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Mixed Configurations in Nuclei*†

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The role of mixed configurations in nuclear states is discussed and illustrated by various examples; it is discussed in particular for β transitions with anomalous ft values and as a qualitative explanation of the empirically-found smooth variations of the first excited states of even-even nuclei. It is shown by various examples that a nucleon of one kind (neutron or proton) with a given j has a "stabilizing" influence on pairs of the other kind with a given j'. This stabilization effect is calculated on simplified assumptions for different j and j'.

INTRODUCTION

It is well known that many properties of nuclei can be traced back to the properties of a single nucleon to a surprisingly good approximation.[1] Thus, the clear-cut division of magnetic moments of odd-neutron or odd-proton nuclei into two groups has been nicely correlated with the single-particle Schmidt lines. Also, the assignment of degrees of forbiddenness of the beta decay of odd-even nuclei based on the assumption that the single odd nucleon alone takes part in the transition proves, in general, to be quite successful. Further, in the case of some gamma transitions, matrix elements calculated on the basis of such a single-particle approximation agree with experimental data within an order of magnitude.[2]

Another immediate result of the single-particle approximation which concerns itself with excited states of odd-even nuclei seems to hold in many cases; namely, the similarity of decay schemes of nuclei which differ by two neutrons or two protons. The smooth "movements" of corresponding levels when one goes in such "families" from one nucleus to the other[3] strengthens one's confidence in the interpretation of the similarities.

It seems, however, that a complete neglect of the structure of the "core" of an odd-A nucleus and its consideration only as the "carrier" of the required central force is an oversimplification of the problem, and a natural further step is to see whether effects due to a "core structure" can be established.[4] The configurations of equivalent nucleons and some cases of non-equivalent nucleons have been considered by various authors.[5–8]

THE ROLE OF MIXED CONFIGURATIONS IN NUCLEAR TRANSITIONS

It is well known that the "degree of forbiddenness" of a β spectrum is usually determined experimentally by its ft value, i.e., by the observed energy and half-life, eventually corrected by some factors which can be deduced from the shape of the spectrum. These ft values have been used to determine the spin and parity changes involved in β transitions, assuming that the transition probability depends, apart from a Coulomb correction, on just two parameters of the initial and final states (spin and parity). However, the matrix elements may depend, to a degree which has sometimes not been sufficiently appreciated, on the detailed structure of the wave functions of the initial and final states. An independent assignment of spin and parity to such states by a method which does not depend on the specific nuclear wave functions might, thus, reveal some wrong assignments of degrees of "forbiddenness," or, what often amounts to the same, some "super-fast" or "super-slow" β transitions. The internal conversion coefficients of γ rays depend only on their multipolarity, their energy, and the atomic number, and do not depend on the nuclear wave functions. One should, therefore, consider the spin assignments made on the basis of a measurement of internal conversion coefficients as more reliable than those made on the basis of ft values, and a disagreement between the two methods should be considered as an "irregularity" in the β-decay matrix elements.

One of the most striking examples of such a "disagreement" is the case of mass number 85 studied by Sunyar *et al.*[9] There, a logft value of 9.15 was found for

* Preliminary reports of this work were given at the Amsterdam Conference on Beta and Gamma Radioactivity [M. Goldhaber, Physica **18**, 1091 (1952)] and at the Rochester meeting of the American Physical Society, Phys. Rev. **92**, 843 (T) (1953).

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[1] For a review of the shell model, see, e.g., B. H. Flowers, Prog. Nuc. Phys. **2**, 235 (1952).

[2] M. Goldhaber and A. W. Sunyar, Phys. Rev. **83**, 906 (1951).

[3] M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).

[4] By "core" here we mean all the nucleons except the odd one and its equivalent nucleons, though sometimes the term is used more loosely to mean all the nucleons except the odd one.

[5] D. Kurath, Phys. Rev. **80**, 98 (1950).

[6] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).

[7] B. H. Flowers, Phys. Rev. **86**, 254 (1952), Proc. Roy. Soc. (London) **215**, 398 (1952) and previous publications.

[8] J. P. Davidson and E. Feenberg, Phys. Rev. **89**, 856 (1953).

[9] Sunyar, Mihelich, Scharff-Goldhaber, Goldhaber, Wall, and Deutsch, Phys. Rev. **86**, 1023 (1952). See also M. Trocheris, J. Phys. **13**, 370 (1952).

a β transition which is definitely "allowed": $g_{9/2} \rightarrow g_{9/2}$. It was suggested[9] that the reason for this behavior might be connected with an extreme lack of overlapping of the initial and final core wave functions which causes this large reduction in the transition probability. It was also noted there that the even-neutron nucleus $_{37}Rb^{85}{}_{48}$ should have different neutron configurations in the ground state ($f_{5/2}$ proton) from that in the excited state ($g_{9/2}$ proton); the effect of the state of the odd particle on the core configuration is discussed below.

An inspection of discrepancies of a similar type in other nuclei (see below) suggests that they can usually be understood in terms of the "purity" of the states involved. This concept of purity of a state can perhaps be most clearly exhibited in the following way:

One knows now that practically without exception the first excited state of an even-even nucleus is a 2+ state.[2,10,11] This is interpreted as evidence for the assumption that this state is an excited state of the ground state configuration,[2,7,12,13] i.e., it is caused by a change of the relative orientation of the j's of the different nucleons, each of them remaining in the same j state which it occupied in the ground state.

