

It should be pointed out that the magnitude of the kinetic energy of the decaying meson can be calculated from the conservation laws, if one is able to measure the energies of the pair components as well as the angles between the direction of propagation of the decaying meson and the pair components. This assumes that one can, at least approximately, draw a line from the star in which the π^0 meson is created to the intersection of the paths of the pair components. No recourse needs to be had in this case to the previously observed energy distribution of charged π mesons; one can measure directly the primary energy of the individual neutral meson.

A detailed report on the calculations leading to the results quoted and to their extension to the decay of moving mesons will soon be submitted to *The Physical Review*.

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¹ R. H. Dalitz, Proc. Phys. Soc. (London) **A64**, 667 (1951).

² Daniel, Davies, Hulvey, and Perkins, Phil. Mag. **43**, 753 (1952); Lindenfeld, Sachs, and Steinberger, Phys. Rev. **89**, 531 (1953); other papers by the Bristol group on the present subject are in the process of publication.

³ J. R. Oppenheimer and L. Nedelski, Phys. Rev. **44**, 948 (1939).

Total Cross Sections for 169-Mev Neutrons

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THE present work has been the first of a series of experiments with high-energy neutrons produced by the 230-cm synchrocyclotron at the University of Uppsala. Neutrons were produced by proton bombardment of an internal Be target, and the experiment consisted of good-geometry transmission measurements in a well collimated neutron beam. A coincidence counter telescope was used to detect recoil protons from a thin polyethylene disk placed in the beam. This counter telescope consisted of three scintillation counters feeding a diode bridge type coincidence circuit.¹ Aluminum absorbers were placed between the second and the third counters in order to set a lower-energy limit to the recoil protons detected, and the effective neutron energy was calculated² to be 169 Mev with a possible error of 4 Mev.

The hydrogen cross section was determined by measuring the attenuation of the neutron beam in samples of polyethylene and graphite containing the same amounts of carbon. Similarly the difference between the deuterium and hydrogen cross sections was found from samples of ordinary and heavy water containing the same amounts of oxygen. The cross sections determined are as follows:

Hydrogen	49.2 ± 1.6 mb
Deuterium - hydrogen	23.1 ± 2.0 mb
Carbon	323 ± 3 mb
Oxygen	430 ± 5 mb

The quoted errors are standard deviations based on the total number of counts. A more detailed account of the work will be published later.

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¹ Dickson, Salter, Gunnill, Dolley, and Cassels, Harwell Atomic Energy Research Establishment Report G/R 1938, 1952 (unpublished).

² A. E. Taylor and E. Wood, Phil. Mag. **44**, 95 (1953).

The Interpretation of Bhabha's Theory of Particles of Maximum Spin 3/2

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BHABHA'S theory¹ of particles of maximum spin $\frac{3}{2}$, characterized by the representation scheme $R_s(\frac{3}{2}, \frac{1}{2})$, has the peculiar feature that the normalization is negative for all states with the higher of the two possible values ($\mu, 3\mu$) of the rest mass, and the application of the customary probability interpretation leads to the difficulty of negative probabilities. For this reason, and doubtless also because of the greater complexity of the noncommutative algebra involved, very little has been published on the results of this equation, apart from Le Couteur's work² on the scattering of mesons by these particles. In order to get some clue about a possible interpretation, it may be helpful if some of the peculiar consequences are brought out in high relief in a very simple, illustrative case. We have therefore studied the magnitudes and behavior of the reflection and transmission coefficients of Bhabha's particles at a discontinuity $V (> 0)$ of the electric potential, the well-known problem treated by Klein for the Dirac electron.³ Similar considerations will apply to other cases with static potential, e.g. Coulomb scattering.

The analysis of this problem⁴ shows that, with incident particles of the lower mass μ and of energy ω , the states of higher mass and negative normalization can occur only in conjunction with states of lower mass such that the total current is conserved in accordance with the equation $1 = R_1 - R_3 + T_1 - T_3$, where $R_1, R_3, (T_1, T_3)$ are the magnitudes of the reflected (transmitted) currents of particles of lower and higher mass, divided by the incident current. For example, with $\omega = 5\mu$ and $eV = 3\mu$, one obtains $R_1 = 52, R_3 = 59, T_1 = 8, T_3 = 0$. This would mean that, apart from ordinary reflection and transmission of the original incident current, the barrier also gives rise to a concomitant creation of pairs of particles of different mass and opposite charge, which are also reflected and transmitted, the energy of each particle being the same as the incident energy. Now it is clear that the energy of these pairs cannot be obtained from the static field, and they cannot, therefore, be considered as real. This indicates that physically interpretable results may be derived from this theory by subtracting the contributions arising from the unreal created pairs and applying the ordinary probability interpretation to the rest. The calculation shows that the ratio of the created to the incident current increases from zero, at $\omega = 3\mu$, to a maximum, which may be of the order of several thousands, and then decreases to the finite limit $eV/2\mu$ for very high incident energies. On the other hand, the proportion of the created particles of lower mass in the reflected and transmitted currents follows no simple pattern. While under conditions of total reflection of the incident current the created particles, too, are totally reflected, in the other extreme case of total transmission (that is, in the limit of very high incident energies) all the created particles go into the reflected current ($R_1 \rightarrow R_3 \rightarrow eV/2\mu, T_1 \rightarrow 1, T_3 \rightarrow 0$), as though they were totally transmitted from the barrier region into free space.

