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A Precision Measurement at 24 500 Volts of the Conversion Constant $\lambda\nu$*

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A precision determination of the wavelength in x-units (Siegbahn scale) at the quantum limit of the continuous spectrum from a tungsten target sealed-off x-ray tube operating under an accurately stabilized and measured applied voltage, $V_H=24\,498.7\pm1.1$ absolute volts (emu) is described. This experiment, often loosely described as a measurement of h/e, is, in the present state of our knowledge of the general physical constants, more properly to be referred to as a determination of the conversion constant $V_A\lambda_s$ between x-ray wavelengths and quantum energy in electron volts. If $V_A=V_H+V_w$ is the sum of the absolute applied voltage V_H and the work function V_w of the tungsten cathode emitter and if λ_s is the quantum limit wavelength on the Siegbahn scale expressed in kilo x-units (kx-units), then $V_A\lambda_s=(h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8$, wherein h is Planck's constant in erg sec, e is the electronic charge in absolute cgs electrostatic units, c is the velocity of light in cm sec^{-1}, and λ_g/λ_s is the conversion factor from wavelengths measured on the Siegbahn nominal scale in x-units to absolute wavelengths in milliangstrom units (10^{-11} cm). Using the method of isochromats, the value obtained in this experiment, with the 2-meter focusing curved quartz crystal spectrometer for the monochromator was $V_A\lambda_s=12\,370.02\pm0.63$ emu·kx-units, the error ±0.63, a relative error of 51 parts per million, being our estimated standard deviation. A comparison with other determinations of this type is given. This value has been used as one of some thirteen precision data of eight different types in DuMond and Cohen's recent least-squares adjustment (November, 1952) to determine the "best" values of five fundamental constants: α, the fine structure constant; c, the velocity of light; e, the electronic charge; N, Avogadro's number; and $\lambda=(\lambda_g/\lambda_s)$, the above mentioned wavelength conversion factor; and to obtain from these a large number of other useful constants.

I. INTRODUCTION

Many precision determinations[1–10] of the wavelength at the quantum limit of the continuous x-ray spectrum from x-ray tubes operating under precisely measured stable dc operating voltages have been made since the first discovery of the inverse photoelectric effect in (1915). This experiment has been variously described as a measurement of Planck's constant h or as a measurement of the ratio h/e, where e is the electronic charge. In the present state of our knowledge of the atomic and other physical constants, the precision with which the electronic charge, e, the velocity of light c, and the conversion factor, (λ_g/λ_s), relating wavelengths expressed on the Siegbahn scale of x-units[11] to absolute wavelengths in milliangstroms are known, is comparable to the precision attainable in the present experiment. It is therefore better to describe this experiment more strictly as a measurement of the conversion constant from quantum energy in electron volts to wavelengths in x-units.

The measurement consists in exciting an x-ray tube with an extremely stable and very accurately measured dc voltage, V_H, and determining with a spectrometer of highest possible resolution the wavelength at the foot of the continuous x-ray spectrum emitted by the tube. Let this quantum limit wavelength, expressed in kx-units (Siegbahn scale), be λ_s. It has been shown,[12] both by theoretical reasoning and by experimental evidence internal to this experiment (coming from determinations made at different voltages), that the quantum energy of the short wavelength limit of the continuous x-ray spectrum corresponds to the energy acquired by the cathode electrons in falling through a voltage $V_A=V_H$

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[11] The wavelengths of a great number of x-ray lines on the Siegbahn nominal scale of wavelengths, in which the unit is now designated as one x-unit (approximately but not exactly 10^{-11} cm), are known by crystal diffraction measurements *relative to each other* with very high accuracy, in many cases one part in 10^6 or better. The constant λ_g/λ_s for converting these wavelengths to the absolute scale in milliangstroms has been determined by a number of workers by ruled grating diffraction at grazing incidence, an outstanding example is the work of J. A. Bearden. (λ_g/λ_s) is probably uncertain to about 3 or 4 parts in 10^5. The abbreviation kx-unit will be used here to denote a unit 1000 times as large as 1 x-unit, hence, approximately but not exactly one angstrom.

[12] The cathode work function correction was first shown to be required in a paper by one of us (see reference 5, also references 8–10). Its validity has been questioned by Ohlin from evidence internal to his experiment but without any theoretical justification of the objection. Later work (references 9 and 10 and the present results) are in accord with the need for this correction. The electron energy converted into x-rays comes in part (chiefly, of course) from the high tension supply which maintains the voltage difference V_H between cathode and anode. A small part of this energy comes also, however, from the cathode filament heating supply which communicates to the thermally emitted electrons enough energy to surmount the cathode work-function barrier V_w. The cooling of the cathode under a heavy demand for such thermally emitted electrons is a well-known phenomenon.

$+V_w$, where V_H is the measured voltage difference between cathode and target, and V_w is the work function of the cathode. The equation,

$$e'V_A=(e/c)V_A=h\nu=hc\lambda_g^{-1}\,10^8$$
$$=hc\lambda_s^{-1}(\lambda_g/\lambda_s)^{-1}\,10^8, \quad (1)$$

expresses the conservation of energy for the case of complete conversion (of electron energy to photon energy) in the transition of a free electron[13] from the region of potential just outside the cathode surface to a potential corresponding to the lowest unfilled conduction level in the target. In Eq. (1), e' is expressed in absolute electromagnetic cgs units, e in absolute electrostatic cgs units, V_A and V_w in absolute electromagnetic cgs units, c in centimeters per second, h in erg seconds, λ_s in kx-units (Siegbahn), and λ_g in angstrom units (10^{-8} cm).

Equation (1) then leads to the following expression for the conversion constant $V_A\lambda_s$, the quantity for which the measurements to be described here are designed to yield a numerical value:

$$V_A\lambda_s=(h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8. \quad (2)$$

The right-hand member of Eq. (2) gives then the function of the fundamental constants h, e, c, and (λ_g/λ_s) which this experiment really measures. The datum obtained in the present experiment has been introduced in this manner as one of thirteen experimentally obtained numerics belonging to eight different types in a least squares adjustment[14] to obtain the best values (as of November 1952) of five fundamental constants, namely, $\alpha=2\pi e^2h^{-1}c^{-1}$, the fine structure constant; N, Avogadro's number; and the constants, c, e, and (λ_g/λ_s) already defined. From these a large number of other useful constants are obtained.

The chief experimental difficulties result from the necessity for (1) extreme stability and very accurate measurement of the applied voltage, and (2) sufficient luminosity and energy resolution in the x-ray spectrometer to give a very accurate delineation of the profile at the foot of the continuous x-ray spectrum. Since the resolution of crystal diffraction spectrometers is such as to give an approximately constant wavelength uncertainty $|\Delta\lambda|$ nearly independent of wavelength, λ the relative energy uncertainty, equal to $|\Delta\nu|/\nu$ or $|\Delta\lambda|/\lambda$, diminishes with increasing quantum energy in inverse proportion. This places a limit on the highest voltage at which it is profitable to do this experiment; said voltage being higher, the higher the resolving power of the spectrometer. The difficulties in making a high-precision measurement as well as the expense for the precision high-voltage high-resistance potential divider also increase rapidly with increasing voltage at which the experiment is done.

There are, on the other hand, certain reasons why it is desirable to do this experiment at several radically different voltages, some of which shall be as high as possible consistent with the high precision now required for a datum to be used in a least-squares adjustment for the atomic constants. These reasons are related to certain sources of both systematic and random error or uncertainty in the experiment which are of the nature of small unknown or ill-defined additive increments to the measured voltage. They are: (1) impurity films of low atomic number atoms which, if present even in very thin layers on the surface of the target, can retard the incident electrons by many volts before they hit the heavy nuclei of the target proper. Such films, because of their low atomic number, do not themselves yield sufficient continuous spectral intensity to detect the "true" short-wavelength limit from unretarded electron encounters with the light atoms. Other energy uncertainties come from: (2) cathode work function correction, (3) thermal emission velocities of the electrons from the cathode, (4) Thomson effect corrections or thermal electromotive forces, (5) potential differences due to the heating current over the emitting surface of the cathode (unless precautions are taken to insure a constant potential cathode), and (6) sources of voltage ripple in the supply. Correction (2) can easily be by far the largest (4.52 volts, for example, in the case of a tungsten cathode). The value obtained for the constant $V_A\lambda_s$ or h/e will be too high if cause (1) is present and uncorrected. Causes (2) and (3), if uncorrected, make these values too low. Causes (4) and (5) may produce shifts in either direction, and causes (5) and (6) may blur the sharpness of definition of the quantum limit of the spectrum in much the same way as does the limited resolving power of the spectrometer. The reasons, then, why it is desirable to perform the experiment at as high a voltage as possible are: (1) that, by so doing, the relative importance of the above mentioned sources of error is diminished, and (2) that, by comparing results of a high-voltage experiment with those from a lower voltage experiment, internal evidence for the validity of correction (2) can be obtained.

A comparison of the results obtained in this experiment over the last three decades shows a more or less steady increase in the measured values of $V_A\lambda_s$ (or of h/e if one recalculates the results using the same value of c and λ_g/λ_s throughout). This is chiefly due to the fact that as better and better spectral resolving power was used, more and more new structural details at the foot of the continuous x-ray spectrum were progressively revealed which had in previous work been ignored. At each stage the practice of extrapolating the slope of the spectral profile at its lowest observed point with a straight tangent to an intercept with the background tended to give a higher limiting frequency (or a shorter

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[13] At the voltage of the present experiment the thermokinetic energy of the electrons just outside the cathode is completely negligible.

[14] J. W. M. DuMond and E. R. Cohen, Revs. Modern Phys. **25**, 693 (1953).