It is also known[11,12] that in the regions of the periodic table in which the single-particle model holds it usually requires more energy to excite the 2+ state in an even-even nucleus than that required for the excitation of the first few excited states of a neighboring odd-even nucleus. In those regions where the 0–2 separation in even-even nuclei is large, one finds as a rule that each (low-lying) excited state in an odd-even nucleus is a ground state of a new configuration and that the spacing between ground states of different (odd) configurations is smaller than the spacing of levels in the same configuration. Conspicuous exceptions to this behavior are the occurrence of low-lying 7/2+ states[2] for the configurations $g_{9/2}{}^{3,\,5,\,\text{or}\,7}$.

For instance, if we consider the two configurations (a) $p_{3/2}{}^2 p_{1/2}$ and (b) $p_{3/2}{}^2 g_{9/2}$, (a) will have a set of states with total $J_a = 1/2,\ 3/2,\ 5/2$, of which $J_a = 1/2$ is the lowest; and (b) will have a set of states with total $J_b = 5/2,\ 7/2,\ 9/2,\ 9/2,\ 11/2$, and $13/2$, of which $J_b = 9/2$ is the lowest. The experimental data now enable one to conclude that the separation in energy between $J_a = 1/2$ and $J_b = 9/2$ is, as a rule, smaller than the separation between different J_a's or between different J_b's. There will be occasions when we are dealing with different but close lying configurations of equal J, and where we may therefore expect a strong configuration interaction. This configuration interaction arises because of the finer details of the interaction between nucleons which are ignored in the zero-order approximation; deviations from the average central potential can be considered as a perturbation which mixes configurations. One is, therefore, led to believe that it might be a poor approximation to think in terms of pure configurations, and that instead of assigning the ground state of a certain nucleus, say, as $(p_{3/2}{}^4 p_{1/2}{}^2 g_{9/2}{}^4)_{J=0}$, one should rather consider it as a mixture of the states $(p_{3/2}{}^4 g_{9/2}{}^6)_{J=0}$, $(p_{3/2}{}^2 p_{1/2}{}^2 g_{9/2}{}^6)_{J=0}$, etc. The existing empirical evidence for a strong pairing energy,[1] makes it unlikely that a considerable amount of a configuration in which a pair is broken [say, $(p_{3/2}{}^3 p_{1/2} g_{9/2}{}^6)$, which has also a state $J=0$] will be mixed in; but it is most probable that the other combinations (those in which pairs are shifted from one state to another) will be mixed considerably. Clearly such mixtures of configurations in the initial and final states will generally tend to change appreciably any transition probability between these two states, and it is only when the states become "pure"—composed of essentially a single configuration—that the transition probability may approximate closely its "intrinsic" value. Depending on whether in these extreme cases the core does or does not change its state in the transition, we shall find transitions that are either very slow or very fast compared to ordinary ones between "average" impure states. Obviously a state is comparatively pure if for some reason the other states which could have mixed with it require a comparatively high energy for their excitation. This could happen, for instance, if the number of nucleons in the configuration considered (protons or neutrons) approaches a magic number, in which case the configurations available for mixtures are energetically far removed.

One also sees that a fair amount of configuration mixing will destroy the symmetry between particles and holes in a subshell below a magic number. For instance, a $g_{9/2}$ state with 41 protons could be a mixture of $p_{1/2}{}^2 g_{9/2}$, $g_{9/2}{}^3$ and perhaps other configurations too; whereas a $g_{9/2}$ state for 49 protons can only be $g_{9/2}{}^{-1}$. Thus, configurations containing one hole in a major shell are expected to be less mixed than those containing one particle beyond a subshell. This, of course is due to the fact that sub-shells are not as tightly closed as the major shells. The absence of symmetry between particles and holes is probably exhibited by the empirically found "movements" of levels,[3] especially the $1h_{11/2} - 2d_{3/2}$ separation, and by the spins of odd-odd nuclei in light elements.[14] It seems[3] that even for three holes the configurations are less mixed than for three particles.

OVERLAP STABILIZATION OF ENERGY LEVELS

A possibility deserving investigation is that one group of nucleons (protons or neutrons) influences the other in such a way as to make a particular configuration energetically more favorable. Such an effect might exist in Rb^{85}, where, as has been noted,[9] the predominant core configuration probably depends on the odd-proton

[10] P. Stähelin and P. Preiswerk, Helv. Phys. Acta 24, 623 (1952).
[11] G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).
[12] de-Shalit, Huber, and Schneider, Helv. Phys. Acta 25, 279 (1952).
[13] Horie, Umezawa, Yamaguchi, and Yoshida, Prog. Theor. Phys. (Japan) 6, 254 (1951).
[14] R. W. King and D. C. Peaslee, Phys. Rev. 90, 1001 (1953).

state. We shall refer to this effect as a stabilization of one configuration by another.

One can obtain a rough idea of the essential features of the stabilization effect by considering the following simple problem: A pair of neutrons in the state $(j_N{}^2)_{J=0}$ interact with an odd proton in a state j_P. The interaction energy clearly depends on what these two states are. It now turns out, as could be expected, that for a given j_P the first-order interaction is biggest if $j_N \approx j_P$, or more precisely if $l_N \approx l_P$. In fact, it is shown in the Appendix that for a neutron-proton interaction of the type $V(|\mathbf{r}_P - \mathbf{r}_N|)$ the above interaction energy is simply

$$\delta E = \int R^2(n_N l_N) R^2(n_P l_P) V(|\mathbf{r}_P - \mathbf{r}_N|) d(\cos\omega) dr_P dr_N,$$

