

$$C=\left(1-\frac{\omega^2(k_-)}{k_-^2}\right)\ln\frac{\omega(k_-)+k_-}{\omega(k_-)-k_-}+2\frac{\omega(k_-)}{k_-},$$
$$D=\left(1-\frac{\omega^2(k_+)}{k_+^2}\right)\ln\frac{\omega(k_+)+k_+}{\omega(k_+)-k_+}+2\frac{\omega(k_+)}{k_+}. \quad (11)$$

In Eq. (10) the terms in the brackets proportional to $1/M$ are antisymmetric in the $+$ and $-$ indices. Therefore, although these terms do not contribute to the total cross section they materially affect the form of the differential cross section. This gives us further insight into why the two values for the cross section, Eqs. (7) and (9), are in such good agreement.

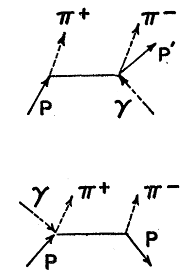

FIG. 4. Additional Feynman diagrams for the process $\gamma+p\rightarrow p'+\pi^++\pi^-$ for pseudovector coupling.

B. Pseudovector Coupling

The interaction energy for pseudovector coupling is given by

$$H=\int H_1d\mathbf{r}+\int H_2d\mathbf{r}+\int H_3d\mathbf{r}+\int H_4d\mathbf{r},$$

where

$$H_1=\frac{ig}{\mu}\bar{\psi}\gamma_5\gamma_\lambda\tau_\alpha\frac{\partial\phi_\alpha}{\partial x_\lambda}\psi,$$
$$H_4=\frac{ieg}{\mu}\bar{\psi}\gamma_5\gamma_\nu A_\nu(\tau_1\phi_2-\tau_2\phi_1)\psi, \quad (12)$$

and H_2 and H_3 are given by Eq. (1).

In addition to the Feynman diagrams for pseudoscalar coupling there are two diagrams arising from the combinations of H_1 and H_4. These are shown in Fig. 4.

The effective S matrix to order eg^2 is

$$S=\frac{eg^2\epsilon_\nu\delta(p_1+K-p_2-k_+-k_-)}{\mu^2[(2\pi)^72E_\gamma\omega(k_+)\omega(k_-)]^{\frac{1}{2}}}\left\{\frac{k_{+\alpha}\bar{u}_f\gamma_\nu(2M\gamma_\alpha+2ip_{1\alpha}-ik_{+\alpha})u_i}{(k_+^2-2p_1k_+)}-\frac{k_{-\alpha}\bar{u}_f(2M\gamma_\alpha+2ip_{2\alpha}+ik_{-\alpha})\gamma_\nu u_i}{(k_-^2+2p_2k_-)}\right.$$
$$-\frac{k_{+\nu}k_{-\lambda}(k_+-K)_\alpha\bar{u}_f(2ip_{2\lambda}+ik_{-\lambda}+2M\gamma_\lambda)\gamma_\alpha u_i}{(k_+K)(k_-^2+2p_2k_-)}+\frac{k_{-\nu}k_{+\alpha}(k_--K)_\lambda\bar{u}_f(2ip_{2\lambda}+i(k_--K)_\lambda+2M\gamma_\lambda)\gamma_\alpha u_i}{(k_-K)(k_-^2+2p_2k_--2p_2K-2k_-K)}$$
$$\left.+\frac{k_{+\alpha}k_{-\lambda}\bar{u}_f(\gamma_\nu\gamma_\beta K_\beta-2p_{2\nu})\gamma_\lambda(2ip_{1\alpha}+2M\gamma_\alpha-ik_{+\alpha})u_i}{2(p_2K)(k_+^2-2p_1k_+)}+\frac{k_{+\alpha}k_{-\lambda}\bar{u}_f(2M\gamma_\lambda+2ip_{2\lambda}+ik_{-\lambda})\gamma_\alpha(2p_{1\nu}+\gamma_\beta\gamma_\nu K_\beta)u_i}{2(p_1K)(k_-^2+2p_2k_-)}\right\}. \quad (13)$$