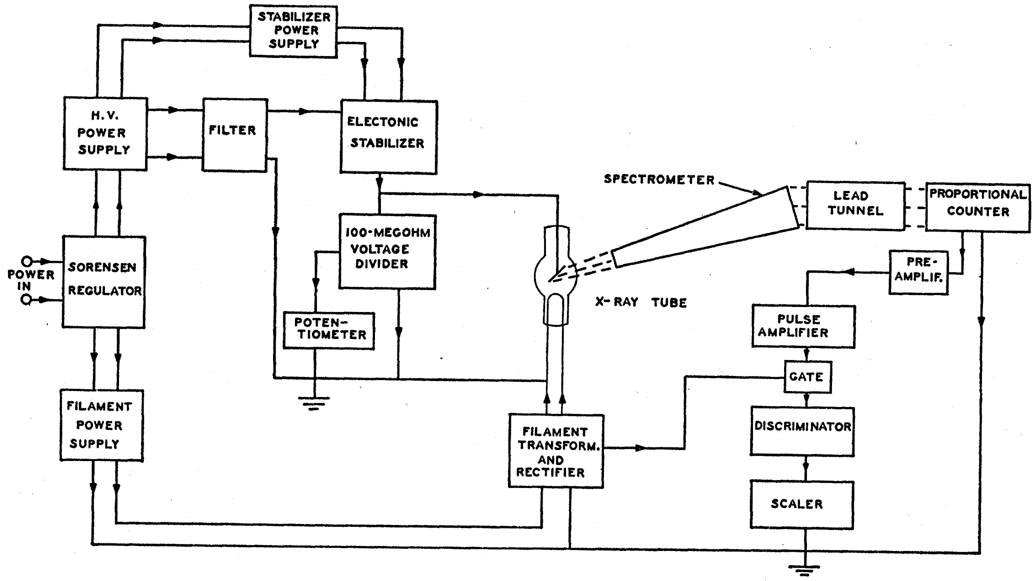

FIG. 1. Block diagram of complete equipment for measuring the conversion constant between quantum energy in electron volts and wavelengths in x-units.

limiting wavelength) than the true quantum limit[15] because the true spectral profile actually fell off to the background more abruptly than did the extrapolated tangent.

On the other hand, the possibility of electron retardation by low atomic number films on the target surface makes it unsafe to regard a *higher* value of $V_A\lambda_s$ (or of h/e) as necessarily a *more reliable* value. All of the later and spectroscopically better resolved experiments[6–10] of this type performed within the last fifteen years have been done with continuously pumped x-ray tubes using organic vapor pumps. It is notoriously difficult to avoid the accumulation of hydrocarbon and other low atomic number deposits in such cases. For this reason we have felt that, in spite of the impressive number of careful recent determinations, there would be a real value in performing one more at as high a voltage as possible consistent with adequate resolution and *made with a completely sealed-off x-ray tube* to remove the last vestige of suspicion regarding accumulation of organic films on the target. It was especially attractive to attempt this because of the availability at this laboratory of the two-meter curved crystal-focusing spectrometer with its exceptionally high luminosity and resolving power.

This work here described has been reported in greater technical detail in the doctorate thesis[16] of one of us, available for private distribution on request, which should be consulted for a more detailed description of the experiment.

II. APPARATUS

The main elements of the apparatus are the x-ray tube, the power supply, the voltage measuring equipment, the monochromator, and the detection and counting equipment. A block diagram of the complete system is shown in Fig. 1.

A. The X-Ray Tube

A "shock-proof" type x-ray tube manufactured by the Eureka X-Ray Tube Corporation of Chicago intended primarily for industrial radiography with a voltage rating of 150 kvp was used. In such a tube the glass vacuum envelope is completely enclosed in an oil-filled metal housing at ground potential which also serves as an x-ray shield. Coaxial high-tension cables enter the housing through large well insulated bushings. The oil not only serves to insulate the high-voltage terminals from the grounded jacket but also cools the x-ray tube target, the heat being removed from the oil by a water jacket on the metal housing. The grounded housing permitted mounting the tube conveniently close to the grounded metal structure of the 2-meter spectrometer[17] without fear of breakdown. It was supported on the upper arm of the spectrometer immediately behind the spherical lead container which usually contains the radioactive gamma-ray source. In place

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[15] This statement is worded to fit the case where the voltage is held constant and the x-ray spectrum is explored to find the quantum limit. If, as in the present case, the more usual method of "isochromats" is used, in which the x-ray monochromator is left at a fixed wavelength and the x-ray intensity is plotted as a function of varying applied tube voltage, exactly the same sort of error is introduced.

[16] Doctoral thesis of Gaelen L. Felt, California Institute of Technology, 1951 (unpublished).

[17] For adequate descriptions of the two-meter curved quartz crystal focusing spectrometer, see J. W. M. DuMond, Rev. Sci. Instr. **18**, 626 (1947); also Muller, Hoyt, Klein, and DuMond, Phys. Rev. **88**, 775 (1952).

of this source a special slit was provided exactly on the focal circle, the tube being aligned so that radiation from the target shining through this slit would illuminate the entire aperture of the curved crystal.

B. The Stabilized Power Supply

The elements composing the stabilized high voltage power supply were: (1) the primary voltage regulator (a 2-kva 220-volt Sorenson type, Model 2000-2S, which regulates its output to 0.1 percent against large fluctuations in both line voltage and load); (2) the main high tension transformer and diode rectifiers which were components of a Philips "Search Ray" outfit built for industrial radiography; (3) the high tension ripple filter; (4) the electronic voltage stabilizer which deamplifies fluctuations of the high voltage (both ripple and secular or accidental fluctuations) by absorbing them on the plate of a vacuum tube (Eimac 3C24); and (5) the x-ray tube filament power supply. We describe some of these elements briefly below. For a more detailed description, the thesis[16] should be consulted.

The H.T. transformer with its oil immersed diode rectifiers contained in the same oil tank was altered by a change in the internal connections so as to convert the output from the original 180-kvp half-wave rectified operation to 75-kvp full-wave operation. An x-ray tube filament heating transformer, already built into this unit to supply power insulated from ground for 90 kvp, was adapted to an entirely different use as a source of power for the electronic stabilizer. The latter was entirely at high tension. The x-ray tube filament ran at essentially ground potential, and its power supply was insulated from ground by an insulation transformer merely to permit the insertion of the milliameter for measuring the tube emission current. Further alterations in the Philips set were made by the addition of an extra step-down transformer and Variac (V_1) in the primary winding to furnish a finer control of the high voltage.

A ripple filter was inserted between the high-voltage output from the diode rectifiers and the electronic stabilizer. This filter consisted essentially of three 0.25μf condensers separated by two large 1000-henry oil-immersed chokes on insulated stands especially designed with air-gap cores to avoid saturation of the iron under dc operation. The high-voltage power supply and filter circuits are shown in Fig. 2.

An electronic stabilizer inside a well shielded case at high potential on an insulating stand was placed between the 50 000-ohm protective series resistor (Fig. 2) at the output end of the high-voltage filter and the "load." The load consisted of the 100-megohm working voltage divider in parallel with either (1) the precision megohm calibrating divider, or (2) the x-ray tube depending on whether the object was to check the 100-megohm ratio or to measure the continuous x-ray spectrum quantum limit.

The stabilizer, whose schematic is shown in Fig. 3, is a three-stage direct-current negative feedback amplifier whose output stage is in series with the load. The control signal is developed across 100 000 ohms, (R_s) at the high end of the 100-megohm divider (R_0 and r_0). The signal is amplified primarily in the first two stages and then applied to the grid of the series control tube (V_5). The main power supply (about 20 watts) for the stabilizer, which must be insulated above ground to

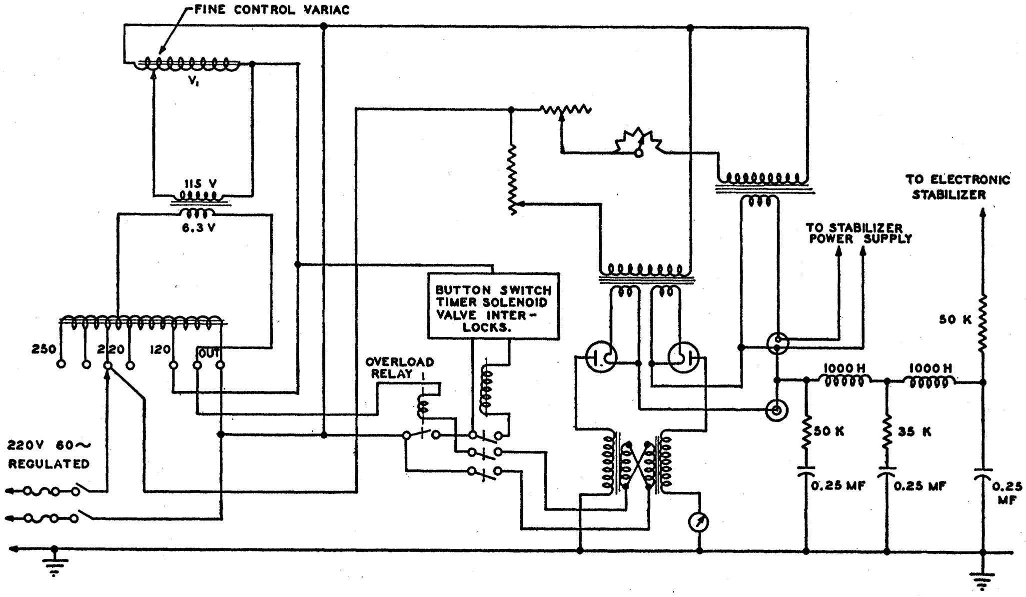
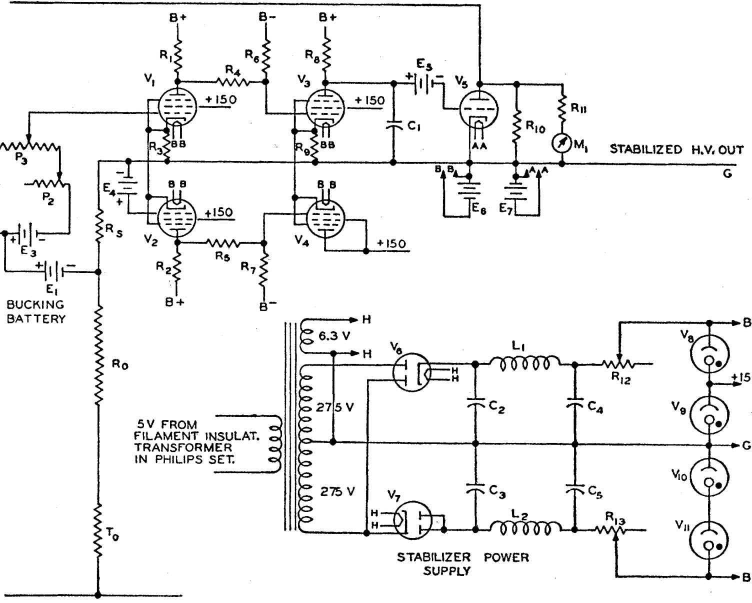


FIG. 2. Two-wave rectified high-voltage power supply and ripple filter. The Sorenson regulator which supplies the regulated 220-volt power is not shown.