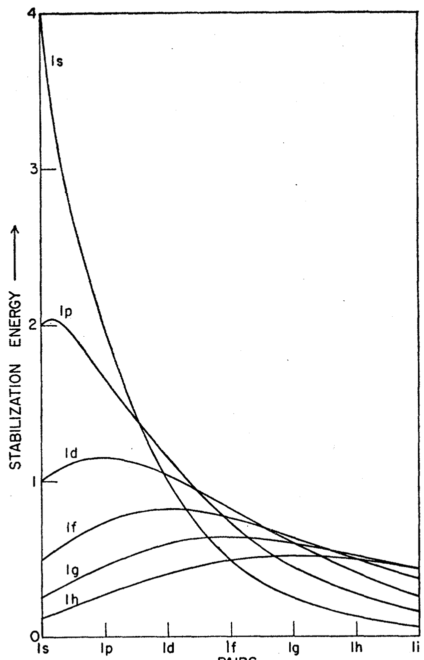

FIG. 1. Stabilization of pairs of nucleons of one kind (neutrons or protons) by a single nucleon of the other kind. The unit of energy is ≈400 kev.

where $\cos\omega = \mathbf{r}_P \cdot \mathbf{r}_N / r_P r_N$ and $R(nl)/r$ is the radial part of the wave function of the nucleon.

For more-or-less reasonable forms of $V(|\mathbf{r}_P - \mathbf{r}_N|)$ this integral would attain its maximum value when $R(n_N l_N)$ and $R(n_P l_P)$ overlap best, i.e., when $l_N \approx l_P$ and $n_N = n_P$. For illustration we show in Fig. 1 a plot of the values of δE for various values of l_N and l_P and $n_N = n_P = 1$. These values were obtained by taking for the wave functions of the nucleons the eigenfunctions of the three-dimensional harmonic oscillator,[6] and assuming a δ interaction, $V(|\mathbf{r}_P - \mathbf{r}_N|) = 4\pi g \delta(\mathbf{r}_P - \mathbf{r}_N)$.

Thus, in the case of Rb^{85}, the $g_{9/2}$ proton in the excited state stabilizes the $g_{9/2}$ neutron pairs so that the neutron configuration of the last pair is predominantly

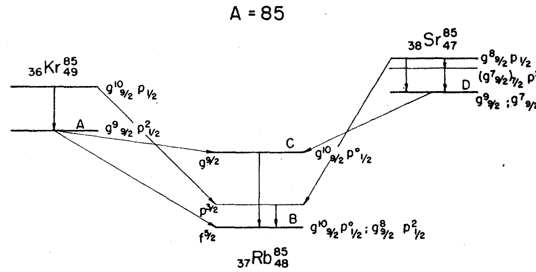

FIG. 2. The main part of the configurations involved in the abnormal β transitions which are important for the present discussion is given in this and the following figures for each level on the right for neutrons and on the left for protons. Whenever two configurations are given, it is meant to imply mixed configurations with comparable amplitudes. For details of the decay schemes, see, e.g., reference 3.

For level C the $g_{9/2}$ proton stabilizes the $g_{9/2}$ pairs of neutrons thus making the neutron configuration mainly $g_{9/2}{}^{10}$. This makes the transition $A \rightarrow C$ very improbable but does not affect $D \rightarrow C$. The other states of Rb^{85} in which there is no stabilization of $g_{9/2}$ pairs have mixed neutron configurations, as is shown by the "normal" behavior of $A \rightarrow B$.

$(g_{9/2})^2$ rather than $(p_{1/2})^2$; the latter would have yielded a considerably lower ft value than the one observed. The case of mass number 85 and the other β transitions with abnormal ft values mentioned in Table I are illustrated in Figs. 2–8.

EVIDENCE FROM β DECAY

Table I contains some of the "abnormal" ft values together with the spin assignments for both initial and final states when they are known from other sources.[3,15–18] The $\log ft$ values should be compared with the average ones for normal transitions, namely 4.5–5.5 for allowed and 6.2–7.2 for first forbidden transitions. It is remarkable that most of the examples of abnormal ft values occur close to magic numbers, and mainly

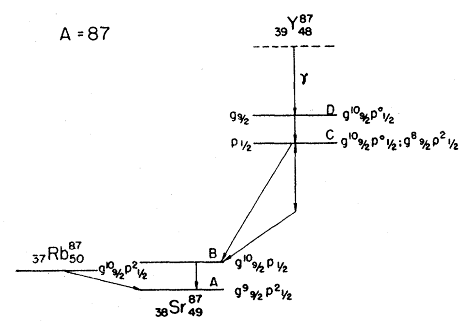

FIG. 3. For level C the $p_{1/2}$ proton stabilizes $p_{1/2}{}^2$ neutrons so that $g_{9/2}{}^8 p_{1/2}{}^2$ predominates for the neutron configuration, and for level D stabilization by the $g_{9/2}$ proton makes $g_{9/2}{}^{10}$ the predominant neutron configuration. Both transitions $D \rightarrow A$ (unobserved) and $C \rightarrow B$ are thus slowed down.

[15] King Dismuke and Way, Oak Ridge National Laboratory Report ORNL-1450 (unpublished).
[16] L. G. Mann and P. Axel, Phys. Rev. **84**, 221 (1951).
[17] Shore, Bendel, Brown, and Becker (private communication); Phys. Rev. **91**, 1204 (1953).
[18] E. J. Konopinski and L. M. Langer, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1953), Vol. 2, p. 261.