Whatever the interpretation of the higher mass states, it is of some interest to note that, in the energy regions where no states of higher mass can be involved (that is, with incident energies less than 3μ and potential energies less than 2μ), the ordinary probability interpretation can be applied without modification. This leads, first of all, to the significant result that in the non-relativistic limit of incident particles of the lower mass, and with small potentials (so that the transmitted kinetic energy is small and positive), the particles actually behave like Dirac electrons. Moreover, as long as no states of higher mass are involved, this unmodified interpretation can consistently be continued even in the relativistic region where significant deviations from the behavior of Dirac electrons occur. In the present illustrative case one finds that, with the values of eV/μ of 0.5 and 1, the reflection coefficient R is always much greater than for the Dirac electron;

for example, with $eV=0.5\mu$ and $\omega=1.6\mu$, one obtains 0.33, as compared with 0.14 for the Dirac electron, while with $\omega=1.85\mu$ the values are 0.16 and 0.03, respectively. Apart from this difference in magnitude, there is also a typical minimum of R with increasing incident energy below $\omega=3\mu$, which is impossible with the Dirac electron, where R drops monotonically to zero for such high values of ω . Similar marked effects in the region of validity of the ordinary interpretation can be found in other more realistic problems (e.g., Coulomb and Compton scattering, details of which will be given elsewhere), and they may thus lead to tests of the theory regardless of questions of interpretation in other regions.

¹ H. J. Bhabha, *Revs. Modern Phys.* **17**, 200 (1945); *Proc. Indian Acad. Sci.* **A21**, 241 (1945).

² K. J. Le Couteur, *Proc. Cambridge Phil. Soc.* **44**, 63 (1947).

³ O. Klein, *Z. Physik* **53**, 157 (1929). Without discussing intensities, the energy regions of total reflection of a particle with two mass states have been considered by G. Pétiau, *Rev. Sci. Instr.* **85**, 135 (1947).

⁴ For the calculation of the required expansion coefficients it is convenient to use a basic orthonormal set of (16-component) solutions that are also eigensolutions of the spin in the direction of motion. This is most readily obtained by transformation from the rest system, and by use of a certain canonical transformation. See W. A. Hepner, *Phys. Rev.* **81**, 290 (1951); **84**, 744 (1951).

The Single-Time Bethe-Salpeter Equation

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IN a recent issue of this journal, Dyson¹ published a note about the Tamm-Dancoff method for treating the interaction of two particles in bound states. In this note he proposes to replace the usual wave function

$$a(N) = (\phi_0, A(N)\Psi), \quad (1)$$

by another one,

$$a(N', N) = (\Psi_0, C(N')A(N)\Psi), \quad (2)$$

and get

$$(W - E_N + E_{N'})a(N', N) = (\Psi_0, [C(N')A(N), H']\Psi) \quad (3)$$

as an equation of motion for $a(N', N)$. The notation here is the same as in Dyson's letter. By comparison of (2) with the paper of Gell-Mann and Low² it can be easily seen that $a(0, 2)$ is the single-time form of the Bethe-Salpeter wave function for two particles in positive free-energy states.

To find an equation for $a(0, 2)$ only, there are two possibilities. First, it can be developed by solving Eq. (3) and eliminating afterwards all components $a(N', N)$ with the exception of $a(0, 2)$. The other way consists in the transformation of the Bethe-Salpeter equation, given in its most general form, into a single-time four-component equation, as was proposed by the author³ and outlined in three papers,^{4,5} the first of them (I) giving the general derivation of the single-time equation. The second paper (II) is concerned with possible transformations of that equation into other forms, and in the third paper (III) the process of renormalization for this single-time equation is described.