T_0	4444.4Ω OIL IMMERSED	
R_0	100 MEG OIL IMMERSED	
R_S	100 K W W RITEOHM	
R_1	150 K WELWYN	PREC.
R_2	" "	"
R_3	25K "	"
R_4	250K "	"
R_5	" "	"
R_6	1 MEG "	"
R_7	" "	"
R_8	250 K "	"
R_9	25 K "	"
R_{10}	125 K 125 W IRC	
R_{11}	10 MEG.	
R_{12}	10 K	
R_{13}	25 K	
P_1	25 K MALLORY W W	
P_2	" "	"
P_3	5 K HELIPOT	10 TURN
C_1	0.1 MF	
$C_2\,C_3\,C_4\,C_5$	4 MF	
$L_1\,L_2$	10 H	
$V_1\,V_2$	6SJ7	
$V_3\,V_4$	6AC7	
V_5	EIMAC 3C24	
V_6	6X5	
V_7	6ZY5	
$V_8\,V_{11}$	VR 105	
$V_9\,V_{12}$	VR 150	
E_1	67.5 V	
$E_2\,E_3$	6 V DRY CELL	
E_4	45 V	
E_5	215 V	
$E_6\,E_7$	6 V STORAGE	
M_1	300 μA	

FIG. 3. The electronic stabilizer.

withstand about 30 kv, was derived as stated, from the unused filament transformer in the Philips set, one end of whose five-volt secondary winding was already tied to the high voltage at the input end of the filter. The pentode heaters and the triode filament were supplied by two storage batteries (E_6 and E_7) of approximately 200-ampere-hour capacity.

At some sacrifice of frequency band width in the stabilizer response (as regards the higher frequencies), a much improved stable operation was obtained by placing a 0.1-μf condenser (C_1) from the plate of the second amplifier tube (V_3) to the stabilizer "ground." The high-frequency response thus sacrificed proved unnecessary after visual examination of the stabilizer output on an oscilloscope trace. The secular stability of the stabilizer comes from the 67.5-volt bucking battery (E_1). No current flowed in this battery under operating conditions. A very delicate setting of the bias of the input grid of V_1 over a twelve volt range was permitted by the combination of two banks of four 1.5-volt flashlight batteries, (E_2) and (E_3), with the two 25 000-ohm Mallory controls, (P_1) and (P_2), ganged so as to give, with the 5000-ohm ten-turn Helipot (P_3), a constant resistance of 30 000 ohms across the 12 volt battery.

Once the various elements of the experimental apparatus had reached thermal equilibrium, the electronic stabilizer could maintain a preset voltage, for several minutes at a time, within limits of 5 parts per million and even under rather poor conditions could hold to a part in 10^5.

The exploration of the isochromat over the entire interesting range in the vicinity of the quantum limit could be made by turning the Helipot (P_3) control rod alone. The high-voltage vernier variac (V_1, Fig. 2) did not need to be reset during the runs except after the bridge measurements for checking the 100-megohm divider ratio.

A very stable and finely controllable power supply for the x-ray tube filament was obtained by a combination of three transformers and two variacs. This circuit is shown in Fig. 4. The selector switch was provided with silver contacts and in series with it was a vernier or fine control given by the output of two variacs stepped down, one through a 12.5-, the other through a 2.5-volt filament transformer.

Across the secondary of the x-ray tube filament transformer were connected the x-ray tube filament and a Selenium rectifier in series. The voltage supplied to the filament was about 9 volts peak and was half-wave rectified so that the filament was heated only during a half-cycle. During the other half-cycle the filament, though still emitting electrons because of its slow thermal response, was electrically essentially at ground potential over its entire length. The potential drop of 0.03 volt through the meter, (MA), under the operating load current of 10 milliamperes was completely negligible. A signal through the lead marked "gate" in Fig. 4 went to the counting equipment which was gated to record counts only during the interval of each cycle while the tube filament was not being supplied with power.

Convenient switching facilities were provided in the high voltage power supply so that the x-ray tube could be disconnected and replaced by the precision megohm divider (to permit checking of the ratio of the working 100-megohm divider) and *vice versa*. A movable copper link on an insulating handle with the ends of the link dipping in mercury cups was used for this.

Under working conditions a residual ripple of about 2 volts remained in the 24 500 volts applied to the x-ray tube. This residue, which the electronic stabilizer, was unable further to reduce, appeared as a result of the transients from the half-wave rectified heating supply for the x-ray tube filament reacting back through the main power supply. Ripple does not, of course, affect the mean value of the high voltage (the quantity we measure) but it does reduce the sharpness of the quantum threshold in a manner similar to the limited resolving power of the monochromator. The dc measurement of the high voltage (by means of resistance divider, potentiometer, and galvanometer) measures the *mean value* of the voltage averaged over a time of the order of the galvanometer period.

The high-voltage power supply and precision high-voltage measuring equipment were in one room, the x-ray tube, monochromator, and counting equipment in another. This arrangement, originally occasioned by space requirements, proved of great value since it reduced many problems of both x-ray shielding and electrical interference and most important of all made possible much better constancy of ambient temperatures.

C. The H.T. dc Voltage Measuring Equipment

The elements of the equipment for the high precision measurement of the high dc voltage applied to the x-ray tube were the 100-megohm working voltage divider (operating continuously in parallel with the x-ray tube), the precision manganin megohm divider (for calibrating the ratio of the 100-megohm working divider), the potentiometer, and the galvanometer.

The working divider, by means of which the voltage applied to the x-ray tube could be continuously observed, consisted of 100 Shallcross 1-megohm Nichrome[18] wire-wound resistors (rated current 1 ma) connected in series with a 100 000-ohm Riteohm wire-wound resistor at the high voltage end (R_0 in Fig. 3), which provided the control signal for the electronic stabilizer, and with a specially wound Manganin low resistance (r_0 in Fig. 3) across which the voltage applied to the potentiometer was developed at the low end of the divider.

The 100 Shallcross resistors were mounted in six Lucite tubes each $1\frac{1}{2}$ inches in inside diameter and 52 inches long and with $\frac{1}{4}$-inch walls. These tubes were held in a horizontal position, one above the other, on bakelite supporting brackets which supported each tube at points 10 inches from either end. The oil-tight metal caps closing both ends of each tube were connected alternately in pairs at either end by $\frac{3}{8}$-inch copper tubing to form a sort of switchback array through which oil was continuously pumped. The resistors were centered in the Lucite tubes by means of Lucite spacers $1\frac{1}{2}$ inches in diameter and $\frac{1}{2}$ inch thick through each of which five holes were drilled. The center hole held a short piece of brass tubing threaded to engage the studs on the ends of the individual Shallcross resistors. The remaining four holes permitted the oil to flow through the tubes.

The practice in using this working resistance divider was to adjust its dividing ratio, by a bridge measurement, so as to coincide with a standard ratio set up with the precision manganin megohm divider. To make the working divider ratio adjustable a 100 000-ohm decade box (not shown in Fig. 3) was inserted between the bottom Shallcross resistor and the 4444.4-ohm low-resistance member r_0 of the divider. Both the 100 000-ohm signal resistor at the high end for the stabilizer and the 100 000-ohm decade box trimmer at the low end were thus included in the nominal 100 megohms of the high-resistance segment of the working divider.

The oil was pumped through the Lucite resistor tubes and through a water-cooled (or heated) heat exchanger by a small motor-driven gear pump backed by a reservoir with a 6-foot head, the entire oil system being carefully shielded from contact with atmospheric water vapor to insure maintenance of a high leakage resistance for the shunt electrical path, offered by the oil, around the Shallcross megohms. The oil flowed into the resistor tubes at the low-voltage end and returned from the high-voltage end to the pump through a long Lucite fall pipe. Owing to the relatively large temperature coefficient of the Nichrome wire the oil temperature was carefully regulated by controlling the water tem-

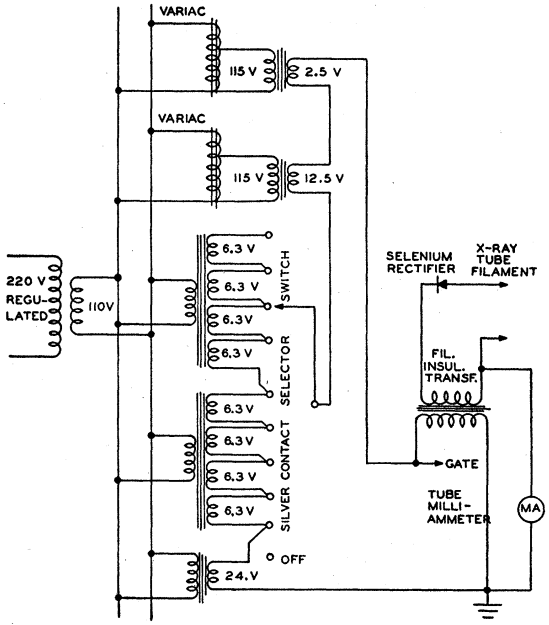


FIG. 4. The filament power supply for the x-ray tube cathode.

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[18] Manganin wire, whose much lower temperature coefficient of resistance would have made it far more suitable for the working divider, would have been prohibitively expensive in sufficient resistance not to overload the power supply with the x-ray tube also operating in parallel.

perature in the heat exchanger to within 0.1°C at 31° by means of a mercury thermal regulator and an electrical heater. The 4444.4-ohm low-resistance manganin segment of the divider was oil-immersed (at room temperature) and was very insensitive to temperature changes compared to the 100-megohm Nichrome segment. Any variations in the divider ratio could therefore be attributed with negligible error to the 100-megohm segment alone.