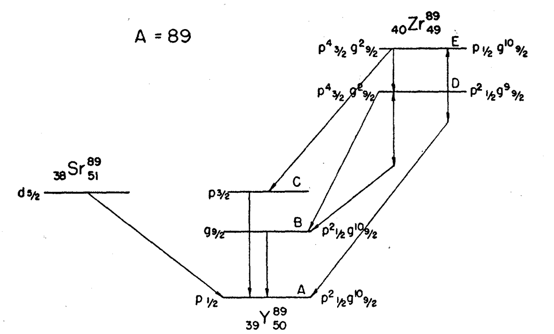

FIG. 4. The big number of $g_{9/2}$'s in the neutron group of Zr stabilize $g_{9/2}{}^2$ for the proton group, though to a lesser degree in D than in E. Thus $D \rightarrow B$ is only slightly affected, and $E \rightarrow A$ is affected more. $E \rightarrow C$ is normal.

from the lower mass number side (holes); the assumption of "purified" states in these cases, therefore, seems to be quite plausible.

Two more examples which should perhaps be added to this table are the odd-odd isotopes $_{81}Tl^{204}{}_{123}$ and $_{81}Tl^{206}{}_{125}$. The β spectrum of $_{81}Tl^{204}{}_{123}$ to the ground state of $_{82}Pb^{204}{}_{122}$ shows the shape characteristic of a ($\Delta I = 2$, yes) transition. The ground state of Tl^{204} would then be $2-$, consistent with the Nordheim rule ($d_{3/2}$ proton and $p_{1/2}$ neutron). However, the transition to the first excited state in Pb^{204} ($2+$) is not observed,[3] $\log ft > 10$, whereas transitions of this type are usually found to have $\log ft$ values of about 6.2 to 7.2. This behavior might be related to the fact that in Pb^{204} the proton shell is filled, so that the $2+$ state arises from an excitation of the neutrons only. On the other hand, if we assume that the neutrons in Tl^{204} first couple to a $p_{1/2}$ state and, therefore, do not contain a $2+$ state of the neutron core, the absence of a transition to the $2+$ state of Pb^{204} could be understood. The assumption that the $2+$ state in Pb^{204} arises from a neutron configuration gets further support from the slowness of the $E2$ transition to the ground state.[2,19,20]

$_{81}Tl^{206}{}_{125}$ has a very low $\log ft$ (5.18), although the transition is expected to be first forbidden. Even if we assume the most favorable spin and parity compatible with the shell model for the ground state of Tl^{206}, $1-$ ($d_{3/2}$ proton and $p_{1/2}$ neutron), we must conclude that the large size of the matrix element is due to an exceptional purity of the wave function near a magic number. Recently Bleuler *et al.*[21] found a very low ft value for the allowed β transition $_{49}In^{112}{}_{63} \rightarrow {}_{50}Sn^{112}{}_{62}$ ($\log ft = 4.03$). Again we probably are dealing with extremely pure wave functions near a magic number.

We thus see that near closed shells, allowed and first forbidden beta transitions occur with the square of the matrix element either close to unity or smaller than the "average" by a big factor, up to $\sim 10^4$. If we turn to the most completely studied isomeric transitions, we see that for $M4$ transitions,[22,23] and perhaps for some odd proton $E3$ transitions,[24] the matrix elements show a tendency to increase as a closed shell is approached. Very small matrix elements, however, are absent among the $M4$ transitions.

EVEN-EVEN NUCLEI

The idea of configuration interactions seems to be especially useful, at least qualitatively, in even-even nuclei. If we assume, as our zero-order approximation, independent groups of protons and neutrons, then each of them can give rise to the characteristic set of levels $0+$, $2+$, $4+$, etc. If there were no interaction between the protons and the neutrons, we would have observed a lowest state $(0_P 0_N)_0$ where $(J_P, J_N)_J$ represents a state in which the protons are coupled to an angular momentum J_P, the neutrons to J_N and both together give J; the next excited states would have been $(0_P 2_N)_2$, $(2_P 0_N)_2$ and $(2_P 2_N)_{0,1,2,3,4}$, etc. The $(0_P 0_N)_0 - (2_P 0_N)_2$ separation would have been exactly the same as the 0–2 separation for the proton group in the absence of the neutron group. If we now consider the effect of the interaction between the protons and the neutrons, different states with the same total angular momentum will interact, with a resulting "repulsion" of levels.[25] Since this repulsion is inversely proportional to the separation between the unperturbed

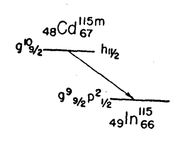

FIG. 5. The $h_{11/2}$ neutron in Cd stabilizes the $g_{9/2}$ protons so that the proton configuration is predominantly $g_{9/2}{}^{10}$, and β decay becomes slower.

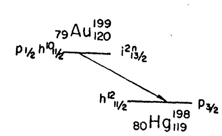

FIG. 6. The $i_{13/2}$ neutrons stabilize $h_{11/2}$ protons both in Au and Hg so that in Au the proton configuration is $p_{1/2}h_{11/2}{}^{10}$ and in Hg it is $h_{11/2}{}^{12}$ with a resulting reduction in the transition probability in the β decay.

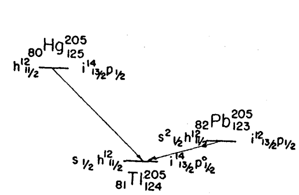

FIG. 7. A similar mutual stabilization of $h_{11/2}$ protons by $i_{13/2}$ neutrons makes the β transitions from Hg^{205} favorable and the electron capture from Pb^{205} unfavorable.

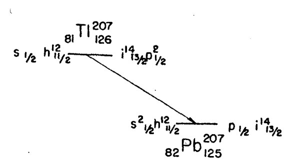

FIG. 8. The configurations are determined uniquely, and it is easy to see that the transition probability should obtain its highest value.