The results show that the wave equation for $a(0, 2)$ can be given by

$$a(0, 2) = fa(0, 2), \quad (4)$$

just as in the Tamm-Dancoff case, where f is an integral operator which can be represented by a totality of "time-ordered graphs" given by Lévy. This totality does not contain vacuum graphs, as Dyson shows. But further on, the analytical meaning of f is rather different from the Tamm-Dancoff case, as is shown explicitly in (II), where the two methods are compared. Here, as well, the totality of graphs is different, and also the energy denominators are different in some cases.

The derivation of (4) from Dyson's equation of motion can be found directly by, first, writing out the interaction operator H' of (3) and eliminating all the $a(N', N)$ with the exception of $a(0, 2)$. Then the graphic representation for all virtual states

and transitions can be constructed in the same way as Lévy did, but here it will contain also the "absent particles" N' with negative energies. These particles N' now can be represented by lines running backward, and by application of the rules of partial fraction decomposition, as given in (I), Eq. (4) is obtained, with an operator f which is identical with the operator $'f_p'+'f_d'$ in Eq. (14) of (II).

As was discussed in (II), something can also be said about the relations between the different wave function definitions given in (1) and (2). As there is a connection between the two vacuum states,

$$\Psi_0 = P \exp\left(-i \int_{-\infty}^0 H' dt\right) \phi_0, \quad (5)$$

there follows from (1) and (2) a relation of the form

$$a(0, 2) = ga(2). \quad (6)$$

The quantity g is calculated in (II). Here it can be found also as an integral operator, representable by Lévy graphs, if one substitutes (5) into (2). The exponential function needs to be expanded, the time integrations to be performed, and H' to be represented by the related creation and annihilation operators. All $a(N)$ for $N \neq 2$ can then be eliminated by the Tamm-Dancoff method and so finally an explicit form for g can be found.

¹ F. J. Dyson, *Phys. Rev.* **90**, 994 (1953).

² M. M. Gell-Mann and F. Low, *Phys. Rev.* **84**, 350 (1951).

³ W. Macke, *Phys. Rev.* **91**, 195 (1953).

⁴ W. Macke, I and II, *Z. Naturforsch.* (to be published).

⁵ W. Macke, III, *Nuovo cimento* (to be published).

The Fission Yields of Ru¹⁰³ and Ru¹⁰⁶

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THE generally accepted values for the fission yields of Ru¹⁰³ and Ru¹⁰⁶ for natural uranium using pile neutrons were obtained by Glendenin and Steinberg.¹ These fission yields have been redetermined for natural uranium irradiated in the Harwell pile. Under the experimental conditions the contribution from fast fission was considered to be negligible. Ru¹⁰⁶ (as Rh¹⁰⁶) and the reference standards Ba¹⁴⁰ and Cs¹³⁷ were determined by β counting. The disintegration rate of Ru¹⁰³ was found by γ counting, assuming the decay scheme proposed by Kondaiah² and Mei,³ and with allowance for the γ 's arising from the Ru¹⁰⁶ present. This allowance was determined by β and γ counting pure Ru¹⁰⁶ separated from old fission products.

An end-window geiger tube was used for β counting. It was calibrated against a "4 π " counter of Isotopes Division, Atomic Energy Research Establishment, and checked using a Ra D-E-F source whose disintegration rate was known from α counting. The Ru¹⁰⁶ aliquots for counting were evaporated on thin plastic film and always contained less than 0.1 mg of carrier. Under these conditions scattering and self-absorption corrections are less than 1 percent.

The γ scintillation counter employed a sodium iodide crystal 1.75 inches in diameter, and 0.75 inch thick. It was calibrated using sources of Au¹⁹⁸ and Cs¹³⁷. These two isotopes were chosen because γ emission from Au¹⁹⁸ is almost wholly 0.41 Mev, and γ emission from Cs¹³⁷ almost wholly 0.66 Mev. The Ru¹⁰³ γ energy of 0.49 Mev is intermediate, and it was assumed that the efficiency of the γ counter for the Ru¹⁰³ γ would thus lie between the values found for the Au¹⁹⁸ and Cs¹³⁷ γ 's. Story⁴ has reviewed the published data and, allowing for conversion, has estimated that 93 percent of β emissions from Au¹⁹⁸ are accompanied by 0.41 Mev γ 's, and 82 percent of the β emissions from Cs¹³⁷ are accompanied by 0.66 Mev γ 's. Using these data, the efficiency of the γ counter was found to be 2.88 percent for Au¹⁹⁸, and 2.73 percent for Cs¹³⁷. The efficiency for Ru¹⁰³ was taken to be 2.8 percent.

Four samples of uranium metal were irradiated, one for seven