The 1-megohm manganin precision divider[19] was entirely designed and built at this Institute to serve as a means of establishing very large resistance ratios to extremely high precision by an ingenious series-parallel method discussed by Wenner[20] and attributed to Lord Rayleigh. It consist of one hundred coils of 10 000 ohms and twelve other coils of low resistance: five 80-ohm, four 100-ohm, and three $133\frac{1}{3}$-ohm coils. The 10 000-ohm coils were wound with No. 36 manganin wire and the low resistance coils with No. 30 manganin. All the coils were wound on enameled brass tubing 1 inch in diameter and 21 gauge. Each 10 000-ohm coil occupies seven inches on each coil form. Great care and attention was paid to the enameling, insulation, winding, baking, and thermal normalizing of these coils to give maximum stability of resistance and freedom from electrical leakage.

The coils were mounted with the bottom ends of the tubes in holes in the false bottom of a spacious oil-tight Lucite box. A Lucite well opening through the false bottom and containing a thermoregulator, a heater, and an impeller shaft was placed in the center of the box with the impeller itself below the level of the false bottom on which the coils were mounted. The entire box was filled with high-resistance oil. The rows of coils were separated by Lucite baffles in such a way that oil from beneath the false bottom was forced up inside each coil form to its top and was then drawn down the outside of the coils before returning to the impeller through the central well.

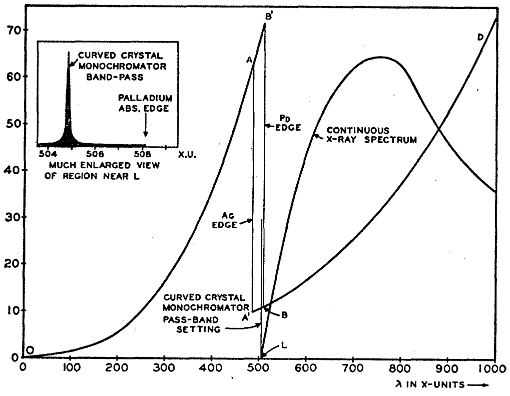

FIG. 5. To illustrate the operation of the filtering technique. Curves *OAA'BD* and *OAB'BD* give, for the silver and palladium filters respectively, as a function of wavelength in x-units the product of the mass absorption coefficient into the effective surface density of each filter. The surface densities of the filters are "balanced" to make the curves coincide in the regions *OA* and *BD*. The insert shows how the wing or tail of the monochromator pass-band, which would otherwise transmit appreciable intensity when integrated over the intense regions of the continuous spectrum, is clipped by applying the Ross balanced filter technique. One can also see why the silver filter alone, curve *A'BD*, is very effective.

The entire unit was thermostated to maintain the oil at the optimum working temperature for the manganin wire. In ±2°C variation from the operating temperature the divider changes by as little as 3 parts per million. The divider was intended to be operated at 10 kv but the design was so generous that operation at 25 kv for limited periods proved entirely feasible. This fortunate circumstance permitted a determination of the 100-megohm divider ratio directly at the working voltage and thereby eliminated the necessity for making an extrapolation by means of the 100-megohm load coefficient.

The potentiometer used to measure the voltage across the low-resistance segment of the working divider was a Rubicon High Precision Type B designed with compensating slide-wire contacts which serve to reduce thermal emf's developed by the usual single slider to a very low value indeed. Since the bulk of the voltage developed across the low segment of the working divider was bucked out by a standard cell, the potentiometer could be used in its 0 to 160-millivolt range with a limit of error of 10 microvolts.

A Leeds and Northrup 2430-D galvanometer was used with the potentiometer for the voltage measurement (but not for calibration of the dividers). With a sensitivity of 0.0004 microampere per mm, one-mm deflection on this instrument represented a deviation of 2 parts per million in the applied high voltage.

D. The X-Ray Monochromator

In the present experiment, monochromatization was effected in two stages, firstly and roughly through the use of filters to suppress the strong integrated effect of the continuous spectrum at wavelengths longer than the immediate quantum limit value and secondly, with high resolving power by means of the 2-meter curved crystal focusing spectrometer. Figure 5 illustrates the operation of the filtering technique and shows to scale the location of the spectrometer band-pass setting between the *K*-absorption edges of the two filters used.

A Ross "balanced filter" arrangement[21] using balanced

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[19] This precision divider or ratio box, on which considerable care and expense was lavished to make it the last word of reliability and accuracy, has been described in much more detail in a separate article by John N. Harris, Rev. Sci. Instr. **23**, 409 (1952).

[20] F. Wenner, J. Research Natl. Bur. Standards **25**, 229 (1940).

[21] In the P. A. Ross method of balanced filters, two filtering foils of materials having adjacent atomic numbers are alternately introduced in the x-ray beam. In a preliminary experiment, by varying the obliquity of the foils in the beam, the absorptions of the x-rays by each foil are carefully balanced to equality for wavelengths longer than the *K*-absorption edge of the foil of lower atomic number and shorter than the *K* edge of the foil of higher atomic number. These are the regions *OA* and *BD* in Fig. 5. Thus

foils of silver ($Z=47$) and palladium ($Z=46$) of optimum thickness was tried but isochromats obtained with the silver foil alone filtering the x-ray beam were found to be as good or perhaps better (because of the limited counting statistics of the present experiment) than those obtained by taking the difference between readings with each filter. A substantial loss in statistical accuracy is suffered with the 2-filter method because the uncertainty in the difference of the readings for the two filters is the square root of the sum of the squares of the separate uncertainties of each reading whereas only the statistical uncertainty of one reading is involved in the single filter method. In the present instance, circumstances beyond our control placed unfortunate time limits and deadlines on the performance of this experiment which limited the accumulation of counting statistics to just about such a degree that the data collected with the two-filter method lost by poor statistics about the same amount or perhaps a little more than they gained by the sharpening of the fillet at the foot of the isochromat. With better statistics the Ross balanced filter method would in all probability have proved superior.

The effectiveness of the single filter of silver is clearly due on the one hand to the low absorption coefficient it offers to radiation of the wavelength for which the crystal monochromator is set, (a wavelength purposely chosen just slightly shorter than the critical wavelength of the K edge of the palladium filter and only some four percent longer than the K edge of silver) and on the other hand to the rapidly increasing absorption coefficient it offers, (because that coefficient is proportional to λ^3) to the longer wavelengths where the continuous spectrum is intense.

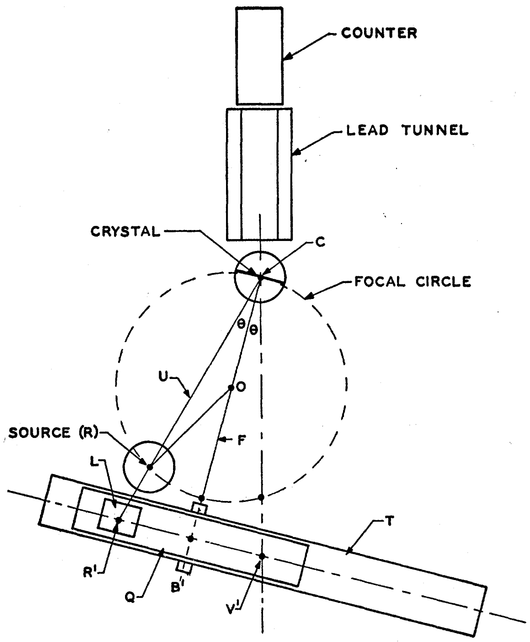

FIG. 6. Schematic lay-out of 2-meter curved crystal spectrometer used as the monochromator. For convenience in identifying the parts the same letters have been used as those in a perspective drawing given in reference 18.

The optimum thicknesses calculated for the foils were

$$X_{\text{Pd}}=0.00109 \text{ inch}, \quad X_{\text{Ag}}=0.00113 \text{ inch},$$

and these were realized with sufficient accuracy for a good balance, first by rolling and then properly adjusting the foil obliquity in the beam.

The two-meter curved crystal spectrometer was used with a curved quartz monocrystalline slab 2 inches square and 2 mm thick mounted in a stainless steel crystal clamp whose convex jaw was precision profiled to a two-meter circular cylindrical shape. The (310) atomic planes of the crystal, for which $d=1177.637 \pm 0.020$ x-unit (Siegbahn scale) at 18°C,[22] are the ones used for the selective reflection and these stood normal to the large faces of the slab and parallel to the generators and to the axis of the cylinder of curvature imposed by the clamp.

Figure 6 shows schematically the general layout of the spectrometer in plan view in sufficient detail to illustrate and clarify the present discussion.

The defining slit is placed at the point labelled "source," R, in Fig. 6, and the x-ray tube mounted close behind the slit on the upper arm, U, furnishes a diverging beam of radiation through this slit which nearly fills the entire curved crystal aperture (about 20 cm^2) at two meters distance. The great advantage of the focusing spectrometer of this type, for x-ray as well as for gamma-ray spectroscopy, lies in the large solid

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the *difference* in the two observed intensities when the two foils are successively introduced into a heterogeneous beam of x-rays measures only the spectral intensity in the region between the two K edges. This isolation of a part of the spectrum, it should be noted, is obtained by an arithmetical not a physical subtraction. Ross and others have discussed the calculation of the optimum filter thicknesses. Because of the large value of the K-absorption jump ratio, it is usually possible to make the smaller of the two subtracted readings only a quarter as large as the larger reading, and for lower atomic number filters even more favorable results are obtainable. No more readings are required than without the foils since the method without filters should properly include a "background reading" with a lead shutter blocking the beam for every regular reading with shutter open.

In conjunction with a high resolution crystal monochromator, the use of the balanced filter method for preliminary rough monochromatization to suppress the integrated effect of leakage of the intense continuous spectrum through the long wavelength wing of the crystal monochromator pass-band was first proposed by one of us to improve this type of experiment, and its advantages were more fully illustrated and discussed in the paper by Panofsky, Green and DuMond (see reference 8). At large distances from its peak the crystal diffraction pass-band curve decays with an inverse square law, while the more or less linear rise of the continuous spectral profile upward from the quantum limit gives an integrated leakage effect through the wing of the pass-band to which radiation spectrally quite remote from the limit can contribute materially.