In the extreme nonrelativistic limit for the nucleon the last two terms in Eq. (13) become unimportant. The differential cross section as a function of the energy of the π^- meson is, in this case,

$$\frac{d\sigma}{d\omega(k_-)}=\frac{2}{137}\left(\frac{g^2}{4\pi}\right)^2\frac{k_-k_+}{\mu^4E_\gamma}\left\{8\left(\frac{k_-^2}{\omega^2(k_-)}+\frac{k_+^2}{\omega^2(k_+)}\right)-\frac{k_-k_+}{\omega(k_-)\omega(k_+)}CD-\frac{4k_-^2(2E_\gamma\omega(k_+)-k_+^2-E_\gamma^2)}{\omega^2(k_-)E_\gamma^2}A-\frac{4k_+^2(2E_\gamma\omega(k_-)-k_-^2-E_\gamma^2)}{\omega^2(k_+)E_\gamma^2}B\right\}, \quad (14)$$

where A, B, C, and D are defined by Eqs. (4) and (11).

The differential cross section is completely symmetric between the $+$ and $-$ indexes and is similar in shape to that obtained for pseudoscalar coupling (shown in Fig. 1 of reference 5). If energy conservation is treated in the same way as discussed in deriving Eq. (7), the total cross section for 400-Mev gamma rays in the center-of-mass coordinate system is

$$\sigma_{pv}(\pi^+,\pi^-)=0.064(g^2/4\pi)^2 \text{ millibarns.} \quad (15)$$

Comparing this to the result obtained for pseuoscalar coupling we see that

$$\sigma_{ps}(\pi^+,\pi^-)/\sigma_{pv}(\pi^+,\pi^-)=18 \text{ for 400-Mev gamma rays.}$$

Thus as stated in the introduction, the cross section derived using pseudoscalar coupling is considerably larger than that obtained from the pseudovector interaction.

Near threshold the differential cross section, Eq. (14), may be integrated exactly in terms of elementary

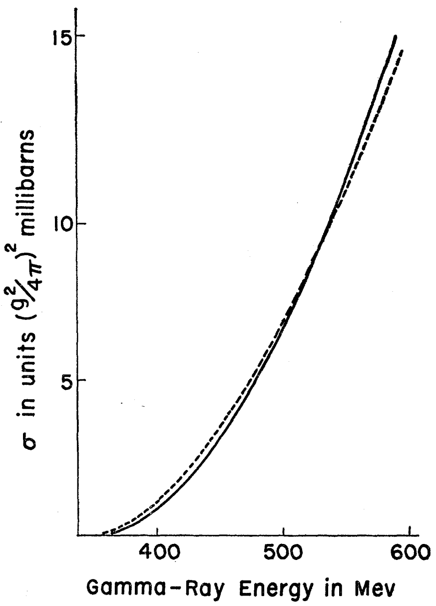

FIG. 5. Total cross section for the photoproduction of (π^+, π^-) pairs as a function of incident gamma-ray energy in the laboratory coordinate system. The dotted curve refers to pseudoscalar coupling and the solid curve to pseudovector. The pseudovector cross section has been multiplied by 15.

functions. Carrying out the calculation, one obtains[10]

$$\sigma \simeq \frac{8\pi}{137}\left(\frac{g^2}{4\pi}\right)^2 \frac{1}{\mu E_\gamma}\left(\frac{E}{\mu}-2\right)^3, \qquad (16)$$

where E is the energy available to the meson pair (in a strict no recoil calculation $E \rightarrow E_\gamma$). Comparing this to the total cross section for pseudoscalar coupling [given by Eq. (4) of reference 5] we find

$$\frac{\sigma_{ps}(\pi^+, \pi^-)}{\sigma_{pv}(\pi^+, \pi^-)} \simeq \frac{1}{3}\left(\frac{2M}{\mu}\right)^2\left(\frac{\mu}{E_\gamma}\right)^2.$$

For $E_\gamma \simeq 2\mu$ this ratio becomes $\simeq 15$.