[19] A. Bohr and B. R. Mottelson, Phys. Rev. **89**, 316 (1953).
[20] A. Bohr and B. R. Mottelson, Phys. Rev. **90**, 717 (1953).
[21] Bleuler, Blue, Chowdary, Johnson, and Tendam, Phys. Rev. **90**, 464 (1953).
[22] S. A. Moszkowski, Phys. Rev. **89**, 474 (1953).
[23] K. Gottfried, thesis, McGill University, 1953 (unpublished).
[24] R. D. Hill, Phys. Rev. **88**, 1428 (1952).
[25] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1951).

TABLE I. β-Transitions with abnormal ft values.

Nucleus		Spin and parity		Expected type of transition	Observed log ft	References
Initial	Final	Initial state	Final state			
$_{36}Kr^{85}{}_{49}$	$_{37}Rb^{85}{}_{48}$	$g_{9/2}$	$g_{9/2}$	allowed	9.03	a
$_{39}Y^{87m}{}_{48}$	$_{38}Sr^{87}{}_{49}$	$g_{9/2}$	$g_{9/2}$	allowed	>8.45	b
$_{39}Y^{87}{}_{48}$	$_{38}Sr^{87}{}_{49}$	$p_{1/2}$	$p_{1/2}$	allowed	7.6	c
$_{40}Zr^{89m}{}_{49}$	$_{39}Y^{89}{}_{50}$	$p_{1/2}$	$p_{1/2}$	allowed	6.85	a
		$p_{1/2}$	$p_{3/2}$	allowed	4.3	d
$_{48}Cd^{115m}{}_{67}$	$_{49}In^{115}{}_{66}$	$h_{11/2}$	$g_{9/2}$	first forbidden	8.8	a
$_{79}Au^{199}{}_{120}$	$_{80}Hg^{199}{}_{119}$	$d_{3/2}$	$p_{1/2}$	first forbidden	7.73	a
$_{80}Hg^{205}{}_{125}$	$_{81}Tl^{205}{}_{124}$	$p_{1/2}$	$s_{1/2}$	first forbidden	5.52	a
$_{82}Pb^{205}{}_{123}$	$_{81}Tl^{205}{}_{124}$	$p_{1/2}$ or $p_{3/2}$	$s_{1/2}$	first forbidden	very big	e
$_{81}Tl^{207}{}_{126}$	$_{82}Pb^{207}{}_{125}$	$s_{1/2}$	$p_{1/2}$	first forbidden	5.16[f]	a
$_{82}Pb^{209}{}_{127}$	$_{83}Bi^{209}{}_{126}$	$i_{11/2}$ or $g_{9/2}$	$h_{9/2}$	first forbidden	5.59	d

[a] See reference 15.
[b] See reference 3.
[c] See reference 16.
[d] See reference 17.
[e] Private communication from C. D. Coryell, based on unpublished results of T. P. Kohman.
[f] The effect of the large Z may be to some extent responsible for the smallness of the ft values of these "first forbidden" transitions (see reference 18), but no general trend with Z is noticeable.

states, one could expect it to be strongest for a pair of 2+ states $(2_P0_N)_2$ and $(0_P2_N)_2$. This leads to mixing of proton and neutron states, and one of the mixed states would be brought nearer to the ground state, while the other will approach the next higher group of states and eventually pass some of them, provided they are of different total J.[25] If, in the zero-order approximation (no p-n interaction), there were no other 2+ state between the $(2_P0_N)_2$ and the $(0_P2_N)_2$ states, then the first two 2+ states of an even-even nucleus would contain only seniority-two components.[7] It would then be difficult to understand the rule found by Kraushaar and Goldhaber[26] for even-even nuclei that for a sequence of states 0+, 2+, 2+ the matrix element for the $E2$ crossover transition (2+→0+) is in many cases considerably smaller than that for the $E2$ fraction in the 2+→2+ transition. If, however, a 2+ state with seniority-four components would already exist between $(0_P2_N)_2$ and $(2_P0_N)_2$ in the zero-order approximation, it would remain the second 2+ state also after considering the p-n interaction, because states of equal J do not cross. If we compare the estimates of the positions of the 2+ states of seniority two and four[7] with the experimentally found energy of ~1 Mev required to excite a pair of neutrons (protons magic) and the energy of ~1.5 Mev required to excite a pair of protons (neutrons magic),[11] we can expect such a situation to exist when we are fairly close to a magic number in either protons or neutrons. Then the experimentally found rule[26] could be explained in terms of the selection rules for the seniority number v ($\Delta v=\pm 2, 0$).[27]

The regularities observed for the 0+−2+ separations in even-even nuclei[10,11] may be described in this way: As one approaches a magic number from either side, the number of different possibilities for states with a total $J=2$ becomes smaller and smaller and this reduces the repulsion of the lowest 2+ state.[11,28,29] At the magic number of one of the kinds of nucleons (protons or neutrons), this kind can no longer be excited to a 2+ state. This naturally results in a maximum for the 0+−2+ separation in that region. The largest number of possibilities of creating a 2+ state occurs in the middle between closed shells with a corresponding minimum in the 0+−2+ separation. The states of $J=0$ which may mix and lead to a depression of the ground state are usually higher and fewer in number than those of $J=2$,[7,29] and their effect may thus be expected to be smaller.[30] It is hard to estimate these effects quantitatively since the situation is very complex.[31] The general trend found experimentally is in agreement with what one might qualitatively expect. The strong mixture of configurations is probably responsible for a "smearing out" of most of the sub-shell effects.[11] For the heaviest elements known at present, the 2+ state is only ~40 kev above the 0+ ground state. As the middle of either the proton shell or the neutron shell has not yet been reached in this region, the possibility cannot be excluded on these considerations alone that the 2+ state eventually may cross the 0+ state to become the ground state.