For a treatment of the Ross balanced filter technique, see P. A. Ross, Rev. Sci. Instr. **16**, 433 (1928); P. Kirkpatrick, Rev. Sci. Instr. **10**, 186 (1939).

[22] Watson, West, Lind, and DuMond, Phys. Rev. **75**, 505 (1949).

angle from which monochromatic radiation is collected with excellent resolving power. It was this feature of high luminosity which permitted us to do this experiment with the intrinsically low power delivered by the sealed off x-ray tube in a very limited time.

The instrument was designed primarily to study much shorter wavelengths, of course. At the wavelength of this experiment the need for the "collimator" (a carefully constructed fan-shaped array of lead partitions) disappears, and since this component entails a sacrifice of more than half the radiation, it was removed and replaced by the lead tunnel shown in Fig. 6.

The upper precision drive-screw in the long carriage, Q, is equipped with a carefully divided drum and vernier which enables one to set the screw to 0.001 of a revolution, corresponding quite closely to 0.001 of an x-unit. Since the kinematics of the instrument are such that displacements of the small carriage, L relative to Q, effected by means of this screw, measure the sine of the Bragg angle, the screw revolutions provide a linear wavelength scale of about one x-unit per revolution (save for very small corrections). Since the spectrometer was to be used as a monochromator in one, and only one, fixed position during the running of the isochromats, the elaborate provisions for exploring a spectrum with this instrument were only of use in establishing the calibration of the fixed monochromator setting. This was done by exploring the profiles of four very accurately known calibration lines lying close to that setting, as will be explained. It turns out that the accuracy of the wavelength calibration of the spectrometer, therefore, scarcely plays any role in the wavelength accuracy involved in the present experiment.

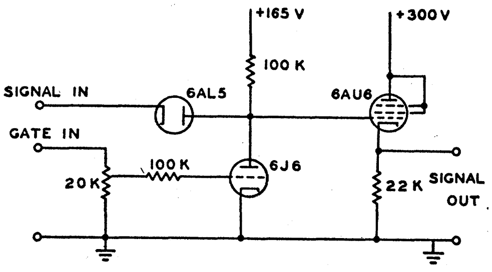

FIG. 7. External discriminator gate. This gate was so phased that counting of x-ray intensity was only recorded during each half-cycle of the rectified filement power supply when no filament current was flowing.

A slit width of 0.004 inch was used corresponding to a spectral spread of about 0.1 x-unit. This together with the small aberrations of focus of the crystal and the intrinsic selective diffraction curve of the quartz lattice gave a monochromator band-pass or "window" half-width at half-maximum intensity of somewhat less than 0.12 x-unit equivalent to an energy difference of somewhat less than six volts.

E. The Detection and Counting Equipment

The intensity of the radiation selected by the monochromator was measured by means of a xenon-filled proportional counter, a preamplifier and linear pulse amplifier, two discriminator channels, and two double decade scalers.

The counter case consisted of a copper plated rectangular arc-welded steel box 8 inches long and with an inside cross section 3 inches square. One of the square ends was fitted with a plate in which the pumping line and four Kovar seals were set. The other end was covered by an aluminum window 0.006 inch thick. Because of the large area of the window, required for admitting the diverging beam of radiation from the entire curved quartz crystal aperture, three thin spring-steel ribs were mounted inside the counter to support the window. Two $\frac{1}{8}$-inch glass rods were mounted $\frac{1}{2}$ inch back from the window, parallel to and in the shadows of the two outer ribs. Four tungsten collecting wires of 0.001-inch diameter were stretched between these glass rods and the four Kovar seals at the back of the counter. These four wires were parallel to each other and each was $\frac{3}{4}$ inch from its nearest side walls. Since the inside walls of the counter were slightly rough, a 0.016-inch polished copper liner was inserted in the case. Great care in the construction to observe cleanliness, especially as regards radioactive contamination, was found necessary to maintain a low background counting rate.

This counter was operated with 14 cm of xenon and 1 cm of CO_2 and it proved to be very stable and reliable. For the 500 x-unit radiation used, its length was sufficient to absorb about 25 percent of the beam.

The counter was shielded from local radiation and soft cosmic ray components by heavy removable lead shields providing four inches of lead on all sides save for the opening for the x-ray beam from the lead tunnel.

The electronic circuits were entirely similar to circuits used at the Los Alamos Scientific Laboratory described by Elmore and Sands[23] and they will not be described in great detail here. The pulse amplifier, a modified version of the Los Alamos Model 100 had a gain of 10^4. Both an integral and a differential discriminator were included in one chassis. The integral discriminator puts out a shaped pulse to a scalar for all input pulses larger than an adjustable predetermined amplitude while the differential discriminator puts out a pulse for each input pulse of amplitude lying between two adjustable limits. A fixed pulse-height window width up to 15 volts was used.

Figure 7 shows the gating arrangement inserted between the pulse amplifier and the discriminator input. This gate prevented counting through both discriminators during the part of each x-ray tube filament heating cycle while current was flowing to heat the filament. The phasing of the gate with the filament heating

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[23] W. C. Elmore and M. L. Sands, *Electronics: Experimental Techniques* (McGraw-Hill Book Company, Inc., New York, 1949).

current was checked with an oscilloscope, the gate being closed for slightly more than a half-cycle. Thus the x-ray cathode was essentially a unipotential one while the counts were being observed.

Each discriminator was followed by a scale-of-one-hundred scaler consisting of two Berkeley decades in tandem each provided with a Mercury mechanical register.

III. CALIBRATION

A. Voltage Calibration

The primary reference voltage consisted of a bank of six standard cells of the saturated type calibrated and certified by the National Bureau of Standards in September, 1947, and guarded with care in the Calibration Laboratory of this Institute. The bank was kept in a well-insulated box whose temperature was controlled within 0.01° at 30.09°C, and periodic intercomparisons of the six cells have shown a maximum deviation of only 3 microvolts for one of the six from the mean of the group.[24]

A second group of four cells of the unsaturated type was also used. One of these was used as the potentiometer current standardizing cell. The remaining three were available for use as bucking cells in the voltage measurement. These unsaturated cells were also kept at constant temperature in an insulated box. They were recalibrated frequently during the course of the experiment using the saturated cell bank as a primary standard. The electromotive force of the bucking cell used in all final runs in opposition to the voltage developed across the low-resistance segment of the 100-megohm working divider (so that the potentiometer measured the difference of these two) was

$$\text{emf S.C. "B"} = 1.019161 \text{ absolute volts}, \qquad (3)$$

with a conservative limit of error of 5 microvolts.

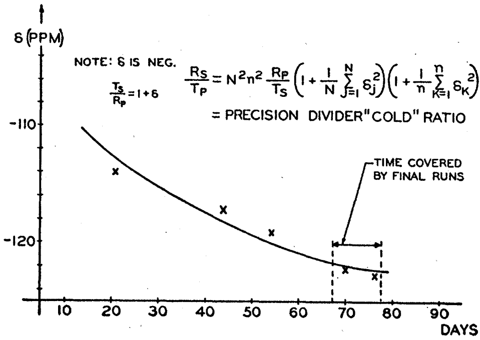

FIG. 8. Slow variation of precision manganin divider "cold" ratio.

The determination of the dividing ratio of the 1-megohm precision divider under working load conditions (24.5 kv) requires a knowledge of the "cold" ratio (essentially at no load) and of the load coefficient. The reader is referred to the thesis[16] for the complete details of how these were determined. The paper[19] on the megohm divider also describes these procedures briefly. The precision megohm divider was purposely designed such that the Rayleigh method described by Wenner[20] could be applied to its calibration. The idea essentially depends upon reconnecting the several closely equal units of which both the high and low resistance divider segments consist, the high from series to parallel, the low from parallel to series, in such a way that the two segments which normally have very different resistances, after reconnection have very closely the same resistance so that their ratio, close to unity, can be measured with extremely high accuracy. The high ratio, when the two divider segments are reconnected in the normal working manner, can then be calculated from this measured ratio with high accuracy. It is easy to show that small deviations from the mean of the individual resistor units in each segment only affect the calculation to the second order.

The method followed for determining the load coefficient of the precision manganin megohm divider was one described by Curtis.[25] By this method not only the load coefficient of the divider but a slow variation of this coefficient with time after first application of the load were determined. By these means the value of the precision manganin divider ratio at the end of five minutes load under working voltage was determined to be

$$\text{p.d.r.} = 22\,503.52 \pm 0.10 \text{(L.E.)}, \qquad (4)$$

where the uncertainty is a very conservative limit of error. Figures 8 and 9 show the slow variations of the precision divider cold ratio over a period of some

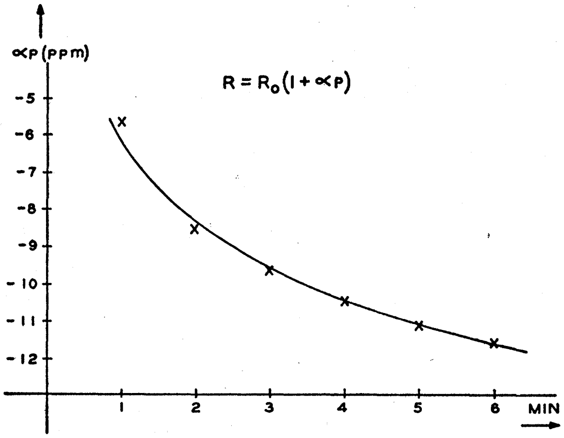

FIG. 9. Time dependence of precision divider load coefficient.

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[24] One may therefore safely assume that the mean of the group has not drifted appreciably from the original mean determined at the time of calibration, three years prior to use in this experiment. This conclusion was based on a letter of December 12, 1949, from G. W. Vinal of the NBS to J. N. Harris discussing such possible drifts of the mean. See also F. B. Silsbee, National Bureau of Standards Circular 475 (1949), pp. 9–11.