To find the recoil corrections to Eq. (14) is tedious, and in view of the non-renormalizability of the pseudovector interaction it is felt that the calculation is not warranted.

The total cross section for the production of (π^+, π^-) pairs as a function of gamma-ray energy is shown in Fig. 5 for both pseudoscalar and pseudovector coupling.

(π^+, π^0) PAIR PRODUCTION

In this section we shall calculate the cross section for the reaction $\gamma + p \rightarrow n + \pi^+ + \pi^0$, assuming the nucleon to be infinitely heavy. Consequently all Feynman diagrams in which the gamma ray interacts with the proton will be neglected. As we shall see, it is convenient to calculate first the cross section for pseudovector coupling.

A. Pseudovector Coupling

The Feynman diagrams to order eg^2 for the process are shown in Fig. 6. The effective S matrix is

$$S = \frac{eg^2\epsilon_\nu \delta(p_1 + K - p_2 - k_+ - k_0)}{2\mu^2[(2\pi)^7 E_\gamma \omega(k_+)\omega(k_0)]^{\frac{1}{2}}}\left\{\frac{k_{0\alpha}\bar{u}_f\gamma_\nu(2ip_{1\alpha} - ik_{0\alpha} + 2M\gamma_\alpha)u_i}{(k_0^2 - 2p_1k_0)} - \frac{k_{0\alpha}\bar{u}_f(2ip_{2\alpha} + ik_{0\alpha} + 2M\gamma_\alpha)\gamma_\nu u_i}{(k_0^2 + 2p_2k_0)}\right.$$
$$\left. + \frac{k_{+\nu}k_{0\lambda}(k_+ - K)_\alpha \bar{u}_f\gamma_\alpha(2ip_{1\lambda} - ik_{0\lambda} + 2M\gamma_\lambda)u_i}{(k_+K)(k_0^2 - 2p_1k_0)} - \frac{k_{+\nu}k_{0\lambda}(k_+ - K)_\alpha \bar{u}_f(2ip_{2\lambda} + ik_{0\lambda} + 2M\gamma_\lambda)\gamma_\alpha u_i}{(k_+K)(k_0^2 + 2p_2k_0)}\right\}. \qquad (17)$$

Converting Eq. (17) to a cross section, we obtain

$$\frac{d\sigma}{d\omega(k_+)} = \frac{16}{137}\left(\frac{g^2}{4\pi}\right)^2 \frac{k_+k_0}{3E_\gamma\omega^2(k_0)\mu^4} \times \left\{2k_0^2 + \frac{k_0^2(k_+^2 + E_\gamma^2 - 2\omega(k_+)E_\gamma)}{E_\gamma^2}A\right\}, \qquad (18)$$

where A is given by Eq. (4).

The differential cross section is shown in Fig. 7 for 400-Mev incident gamma rays in the laboratory coordinate system. If energy conservation is treated in the same way as discussed in deriving Eq. (7), we find

$$\sigma_{pv}(\pi^+, \pi^0) = 2.22 \times 10^{-2}(g^2/4\pi)^2 \text{ millibarns.}$$

Comparing this to the result for (π^+, π^-) pair production at the same energy and with the same coupling we see that

$$\sigma_{pv}(\pi^+, \pi^-)/\sigma_{pv}(\pi^+, \pi^0) = 2.9.$$

One may qualitatively understand the shift of the differential cross section to lower π^+ energies as follows:

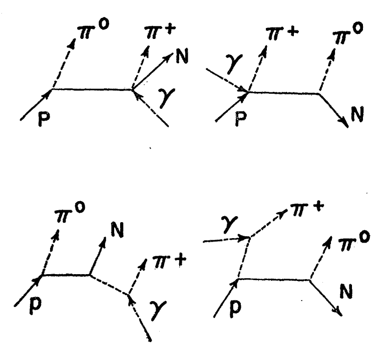

FIG. 6. Feynman diagrams for the process $\gamma + p \rightarrow n + \pi^+ + \pi^0$ assuming pseudovector coupling.