A somewhat more quantitative idea of the amount of configuration mixing of different states in even-even nuclei may be gained by comparing the rate of β decay from a 1+ state in an odd-odd nucleus to the ground state ($J=0$) and the first excited state ($J=2+$) of the even-even daughter. This ratio, very much like the β^+/K capture ratio, should be independent of the

[26] J. J. Kraushaar and M. Goldhaber, Phys. Rev. **89**, 1081 (1953).
[27] G. Racah, Phys. Rev. **62**, 438 (1942).
[28] P. Preiswerk and P. Stähelin, Physica **18**, 1118 (1953).
[29] H. J. Maehly and P. Stähelin, Helv. Phys. Acta **25**, 624 (1952).
[30] B. J. Hogg and H. E. Duckworth have kindly informed us of their recent mass measurements [Phys. Rev. **91**, 1289 (1953)] which indicate the existence of a broad maximum of the total binding energy in the region in which the 2+−0+ separation reaches a broad minimum (approximately in the middle of the proton (50–82) and neutron (82–126) shells).
[31] A simplified case has been treated quantitatively by A. Temkin and A. de-Shalit (unpublished).

nuclear matrix elements if both states of the even-even daughter would belong to the same configuration. Its evaluation is then quite straightforward by using Racah's tensor-operator formalism.[27] With Gamow-Teller selection rules one obtains for the transition probability from the state $(j_n j_p J_i)$ to the state $(j_p^2 J_f)$, where j_p and j_n are the angular momenta of the odd proton and the odd neutron, respectively, J_i is the total angular momentum of the initial state in the odd-odd nucleus, and J_f is the total angular momentum of the final state in the even-even nucleus:[32]

$$|M|^2=(2J_f+1)(2j_n+1)\cdot|M_1|^2\cdot|W(j_pJ_fj_nJ_i;j_p1)|^2,$$

where $|M_1|^2$ is the transition probability for a single neutron j_n to a proton j_p and W is a Racah coefficient.[27] The ratio in question will then be

$$R=\frac{|M(1\rightarrow2)|^2}{|M(1\rightarrow0)|^2}=5\cdot\left|\frac{W(j_p2j_n1;j_p1)}{W(j_p0j_n1;j_p1)}\right|^2=15(2j_p+1)\times W(j_p2j_n1;j_p1)|^2.$$

TABLE II. Ratio of ft values for β transitions $1^+\rightarrow0^+$ and $1^+\rightarrow2^+$.

Parent nucleus	Daughter nucleus	$\log\frac{ft(1\rightarrow0)}{ft(1\rightarrow2)}$ (exp)	j_n	j_p	$\log R$ (theor)	Remarks
${}_5B^{12}{}_7$	${}_6C^{12}{}_6$	-0.62	1/2	3/2	$+0.40$	
${}_{15}P^{34}{}_{19}$	${}_{16}S^{34}{}_{18}$	$+0.41$	1/2	3/2	$+0.40$	spin of excited state not known
${}_{29}Cu^{66}{}_{37}$	${}_{30}Zn^{66}{}_{66}$	-0.07	3/2	5/2	$+0.15$	spin of excited state may be 1+ or 2+ (11)
${}_{45}Rh^{106}{}_{61}$	${}_{46}Pd^{106}{}_{60}$	-0.56	7/2	9/2	-0.26	
${}_{47}Ag^{110}{}_{63}$	${}_{48}Cd^{110}{}_{62}$	$+0.62$	7/2	9/2	-0.26	
${}_{53}I^{128}{}_{75}$	${}_{54}Xe^{128}{}_{74}$	-0.70	3/2	5/2	$+0.15$	

Table II contains some experimental data on this ratio together with the theoretical predictions for various reasonable spin assignments. The lack of agreement between "theory" and experiment probably arises from the erroneous assumption that the 0+ and the 2+ states are "pure" states of one and the same configuration. It looks as if both states are very well mixed with proper states of other configurations.

SOME SPECIAL EXAMPLES

There are a number of other regularities in nuclear spectroscopy which might possibly be explained along the general lines developed here. Since this paper was meant mainly to suggest a possible direction in which the strict shell model might successfully be modified, we shall say only a few words on some additional applications of present ideas.

It was pointed out[3] that a plot of the $p_{1/2}-g_{9/2}$ separation for odd-proton nuclei as a function of the neutron number has a very pronounced minimum at $N=50$ (see reference 3, Fig. 75). If we assume (as is generally and consistently done) that the last pair of neutrons filling the 50-shell is $p_{1/2}{}^2$, this minimum is explained by the extra stabilization given to the $p_{1/2}$ proton by this neutron pair. The next pair of neutrons will tend to fill in $g_{7/2}$ states, thus depressing the $g_{9/2}$ proton state again.

Inspecting the $h_{11/2}-d_{3/2}$ separation in, say, ${}_{52}Te^{129}{}_{77}$, ${}_{54}Xe^{131}{}_{77}$, ${}_{56}Ba^{133}{}_{77}$ (reference 3, Fig. 78), one sees that it becomes harder and harder to excite the $h_{11/2}$ state as the number of protons increases. If the $h_{11/2}$ state is interpreted as arising from the breaking of an $h_{11/2}$ pair,[3,12] one might interpret this behavior as due to the increasing stabilization of $h_{11/2}$ pairs as $g_{7/2}$ protons fill in. An alternative interpretation is to say that $d_{5/2}$ protons are filled in this set of nuclei with a corresponding increase in the binding energy of the $d_{3/2}$ neutron states. This last example is typical of many cases in which no unique interpretation is suggested. More information on the nature of the states involved (such as their magnetic and electric moments) and better calculations could perhaps decide the relative importance of each configuration.