[25] H. L. Curtis, *Electrical Measurements* (McGraw-Hill Book Company, Inc., New York, 1937); see also reference 19 for a description of this method.

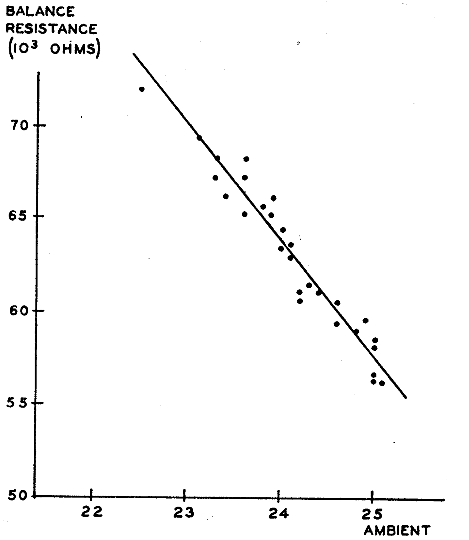

FIG. 10. Dependence of 100-megohm nichrome working divider on ambient temperature.

months and the time dependence of the load coefficient over five minutes. The latter is seen to indicate a change of about one part per million per minute after five minutes loading.

The procedure in the "calibration" of the 100-megohm working divider should in the strict sense be described as an *adjustment* rather than a calibration. Before and after every run of the isochromat the x-ray tube was disconnected from the high-tension supply and the 1-megohm precision divider was put in its place in parallel with the 100-megohm working divider. A Leeds and Northrup 2284-X galvanometer with a sensitivity of 0.25 microvolt per mm at 1 meter and with a circuit resistance of 1100 ohms was used in all of the divider calibration measurements. With this instrument connected across the intermediate terminals between high and low segments of each divider, the 100 000-ohm decade box trimming resistance at the low-voltage end of the 100-megohm segment of the working divider was adjusted to balance this bridge arrangement so that the working divider ratio was made just equal to the precision megohm divider ratio (5-minute loading ratio). In this adjustment a 1.5-mm deflection corresponded to 1 part per million variation in the working divider ratio. After each run on the isochromat was completed (usually $3\frac{1}{2}$ to $4\frac{1}{2}$ hours was required for a run) the working divider ratio was again checked against the manganin precision divider. Except in one or two runs this final bridge balance differed from the initial by less than 5000 ohms (5 parts in 10^5). Corrections were made to the working divider ratio for intermediate potentiometer settings based on a knowledge of the ambient room temperature measured every half hour during each run. Figure 10 shows the measured dependence of the 100-megohm working divider ratio on ambient room temperature. The ordinates give the amount of compensation required in the 100 000-ohm decade balancing resistance as a function of ambient room temperature. The drift of about 5000 ohms per ambient degree corresponds to about 50 parts per million and by using this curve and the recorded room temperatures a correction to each setting of the potentiometer during the isochromat run could be made which in the worst cases, for runs in which temperature variations were greatest, gave an accuracy of the voltage measurement within limits of 2 parts in 10^5.

TABLE I. Wavelengths of calibration lines on the Siegbahn scale (x-units).

Ag $K\beta_3$	496.65 ± 0.005
Ag $K\beta_1$	496.01 ± 0.005
Sn $K\alpha_2$	494.02 ± 0.005
Sn $K\alpha_1$	489.57 ± 0.005

The load coefficient of the 100-megohm working divider was also measured to ascertain how accurately the high voltage had to be adjusted during the bridge measurements for calibrating the working divider before and after each run. These studies showed that a voltage setting within 250 volts of the working value (24 500 volts), i.e., to one percent amounted to a limit of error of only 5 parts per million from this cause.

B. Wavelength Calibration

The wavelength of the pass-band at which the monochromator was set for running the isochromats was, as already stated, selected so as to fall between the *K* edges of the Ag and Pd filters just to the short-wavelength side of the Pd *K* edge. The mean value determined for the setting was

$$\bar{\lambda}_{SWL}^{(S)}=504.833\pm0.008 \text{ x-units (Siegbahn) at } 24^{\circ}\text{C}. \quad (5)$$

This wavelength was determined by using four already known and carefully measured[26] x-ray lines as local reference points on the spectrometer wavelength screw. These lines and their wavelengths are listed in Table I.

These four lines were produced by fluorescent excitation using back illumination in foils of silver and tin placed behind the spectrometer resolving slit and in front of the x-ray tube. They were excited by the continuous radiation from the same tungsten target x-ray tube as was used for the isochromat runs. The tube was slightly displaced for this purpose so that direct primary radiation transmitted through the foil and slit would not hit the curved crystal. The tube was mounted on a small transversely displaceable carriage supported on the upper beam of the spectrometer and provided with

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[26] Y. Cauchois and H. Hulubei, *Longuerers d'Onde des Emissions X et des Discontinuites d'Absorption X* (Hermann et Cie, Paris, 1947). The silver wavelengths in question were measured by Kellström and the tin wavelengths of Stenman. G. Kellström, Z. Physik **41**, 516 (1927); John Stenman, Z. Physik **48**, 349 (1928).

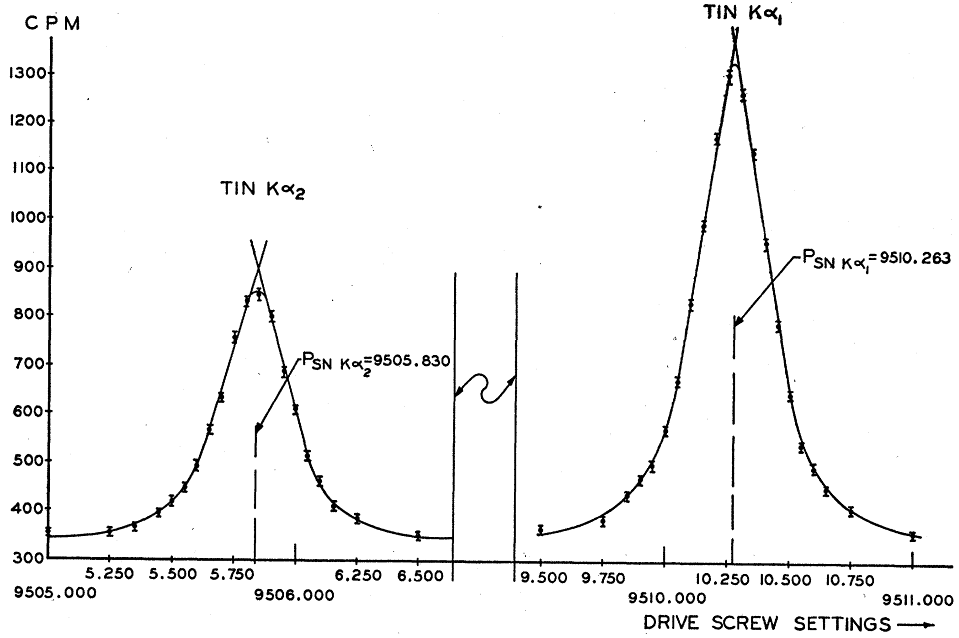

FIG. 11. Fluorescent tin $K\alpha_{1,2}$ doublet used in precision calibration of spectrometer for the monochromator setting. One unit on the scale of the wavelength screw settings corresponds to one x-unit. The $K\alpha_1$ line is seen to have a half-width at half maximum of 0.18 x-unit. The instrumental contribution is about 0.12 x-unit or the equivalent of 6 volts.

a micrometer screw to facilitate this transverse displacement. Most of the weight of the tube was supported by a counterbalancing sling to minimize any possible deflection of the spectrometer slit setting which might be occasioned by this slight shift in the weight of the tube on the spectrometer beam. The fluorescence line profiles were spectrally explored with the tube displaced both to left and to right and no shift in the lines was detected as a result of this process.

Optimum thicknesses for the fluorescer foils were computed using the approximate formula:[27]

$$t_{\text{opt}}=(\mu_1-\mu_2)^{-1}\ln(\mu_1/\mu_2), \tag{6}$$

wherein μ_1 is the mean linear absorption coefficient in the foil for a wavelength just short of its K edge and μ_2 is the linear absorption coefficient of the fluorescer for its own characteristic K line, the line to be used. Essentially μ_1/μ_2 is the K-jump ratio. This leads to thicknesses of 0.0015 and 0.003 inch, respectively, for silver and tin.

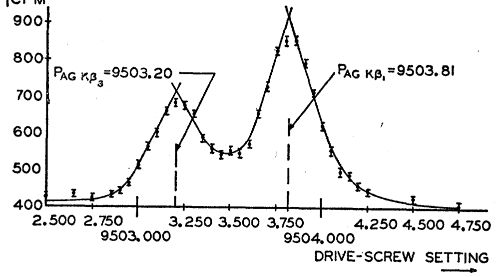

FIG. 12. Fluorescent silver $K\beta_{1,3}$ doublet used in precision calibration of monochromator setting. One unit on the scale of drive screw-settings corresponds to one x-unit.

Examples of the profiles obtained for these fluorescence lines with the primary tube operated at 35 kv and 10 ma are shown in Figs. 11 and 12. Several such runs were taken and the mean values of the spectrometer screw settings (for the intersections of straight lines tangent to each side of a line profile) yielded the results shown in Table II. A change of one unit (one turn) in the screw reading corresponds quite closely to a change of wavelength of one x-unit.

The mean screw calibration factor used was

$$\text{S.F.}=1.00029\ \text{turns/x-unit}, \tag{7}$$

as obtained in other recent calibration work on the spectrometer.

The estimated probable error in the location of the lines as given above is based on examination of the

TABLE II. Nominal spectrometer screw readings at calibration lines.

Line	Mean reading of wavelength screw[a]
Ag $K\beta_3$	9503.20 ±0.020
Ag $K\beta_1$	9503.81 ±0.015
Sn $K\alpha_2$	9505.830±0.010
Sn $K\alpha_1$	9510.263±0.005

[a] Turns of the screw are read with a Veeder-Root counter coupled to the shaft. The position at which this counter reads "zero" is quite closely, but not absolutely exactly, at the zero wavelength setting of the instrument. For reflections on one side of the crystal planes, save for the small additive zero corrections, turns of the screw indicate nominal x-units directly (before corrections for screw constant and for periodic and secular errors). For reflections on the other side of the crystal planes the reading of the Veeder-Root counter must be subtracted from 10 000 (its maximum and therefore also its zero reading) to obtain the corresponding nominal reading in x-units.