[10] Equation (16) has the same form as the result given by K. A. Brueckner and K. M. Watson, reference 4.

the eg term in the meson-nucleon coupling tends to emit the π^+ meson in an S state. On the other hand, the gradient coupling emits the mesons in a P state. Since the π^0 is emitted through the derivative coupling, it always has angular momentum >0. The π^+, however, is produced sometimes in an S state by the eg term in the interaction and sometimes in a P state due to its direct interaction with the gamma ray. Since it is more difficult to emit low-energy mesons in angular momentum states with $l\neq 0$, it follows that the differential cross section should be shifted to higher π^0 energies (lower π^+ energies).

Another interesting feature of the differential cross section is the fact that near the upper end of the π^+ spectrum the cross section goes to zero with zero slope. This may be understood by noting that the gradient coupling brings down an extra factor k_0 which when combined with the k_0 arising from the density of final states leads to a zero slope of the differential cross section for $k_0\rightarrow 0$.

B. Pseudoscalar Coupling

It is well known that the pseudoscalar interaction may be transformed by the Dyson transformation into one large term which is bilinear in the meson field,[11]

$$2M\left(\frac{g^2}{\mu^2}\right)\int \bar{\psi}\psi\phi_\alpha{}^2 d\mathbf{r}, \tag{19}$$

plus other terms, one of which is the usual pseudovector interaction. It is obvious that the core term, Eq. (19), cannot contribute to the (π^+, π^0) cross section since it

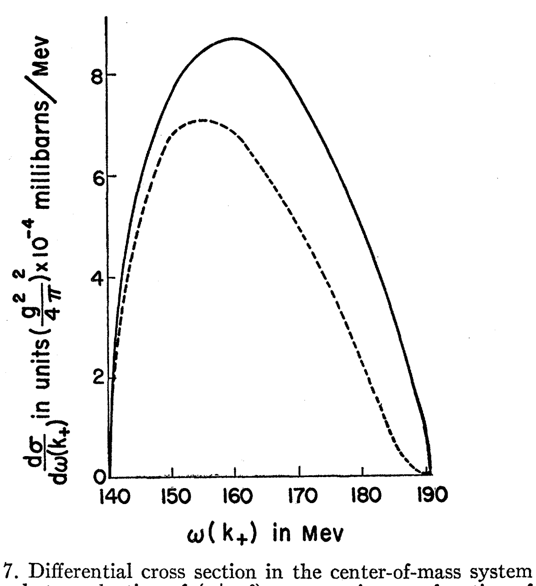

FIG. 7. Differential cross section in the center-of-mass system for the photoproduction of (π^+, π^0) meson pairs as a function of the energy of the emitted π^+ meson. The curve is plotted for 400-Mev gamma rays in the laboratory coordinate system. The nucleon is treated as being infinitely heavy. The dotted curve refers to pseudovector coupling; the solid curve to pseudoscalar.

[11] See for example S. D. Drell and E. M. Henley, reference 3.

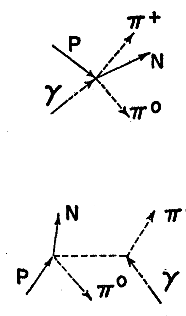

FIG. 8. Additional Feynman diagrams for the process $\gamma+p\rightarrow n+\pi^++\pi^0$ for pseudoscalar coupling.

contains no mechanism for changing the nucleon charge. Consequently, to calculate the (π^+, π^0) cross section for pseudoscalar coupling we must consider nucleon recoil. This follows from the fact that to leading order there is a cancellation of the S matrices associated with the two Feynman diagrams in which the gamma ray interacts with the charged meson. However, if we use the transformed pseudoscalar coupling,[11] we may calculate the cross section without considering recoil since the zero contribution from the core term has already been separated out.

In the transformed pseudoscalar interaction, aside from the usual derivative coupling, the term

$$H'=\frac{ieg^2}{\mu^2}\int \bar{\psi}\gamma_\nu A_\nu\phi_3(\tau_1\phi_1+\tau_2\phi_2)\psi d\mathbf{r} -\frac{g^2}{2\mu^2}\int \phi_{\alpha'}\frac{\partial\phi_\alpha}{\partial x_\lambda}\bar{\psi}\gamma_\lambda[\tau_\alpha, \tau_{\alpha'}]\psi d\mathbf{r} \tag{20}$$

may contribute to the process. The additional Feynman diagrams arising from these terms are shown in Fig. 8.