A similar effect might be responsible for the different spins of ${}_{51}Sb^{121}{}_{70}$ and ${}_{51}Sb^{123}{}_{72}$. These isotopes have a single odd proton outside a closed proton shell. The ground state of Sb^{121} is $d_{5/2}$, whereas that of Sb^{123} is $g_{7/2}$. This could perhaps be explained by the stabilization of the $g_{7/2}$ state due to the two extra neutrons going into $h_{11/2}$ orbits which start not later than for 71 neutrons. On the same argument, one might have expected that the $g_{7/2}$ state is more stable than the $d_{5/2}$ state in ${}_{53}I^{125}{}_{72}$ and ${}_{53}I^{127}{}_{74}$. Actually, however, the observed ground state is 5/2+ for these two isotopes, whereas it becomes $g_{7/2}$ for the higher isotopes. On the stabilization principle, we should therefore expect a considerable amount of $(g_{7/2}{}^3)_{5/2}$ state in I^{125} and I^{127}. The unusually large deviation of the magnetic moment of I^{127} from the Schmidt value for a $d_{5/2}$ proton might be explained in this way. A somewhat smaller deviation might be expected for I^{125}.

One of the properties of nuclei which is sensitively affected by the mixing of different configurations is the g factor. The effect of mixing different configurations in the "odd group" alone is to cause a deviation from the Schmidt lines which is proportional to g_s-g_l, where g_s and g_l are the spin and orbital g-factors of a single nucleon in the odd group.[33] As (g_s-g_l) is 4.6 for protons and -3.8 for neutrons, this would meaa that the distribution of the deviations of odd-P nuclei from the Schmidt lines should be similar to that of the odd-N ones, enlarged by the factor 4.6/3.8. Existing data seem to favor these results and give further support to the hypothesis of mixed configurations. Different types of

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[33] A. de-Shalit, Phys. Rev. 90, 83 (1953).

mixed configurations where the core is excited to a 2+ state have been considered by Davidson and Feenberg.[8]

As magnetic moments of excited states are measured, valuable information on the type of mixing will become available. Thus, for instance, it is easily seen from the Landé formula that all states of a configuration of equivalent nucleons, irrespective of their number, have the same g factor. A measurement for an even-even nucleus of the g factor of a 2+ excited state, and especially of its sign, will thus show how pure these configurations are.

DISCUSSION

Nuclear models have gone from the extreme of the early version of the one-particle model to the other extreme of Bohr's liquid-drop model, then to the revised version of the one-particle strong spin-orbit coupling model, and quite recently several compromises have been considered.[1] In particular, the models, which treat the bulk of nucleons in a nucleus on a collective basis and add the effects of individual particles,[34–36] have many attractive features because calculations carried out on these models have been able to explain several nuclear properties for which the single-particle model failed to account. The approach which we have suggested here as the next approximation to the one-particle model may be more advantageous near closed shells where the relative purity of wave functions plays an important role. In the middle of shells, the existence of large mixtures of configurations may have an averaging effect equivalent to the assumption of a deformable core which is the starting point in the collective model. The relative usefulness of the various models may depend, therefore, very much on the particular nucleus and the phenomenon treated on the one hand, and the ability to overcome the mathematical difficulties of the necessarily complex situations on the other hand.

We should like to thank many of our colleagues who have helped us with valuable discussions, in particular D. R. Hamilton, Gertrude Scharff-Goldhaber, I. Talmi, and J. Weneser.

APPENDIX

To calculate the diagonal matrix elements of the interaction in a state JM between a pair of protons (j_P^2) with total $J_P=0$ and a neutron j_N, we note that $V_{10}+V_{20}$ can be written in the form[27]

$$V=V_{10}+V_{20}=\sum_{k,\kappa}[v_k(r_1,r_0)C_\kappa^{(k)}(1)C_\kappa^{(k)}(0)+v_k(r_2,r_0)C_\kappa^{(k)*}(2)C_\kappa^{(k)}(0)],$$

where V_{10} is the interaction between the first proton and the neutron, and V_{20} has a similar meaning;

$$C_\kappa^{(k)}(i)=\left(\frac{4\pi}{2k+1}\right)^{\frac{1}{2}}Y_{k,\kappa}(\theta_i,\varphi_i),$$

and

$$v_k(r_i,r_0)=\frac{2k+1}{2}\int V_{i0}P_k(\cos\omega)d(\cos\omega),$$

$$\cos\omega=\mathbf{r}_i\cdot\mathbf{r}_0/r_ir_0.$$

Denoting

$$F^{(k)}\equiv\int R_P^2(n_Pl_P)R_N^2(n_Nl_N)v_k(r_Pr_N)dr_Pdr_N,$$

where R/r is the radial part of the wave function, we find that

$$((j_P^2)J_Pj_NJM|V|(j_P^2)J_Pj_NJM)=\sum_{k,\kappa}F^{(k)}(J_Pj_NJM|C_\kappa^{(k)*}(P)C_\kappa^{(k)}(0)|J_Pj_NJM)$$