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[27] The approximation is based on the assumption that only that part of the primary continuous spectrum harder than and very close to the K edge is effective in exciting the foil.

measurements themselves and on the discrepancies arising from the determination of the line location (1) by means of intersecting tangents and (2) by finding the position of the midpoint of the full width at half-maximum intensity. The Sn-$K\alpha_1$ line was much the sharpest and was, therefore, given more weight in determining the wavelength of the monochromator setting for the isochromats. That setting was at a screw reading of 9495.000; i.e., slightly more than 8 x-units from the nearest reference line (Ag $K\beta_3$) and a little over 15 x-units from Sn $K\alpha_1$. In order to locate to 1 part in 10^5 an extrapolated position 15 x-units from a reference point in the region of 500 x-units, one need know the screw calibration factor to only one part in 3000. Thus our knowledge of the decimal digits in the above screw correction factor is only just marginally needed at all. Within the precision with which they could be located, the positions of all four fluorescent lines were entirely consistent with their above listed wavelengths and with the screw calibration factor.

Small changes in room temperature leave the angular geometry of the two-meter spectrometer practically unchanged, so long as no serious local differences in temperature exist, because the entire metal structure has everywhere a quite uniform thermal expansion coefficient. The chief effect of ambient temperature changes on the monochromator pass-band setting come therefore from thermal expansion of the curved quartz crystal lattice. For the (310) planes used this introduces a deviation of 1 part in 10^5 per degree centigrade. For this reason the temperature of the quartz crystal was recorded during isochromat runs and also fluorescent calibration line runs, and all data were referred to 24°C.

IV. OBSERVATIONS AND RESULTS OF THE EXPERIMENT

A. Procedures

Procedures were carefully worked out for the various phases of this precision experiment after considerable preliminary experimentation. These cannot be treated in detail here.[28] Each isochromat run consisted of a preheating stage to stabilize the x-ray tube, the electronic stabilizer and the 100-megohm working divider. To shorten the time for this the practice was adopted of keeping these three components including the oil circulation systems warmed up almost continuously. The stabilizer, for example, when not in operation at 25 kv, was kept switched to an auxiliary power supply. The warming up of the x-ray tube before each run then remained the most time-consuming factor. After about 30 minutes to warm up the electronic counting equipment to stability and while the x-ray tube was still being warmed up, a check of the counting, by comparing the results of the two discriminators, would be made. The setting of the "window" in the differential discriminator relative to the pulse height distribution curve would be checked for any possible shifting and remedied if required. In the course of a run such a shift, which occurred very rarely after sufficient warming, could be recognized and remedied. The integral discriminator which passed all pulses of amplitude greater than 20 volts was virtually insensitive to the small changes which could introduce erratic results in the differential channel and it served, therefore, as a valuable monitor, particularly so when scanning the pulse-distribution curve with the differential discriminator during the x-ray tube warm-up period. The x-ray warm-up period was also used as a good time for zeroing the galvanometers and standardizing the potentiometer current. Such economies of time were valuable in order to complete an isochromat run properly in a day's work.

Counting periods were divided into 5-minute intervals. At a minimum counting rate in the integral discriminator channel of about 200 to 225 counts per minute, a fairly satisfactory sample could be obtained in 5 minutes and a neighboring point could be reached before any large change in conditions could take place.

The sequence of counting through the two filters and changing the voltage by increments followed one of two patterns. In all cases the potentiometer settings were the same, but in some of the runs double readings through the silver filter were taken at points lying within about 15 volts on either side of the quantum limit. In this way, though the background readings through the palladium filter were more widely separated in time, much more information could be obtained in this highly important region near the limit. Furthermore, since readings through the palladium filter were intended (and proved to serve) primarily as background readings and were not themselves strongly involved in locating the limit, these readings were taken only at alternate voltage settings except in the case of the first six runs.

The data were collected in the course of 16 "final" runs across the continuous spectrum quantum limit. In addition to the counting data the following information was recorded for each and every run: (1) balance resistance required in the 100-megohm working divider, (2) x-ray tube current, (3) xenon counter voltage, (4) discriminator bias settings, (5) electromotive force of the bucking standard cell, (6) ambient temperature in the power supply room, (7) temperature of the quartz crystal. The temperatures were read every half-hour during the run.

B. Reduction of the Data

The data taken on the 16 isochromats were combined to give a single composite isochromat with better statistics. Although the same potentiometer settings were used in all 16 runs the actual voltage applied to the tube corresponding to these settings in a few cases differed by as much as 4 or even 5 parts in 10^5 by reason of the various corrections required. Such extreme cases were

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[28] For an adequate description, see reference 16.

infrequent but these departures though usually small constituted the chief reason for adopting the following adjustment procedure. A preliminary rough isochromat was plotted by combining all the counting data at the same potentiometer setting ignoring the necessity for the small corrections to the potentiometer settings needed to obtain applied tube voltages precisely. The slope of this rough preliminary composite isochromat then furnished the necessary information to correct the number of counts observed on each of the 16 isochromats to what would have been obtained if the desired voltage for that setting had been applied to the tube instead of the actual voltage that was applied. None of the voltage deviations calling for this correction were as large as 3 parts in 10^5 in the critical region of the isochromat. An additional correction of this same type was made because of temperature variations of the quartz crystal from run to run. (The crystal temperature was usually quite constant during a run). A voltage rather than a wavelength correction was applied to meet this situation. The wavelength selectively diffracted by the crystal at the fixed angular setting of the spectrometer will change by 1 part in 10^5 per degree centigrade, and clearly one may consider this equivalent to a change of the voltage in the same proportion. The total variation in crystal temperature over all runs combined was about 2.5°C and the variation during a run was rarely as great as 1°C so that this was a very unimportant correction. All such voltage corrections proved to be completely "swamped" by the error (chiefly due to restricted statistics) in locating the limit from the plotted isochromats.

No data were discarded on the grounds of large statistical deviations alone, since we consider this practice, which tends to reduce the variance of the data artificially, a very dangerous one bordering on dishonesty. Only good and sufficient *a priori* reasons such as a known fluctuation in applied counter voltage or a shifting of the peak of the differential discriminator pulse-height distribution curve would justify discarding of data. It was finally decided not to discard any data taken through the integral channel of the discriminator on any of the 16 runs.

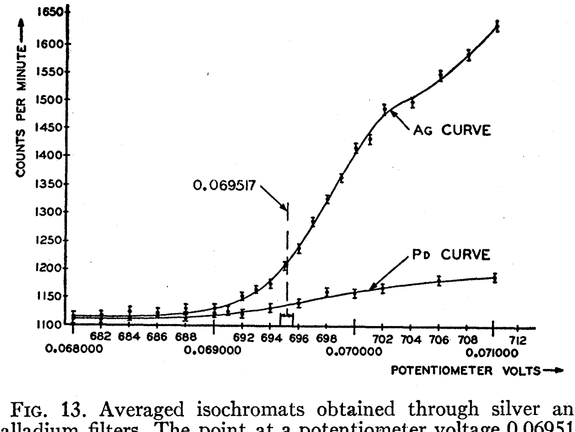

FIG. 13. Averaged isochromats obtained through silver and palladium filters. The point at a potentiometer voltage 0.069517 is the point of maximum bending of the "best" smoothed curve through the silver filter and the spread on either side of this point, indicated by the arrows, is the estimated uncertainty based on the deviations from this position obtained when the points of maximum bending were determined for intentionally poorly smoothed curves through the same filter. To the abscissa readings here shown must be added the potential of the bucking standard cell to obtain the total potential across the low-resistance segment of the working divider.

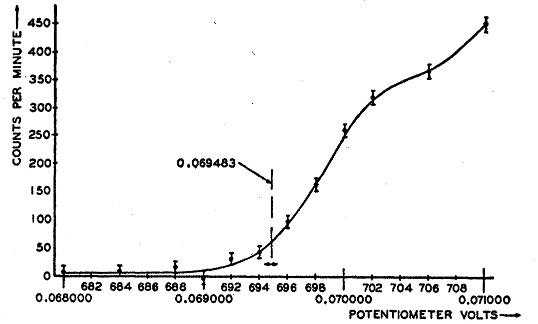

FIG. 14. The difference isochromat. This is the difference between the Ag and Pd curves of Fig. 13. The abscissa point at a potentiometer voltage of 0.069483 is the point of maximum bending of the "best" smoothed curve plotted through these difference observations and the spread indicated by the arrows is the estimated uncertainty based on points of maximum bending determined for intentionally poorly smoothed curves through the same points. Here as in Fig. 13, the bucking standard cell voltage must be added to the potentiometer readings given as abscissas.

C. The Isochromats

Figures 13 and 14 show all the data adjusted to the same abscissae for each run and plotted against potentiometer microvolts as abscissas. Each ordinate plotted gives the mean number of counts observed in 5 minutes averaged for from 16 to 22 separate measurements. The indicated spread about the mean value for any ordinate is given by the square root of the total number of counts at that setting divided by the number of separate 5-minute observations made at that point (16 or, for silver points in the middle of the curve, 22). Figure 13 gives the two composite isochromats obtained through the silver and the palladium filters separately, while Fig. 14 gives the difference of these two curves.

The data were plotted to large scale and smooth curves were drawn through these points with a spline. The smoothed ordinates were then read off numerically and the third derivative of this curve was computed by a method of numerical differentiation based on the calculus of finite differences given by Whittaker and Robinson.[29] The method consists essentially in the fitting of a high degree polynomial to an array of adjacent points read from the graph at equal abscissas intervals and a simultaneous differentiation of the

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[29] E. T. Whittaker and G. Robinson, *Calculus of Observations* (Blackie and Sons, Ltd., London, 1924), p. 65.