The S matrix associated with these two diagrams is

$$S=\frac{ieg^2\epsilon_\nu\delta(p_1+K-p_2-k_+-k_0)}{2\mu^2[(2\pi)^7E_\gamma\omega(k_+)\omega(k_0)]^{\frac{1}{2}}} \times\left\{\bar{u}_f\gamma_\nu u_i-\frac{2k_{+\nu}k_{0\alpha}\bar{u}_f\gamma_\alpha u_i}{(k_+K)}\right\}. \tag{21}$$

Combining Eqs. (17) and (21) we obtain the total effective S matrix to order eg^2 for pseudoscalar coupling. Converting this to a cross section we find

$$\left(\frac{d\sigma}{d\omega(k_+)}\right)_{ps}=\left(\frac{d\sigma}{d\omega(k_+)}\right)_{pv} +\frac{16}{137}\left(\frac{g^2}{4\pi}\right)^2\frac{k_+k_0\omega^2(k_0)}{\mu^4E_\gamma{}^3}A, \tag{22}$$

where $[d\sigma/d\omega(k_+)]_{pv}$ is given by Eq. (18) and A is defined by Eq. (4).

The differential cross section for 400-Mev gamma rays is illustrated in Fig. 7. The total cross section has the value

$$\sigma=3.12\times 10^{-2}(g^2/4\pi)^2 \text{ millibarns.}$$

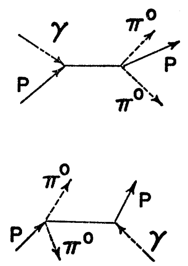

FIG. 9. Feynman diagrams for the process $\gamma+p\rightarrow p'+\pi^0+\pi^{0'}$ assuming pseudoscalar coupling.

From this we see that the ratio of the pseudoscalar to pseudovector cross section is 1.41 for 400-Mev gamma rays. The ratio of the (π^+, π^-) to (π^+, π^0) cross section for pseudoscalar coupling and the above energy is

$$\sigma_{ps}(\pi^+, \pi^-)/\sigma_{ps}(\pi^+, \pi^0)=37.$$

It is interesting to note that the most pronounced difference between pseudoscalar and pseuovector coupling in the production of a (π^+, π^0) meson pair is that the slope of the differential cross section at the upper end of the π^+ spectrum is zero for pseudovector coupling and infinite for pseudoscalar coupling. If Eq. (8) is used for the density of final states the slope of the differential cross section at the upper end of the π^+ spectrum becomes finite (but not zero) for pseudscalar coupling but remains zero for the pseudovector interaction.

(π^0, π^0) PAIR PRODUCTION

For completeness we mention briefly the cross section for the photoproduction of a (π^0, π^0) meson pair assuming pseudoscalar coupling. From the discussion given in reference 5 we see that in the low energy limit the entire contribution to this cross section comes from the core term, Eq. (19). The Feynman diagrams for the process are shown in Fig. 9.

The effective S matrix is given by

$$S=\frac{Meg^2\epsilon_\nu\delta(p_1+K-p_2-k_0-k_0')}{\mu^2[(2\pi)^7 2E_\gamma\omega(k_0)\omega(k_0')]^{\frac{1}{2}}}\times\left\{\frac{\bar{u}_f(2p_{1\nu}+\gamma_\lambda\gamma_\nu K_\lambda)u_i}{(p_1K)}-\frac{\bar{u}_f(2p_{2\nu}-\gamma_\nu\gamma_\lambda K_\lambda)u_i}{(p_2K)}\right\}. \quad (23)$$

Converting Eq. (23) to a cross section, we obtain

$$\frac{d\sigma}{d\omega(k_0)}=\frac{64}{137}\left(\frac{g^2}{4\pi}\right)^2\frac{k_0k_0'}{3\mu^4E_\gamma^3}(k_0^2+k_0'^2). \quad (24)$$