where

$$C_\kappa^{(k)}(P)=C_\kappa^{(k)}(1)+C_\kappa^{(k)}(2).$$

Since $C^{(k)}(P)$ is a tensor of degree k with respect to J_P, we see immediately that for $J_P=0$ only the term with $k=0$ will contribute and since $C^{(0)}(0)=1$ and $C^{(0)}(P)=2$, we finally get

$$\langle|V|\rangle=2F^{(0)}.$$

For the interaction between $2r$ protons coupled to $J_P=0$ and n neutrons, one gets in a similar way

$$\langle|V|\rangle=2r\cdot n\cdot F^{(0)}.$$

For the numerical evaluations of these results we assume an interaction between the protons and the neutrons of the type

$$V_{NP}=4\pi g\delta(\mathbf{r}_N-\mathbf{r}_P)=g\sum_k\frac{\delta(r_N-r_P)}{r_Nr_P}(2k+1)P_k(\cos\omega),$$

and take as the wave function those obtained for a nucleon moving in a harmonic oscillator potential,[37] $[(\hbar\nu)^2/2M]r^2$. $F^{(0)}$ is then obtained in the form:

$$F^{(0)}=g\int_0^\infty R^2(n_Pl_P)R^2(n_Nl_N)\frac{dr}{r^2}.$$

For the special case of $n_P=n_N=1$, one finds

$$F^{(0)}=g\left(\frac{\nu^3}{2\pi}\right)^{\frac{1}{2}}\frac{(2l_P+2l_N+1)!!}{2^{l_P+l_N-1}(2l_P+1)!!(2l_N+1)!!}.$$

The value of $g(\nu^3/2\pi)^{\frac{1}{2}}$ can be determined by comparing the observed 0–2 separation for double-magic-plus-or-

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[35] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).
[36] K. W. Ford, Phys. Rev. **90**, 29 (1953).
[37] See, for instance, reference 6.

minus-two even-even nuclei with theoretical predictions. In the δ limit this separation is found to be[38]

$$\tfrac{1}{2}(2j+1)^2[(j\tfrac{1}{2}j-\tfrac{1}{2}|jj00)^2-\tfrac{1}{5}(j\tfrac{1}{2}j-\tfrac{1}{2}|jj20)^2]F^{(0)}$$

for a pair of nucleons in the state (l, j). Comparing this expression with the experimental data[11] on Ca^{42} and Pb^{206}, two nuclei which differ by one nucleon pair from double-magic nuclei, one obtains

$$g(\nu^3/2\pi)^{\frac{1}{2}}\approx 800 \text{ kev}.$$

The same value is obtained by considering the pairing energy.[39]

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Decay of $_{66}Dy^{165m}$(1.2 min) and $_{66}Dy^{165}$(2.3 hr)

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The activities induced by neutron capture in Dy^{164} have been studied with 180° photographic internal conversion electron spectrometers and a scintillation coincidence spectrometer. The metastable transition energy is 108.0±0.2 kev. Other gamma rays of approximately 160, 360, and 515 kev are associated with the 1.2-min activity and appear to follow beta decay from the metastable level. Gamma rays of 94.4±0.2, 279.4±0.8, 361.2±1.0, 634±3, 710±20, and 1020±30 kev follow the 2.3-hr beta decay from the ground state. Coincidences are observed between members of the pairs (279)–(710) and (361)–(634). The 94-kev gamma ray is coincident with a beta transition of about 1.2 Mev, while the other gamma radiations are coincident with a softer beta component (∼0.3 Mev).

INTRODUCTION

IN 1935, Marsh and Sugden[1] and, independently, Hevesy and Levi[2] reported that a very strong beta activity was produced when Dy was exposed to neutrons from a Ra–Be source. They found the half-life to be about 2.5 hr. A recently reported value is 2.310 ±0.002 hr.[3] Several measurements of the beta energy using cloud chamber and absorption techniques have been made.[2,4–8] The values reported from these investigations range from 1.1 to 1.9 Mev. Two spectrometer measurements have listed the maximum beta energy as 1.18 Mev[9] and 1.24 Mev.[10] In addition to the 1.24-Mev beta ray, Slätis[10] has resolved two lower-energy components of 0.42 and 0.88 Mev. Meitner[6] reported gamma radiation with an average energy of about 0.6 Mev to be associated with this Dy activity. From a study of the internal conversion electron spectrum and the spectrum of electrons from secondary radiators, Slätis[10] concluded that gamma transitions of 0.91, 0.36, and 0.76 Mev were present. With the postulation of one additional unresolved beta component, he was able to propose a reasonable level scheme. Another measurement of the gamma-ray energies has been made by Miller and Curtiss,[11] who report energy values of 0.37 and 1.0 Mev. Clark[9] has set an upper limit of 1.1 Mev for the gamma energy and has also detected beta-gamma and gamma-gamma coincidences.

A short-lived Dy activity with a half-life of 1.25 min was first reported by Flammersfeld.[12] Electrons with an energy of approximately 130 kev were detected. These were interpreted as arising from internal conversion of an isomeric transition in Dy^{163}. Later work by Inghram *et al.*[13] has established that this, as well as the 2.3-hr activity, is associated with Dy^{165}. The cross sections for production of the 1.25-min and 2.3-hr activities were observed to be approximately equal, indicating that only the metastable state is formed directly in the capture process. Since growth of the 2.3-hr activity had not been observed,[12] it was suggested that a small percentage of the decay of the metastable state was by emission of a beta particle. In the present research some additional evidence for the existence of such a transition has been found.

The conversion electron spectrum of this activity has been investigated with spectrometers by Hole[14] and Caldwell.[15] The former noted that conversion was pre-

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