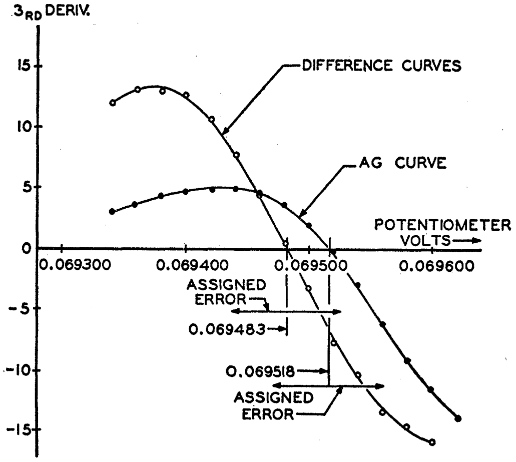

FIG. 15. Third derivatives of the "best" smoothed isochromats for the silver curve of Fig. 13 and for the difference curve of Fig. 14, obtained by a numerical differentiation process based on the theory of finite differences. The "zero" of each curve gives the point of maximum bending of the isochromat. By differentiating intentionally poorly smoothed isochromats in this same way the deviations of the zero points of these gave the spread from which the assigned error, indicated by the arrows, was obtained.

resulting polynomial. The curves of Fig. 15 are the third derivatives of the isochromats of Figs. 13 and 14. The point where the third derivative vanishes corresponds to the point of maximum bending of the isochromat, and this has been shown by one of us[30] to be a reliable criterion for finding the true threshold position at the quantum limit.

The virtue of the above described method of numerical differentiation is that by selecting abscissas rather widely spaced (30 or 40 microvolts on our graphs) one can include a large region of the curve in the calculation of the third derivative at the center of this region. Our calculations were based on regions 500 microvolts wide. On the other hand, this method of locating the limit gives a deceptive appearance of precision because it is in reality no more precise than the smoothing of the curve is reliable.

As always, the "best" smooth curve which can be drawn through points subject to statistical uncertainty, when little is known of the nature of the curve, must be somewhat a matter of opinion. For this reason, several curves were drawn through the measured points and differentiated before the final curve was selected. We found that the location of the limit varied by about 3 parts in 10^5 among the several "good" curves. In order to form a satisfactory estimate of probable error, we drew in addition several curves which were intentionally not good in that they ignored some of the data while remaining close to the statistically determined ranges of the rest. The third derivatives of these exhibited shifts in the limit point of as much as 7 parts in 10^5. (It should not be forgotten that the emf of the bucking cell, 1.019161 absolute volts, must be added to the potentiometer readings plotted as abscissas in Figs. 13 and 14 to obtain the total figure proportional to the applied high voltage.)

The limit points (shown in Fig. 15) which we consider the "best" for each of the two isochromats, (1) the silver isochromat and (2) the silver-palladium difference isochromat, are seen to differ by only about 35 parts per million.

The striking features of the true spectrum, the peaks and valleys slightly above the quantum limit found at lower voltages by Ohlin,[6] by Bearden and Schwarz,[9] and by Bearden, Johnson, and Watts[10] are clearly evident in these curves though they are less accentuated than in the longer wavelength isochromats.

D. Calculation of Final Results and Estimation of Errors

The data described above in Sec. III B, yield the results shown in Table III on which the calculation of the monochromator window wavelength and its uncertainty are based. Columns 2 and 3 list the separation in screw turns, ΔS, and in x-units, $\Delta\lambda$, between each fluorescence line and the monochromator setting, 3 being calculated from 2 using the screw factor S.F. $=1.00029\pm0.0003$ turns per x.u.[31] Column 4 gives the accepted wavelengths of the lines and 5 is the monochromator window wavelength computed by adding $\Delta\lambda$ and λ, columns 3 and 4. The estimated probable errors in column 3 are computed in the usual quadratic way from the relative error in location and the relative error in screw factor and those of column 5 are computed from the quadratically compounded absolute errors of 3 and 4. The determination of the error of the above weighted mean value of the monochromator window wavelength differs insignificantly whether calculated by internal or by external consistency.

A weighted average of the potentiometer readings corresponding to the points of maximum bending of the two "best" curves, $v_{\mathrm{Ag}}=0.069517$ abs. volts and $v_{\mathrm{dif}}=0.069483$ abs. volts (for the "silver" and the "difference" isochromats respectively) was taken with respective weights 1.4 and 1.0. These were chosen because the statistical counting rate uncertainties near the

TABLE III. Calculation of mean value of monochromator window wavelength.

1 Line	2 ΔS in screw turns	3 $\Delta\lambda$ in x-units	4 λ of line in x-units	5 Monoch. window wavelength in x-units
Sn $K\alpha_1$	15.263 ± 0.005	15.259 ± 0.0067	489.57 ± 0.005	504.829 ± 0.0083
Sn $K\alpha_2$	10.830 ± 0.010	10.827 ± 0.0105	494.02 ± 0.005	504.847 ± 0.0116
Ag $K\beta_1$	$8.81\ \pm0.015$	8.807 ± 0.015	496.01 ± 0.005	504.817 ± 0.016
Ag $K\beta_3$	$8.20\ \pm0.02$	8.198 ± 0.02	496.65 ± 0.005	504.848 ± 0.021
Weighted mean value of $\bar{\lambda}_{SWL}{}^{(S)}$ in x-units $=\lambda_S=504.833\pm0.008$				

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[30] First discussed in reference 5. A better proof by Panofsky is given in reference 8.

[31] The error ±0.0003 in S.F. represents a very liberal increase above the value determined for the shorter wavelength regions near the center of the screw. Its effect on the calibration is negligible however because the increments ΔS are relatively so small.

limit were very nearly such as to call for this ratio. The potentiometer reading adopted was, therefore,

$$v_p = 0.069503 \pm 0.000045. \tag{8}$$

The estimated probable error is based on our finding that the "poor" curves could shift the low voltage (standard bucking cell voltage added to v_p above) by as much as 70 parts per million. We regard ±0.000045 volts which is plotted on Figs. 13, 14, and 15 as a generous estimate for the probable error in determining the point of maximum bending. It is admittedly little more than an educated guess.

Adding to v_p above the bucking standard cell voltage, $v_{sc} = 1.019161 \pm 0.000005$ (L.E.) abs. volts, one obtains for the voltage across the low resistance arm of the working divider

$$v_L = 1.088664 \pm 0.000046 \text{ abs. volts,} \tag{9}$$

in which the contribution to the probable error from the limits of error of the bucking standard cell and the potentiometer measurement play a completely negligible role even if they are treated as probable errors instead of limiting errors.

The conversion from low to high voltage is accomplished by means of the precision divider ratio used to set the 100-megohm divider

$$\text{p.d.r.} = 22\,503.52 \pm 0.10 \text{ (L.E.).} \tag{10}$$

The error limit in this ratio is generous but it still contributes insignificantly to the error of the nominal high voltage, V_N, obtained from the product of (9) and (10).

$$V_N = 24\,498.7 \pm 1.0 \text{ abs. volts.}$$

The effects of ripple, uncertainty in the 100-megohm divider (1 part in 10^5) and uncertainty from possible errors in the electronic stabilizer (also estimated at 1 part in 10^5) raise the latter error estimate to ±1.1 absolute volts and give the following value for V_H, the high voltage applied to the tube:

$$V_H = 24\,498.7 \pm 1.1 \text{ abs. volts.} \tag{12}$$

To this must be added the cathode work function, taken as

$$V_w = 4.52 \text{ abs. volts,} \tag{13}$$

to obtain the voltage V_A corresponding to the maximum electron energy available to be converted into photon energy. Clearly any error in tungsten work function at the voltage of this experiment would have to be of the unlikely order of 20 percent before the final error would be appreciably altered. One obtains then for the final voltage,

$$V_A = 24\,503.2 \pm 1.1 \text{ abs. volts.} \tag{14}$$

TABLE IV. Comparison of three independent measurements of $V_A\lambda_s$.

(Felt, Harris, and DuMond)	$V_A\lambda_s = (h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8$ $= 12\,370.02 \pm 0.63$ emu·kx-units
Bearden, Johnson, and Watts)[a]	$V_A\lambda_s = (h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8$ $= 12\,371.03 \pm 0.48$ emu·kx-units
(Bearden and Schwarz)[b]	$V_A\lambda_s = (h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8$ $= 12\,370.77 \pm 1.03$ emu·kx-units

[a] See reference 10.
[b] See reference 9.

From this, one computes immediately the value of

$$V_A\lambda_S = (h/e)c^2(\lambda_g/\lambda_s)^{-1}\,10^8$$
$$= 12\,370.02 \pm 0.63 \text{ emu}\cdot\text{kx-units.} \tag{15}$$

This last value, the conversion constant between electron volts and wavelengths in kx-units (Siegbahn), is the numerical result which should strictly be regarded as the output value of the present experiment. To avoid confusion in the literature we shall resist the temptation of combining it with other measured values (c^2 and λ_g/λ_s) to calculate h/e for instance. We confine ourselves to comparing in Table IV our value of the voltage—wavelength conversion constant with values obtained in two other recent sets of experiments.

It is a pleasure to acknowledge our indebtedness to Dr. David A. Lind for his generous assistance and advice throughout this experiment; to Henry N. Beets and John C. Scott (North American Philips and Eureka Company representatives) for donating the x-ray tube used in all of the final work and for their prompt and much needed cooperation when an unsuspected minute water leak into the oil jacket of the first (newly purchased) "shockproof" tube housing caused a most disconcerting, expensive, and time-consuming series of tube failures at a critical time in the experiment; and to Martin Walt and Gerald Jacobs for their diligent and able assistance, both in the construction of the 100-megohm precision manganin divider and, in the case of Mr. Walt, also during the performance of the experiment. Some of the expensive components used in this experiment, such as the 0.25-μf H.T. condensers, the 1000-henry chokes, and the 100-megohm units of the nichrome wire-wound working resistance divider were purchased at an earlier date with funds donated by Dr. Leon L. Watters of New York City, known as the Frances Hayes Watters Memorial Research Fund, and it is a pleasure to express our gratitude to this donor as well as to our sponsors, the U. S. Office of Naval Research.