The total cross section for 400-Mev incident gamma rays in the laboratory coordinate system is

$$\sigma_{ps}(\pi^0, \pi^0)=0.059(g^2/4\pi)^2 \text{ millibarns}.$$

Comparing this to the other possible pair production cross sections at the same energy and with the same coupling, we see that

$$\sigma_{ps}(\pi^+, \pi^-)/\sigma_{ps}(\pi^0, \pi^0)=19.5,$$
$$\sigma_{ps}(\pi^+, \pi^0)/\sigma_{ps}(\pi^0, \pi^0)=0.53.$$

Since the core term does not appear in the pseudovector interaction, it follows from Eq. (5) that

$$\sigma_{pv}(\pi^0, \pi^0)\sim(\mu/M)^2\sigma_{ps}(\pi^0, \pi^0).$$

The explicit evaluation of the pseudodvector (π^0, π^0) cross section is laborious and will not be given here.

DISCUSSION OF RESULTS

From the preceding work we see that information on the meson-nucleon interaction may be obtained by studying the photoproduction of meson pairs when a gamma-ray bombards a hydrogen target. We shall briefly discuss possible experiments which may help in determining the type of coupling.

A. Absolute Value of the (π^+, π^-) Cross Section

To obtain a theoretical value for the pair-production cross section, we shall assume that the meson-nucleon coupling constant in the derivative coupling theory has a value[12] $g^2/4\pi=1/15$. Under this assumption the total pair-production cross section for 400-Mev gamma rays is

$$\sigma_{ps}(\pi^+, \pi^-)=5.1\times10^{-30} \text{ cm}^2,$$
$$\sigma_{pv}(\pi^+, \pi^-)=2.8\times10^{-31} \text{ cm}^2.$$

For this choice of coupling constant and gamma-ray energy the ratio of the single meson photoproduction cross section[13] to the pair cross section is

$$\sigma_{ps}(\pi^+)/\sigma_{ps}(\pi^+, \pi^-)\simeq35, \quad \sigma_{pv}(\pi^+)/\sigma_{pv}(\pi^+, \pi^-)\simeq650.$$

For 500-Mev gamma rays the above ratios become approximately 7 and 100, respectively.

The major contribution to the (π^+, π^-) cross section for pseudoscalar coupling comes from the important pair term, Eq. (19). However, the effective value of this term may be smaller, as indicated by scattering experiments.[14] In this case the pair-production cross section for the pseudoscalar interaction would be decreased. Further, it is obvious that given an experimental value for the cross section, a coupling constant may be found which makes either form of the interaction fit the results. Thus with this type of test we can at best surmise the the answer to our problem.

B. Ratio of the (π^+, π^-) to (π^+, π^0) Cross Section

A sensitive test, dependent only on the assumption that the meson-nucleon coupling constant is small, is provided by a measurement of the ratio of the (π^+, π^-) to (π^+, π^0) cross sections. According to our results, in the low energy limit this ratio should be $\sim(M/\mu)^2$ if pseudoscalar coupling is operative and should be ~1

[12] M. M. Lévy, Phys. Rev. **86**, 806 (1952); **88**, 72 (1952). Lévy's analysis gives a value for the pseudoscalar coupling constant. Converting this by Eq. (5), one finds that the value of the interaction constant in the derivative coupling theory is $g^2/4\pi=1/18$. A Klein [Phys. Rev. **89**, 1158 (1953)] and Drell, Huang, and Weisskopf [Phys. Rev. **91**, 460 (1953)] have indicated that an error exists in this work.

[13] L. L. Foldy, Phys. Rev. **76**, 372 (1949).

[14] See for example S. D. Drell and E. M. Henley, reference 3.

if the pseudovector interaction is involved. However, this difference is a direct consequence of our perturbation approach with weak coupling theory. If an intermediate or strong coupling calculation were made, the important core term which provides the entire contribution to the low energy (π^+, π^-) cross section in the pseudoscalar theory might be damped out[15] and the two coupling schemes would give approximately the same ratio for these cross sections.

C. Measurement of the Differential (π^+, π^0) Cross Section

Detailed experimental data on the shape of the (π^+, π^0) cross section near the upper end of the π^+ spectrum should indicate the type of interaction between mesons and nucleons.

[15] The damping of the pair term arising from one particular class of Feynman diagrams has been shown by Brueckner, Gell-Mann. and Goldberger, Phys. Rev. **90**, 476 (1953).

D. Other Possibilities

It is quite possible that a measurement of the differential (π^+, π^-) cross section either as a function of the angle between the emitted mesons or as a function of the energy of the π^- meson may give information on the type of interaction operative.

The author would like to express his sincere appreciation to Dr. S. D. Drell who originally suggested this investigation and who was a constant source of encouragement and stimulation during the couse of the work. Thanks are also due Prof. L. I. Schiff for many helpful suggestions. During the latter stages of this work the writer benefited greatly from many interesting discussions with Dr. D. R. Yennie.

PHYSICAL REVIEW VOLUME 92, NUMBER 5 DECEMBER 1, 1953

A Hypothesis Concerning the Relations among the "New Unstable Particles"*

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(Received August 17, 1953)

An attempt is made to systematize the present knowledge of phenomena concerned with the production, absorption, and decay of the "new unstable particles" in terms of only one "new" particle and its "compounds" with nucleons and π mesons.

THE great variety of new particles which cosmic-ray experiments have revealed in the last few years invites attempts to search for some unifying principle. We should like to put forward here a hypothesis which we are not able to work out in its complete form, but which we have found useful in correlating many phenomena. We shall start from some of the general ideas of Nambu, Oneda, Pais, and others[1] on pair production of V particles and make a number of tentative but specific assumptions. We find in this way that the fairly well-established experimental data are consistent with the following scheme which is designed to unify our knowledge of the "new unstable particles" in terms of only one "new" particle.

ASSUMPTIONS

(1) We shall assume that besides the π mesons there exists one other particle, fundamental in nucleon-nucleon interactions, which we shall call here the η meson. We shall assume that this is the particle which has sometimes been called phenomenologically V_2^0 or V_4^0, and which decays into two π mesons:[2]

$$\eta \rightarrow \pi^+ + \pi^- + 210 \text{ Mev}.$$

Thus, its mass is $m_\eta = 962 m_e$. If we take this decay scheme for granted, the η meson is a boson.[3] It further follows that the spin and parity of η are either both even $(0+, 2+, \cdots)$, or both odd $(1-, 3-, \cdots)$.

(2) Following previous considerations,[1] we shall assume that η mesons are created in pairs either through nucleon-nucleon or pion-nucleon collisions. (This assumption seems necessary to reconcile their comparatively long lifetime, of $\sim 10^{-10}$ sec, with their comparatively copious production.)

(3) We shall assume that η is a particle of isotopic spin $T=0$. (Reasons for this assumption are given below in 4b.)

(4) We shall assume that η can form "compounds" with either a nucleon or a pion.

(a) We shall assume that the compound of η and a neutron is a V_1^0:

$$V_1^0 = n + \eta - W_1^0,$$

and that the compound of η and a proton is a V_1^+:

$$V_1^+ = p + \eta - W_1^+,$$

where the binding energy $W_1^0 \approx W_1^+ \approx 310$ Mev is calculated from the energy release found in the decay of the V_1^0 (and V_1^+). The fundamental decay of a

* Under the auspices of the U. S. Atomic Energy Commission.

[1] For references, see A. Pais, Phys. Rev. **86**, 663 (1952).

[2] C. C. Butler in *Progress in Cosmic Ray Physics*, edited by J. G. Wilson (Interscience Publishers, Inc., New York, 1952), Chap. 2, pp. 65–123; Thompson, Buskirk, Etter, Karzmark, and Rediker, Phys. Rev. **90**, 329 (1953).

[3] R. W. Thompson *et al.* (see reference 2) emphasize that the present experimental evidence does not exclude the possibility of a decay $V_4^0 \rightarrow \pi + \mu$.