

The Universal Fermi Interaction*

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On the basis of the hypothesis that the same form of interaction acts among any spin- $\frac{1}{2}$ particles, it is interesting to apply the interaction law found for β decay to the muon processes. The application is beset by two types of ambiguity. The first is due to the uncertainty in measured values of coupling constants, and particularly their signs. The second arises from the various ways in which the correspondence between the particles of μ and β decay may be taken. Arguments are presented that the unique correspondence established if two *like* neutrinos are ejected in μ decay is the correct one.

It is argued that previous interpretations of the Universal Fermi Interaction have been unjustifiably broad. Only processes in which two normal particles (*vs* antiparticles) are annihilated, and two created, should be expected. The positive muon must be treated as the normal-particle (if the neutron, proton and negatron are) in order to avoid the expectation that muon capture by a proton may yield electrons, contrary to experimental facts. The conclusion that two like neutrinos are ejected in μ decay follows immediately.

1. β AND μ DECAY

IN a recent paper,¹ a "phenomenological derivation" of the law of β decay led to the conclusion that it must be an "STP combination." This means that the interaction energy density is a linear combination of the well-known scalar (S), tensor (T) and pseudo-scalar (P) forms. Experimental evidence was used to conclude that all three forms must be present and that the polar vector (V) and axial vector (A) forms should not be included.

The weakest point of the "derivation" was that only one piece of evidence, the singular spectrum of RaE, could be offered that the TP combination must be part of the correct law. A VA , ST , or VAP combination would otherwise do just as well as the STP combination. Since the publication of that result there has been experimental confirmation that the T component is an essential part of the law. This was shown by improved measurements² on the correlation between the nuclear recoil and the β emission of He⁶. A similar measurement³ on O¹⁴ is awaited with interest; it should provide an unambiguous check on the choice of the S or the V component in the coupling.

Meanwhile, the analysis⁴ of RaE which led to the choice of the TP components receives further support. Konopinski and Langer⁵ pointed out that the Petschek-Marshak analysis of the RaE spectrum yields the peculiar deviation from a statistical shape, (which is the shape to be expected normally) through the admission of an accidental, destructive interference between the T and P contributions. A corresponding slowing down of the RaE decay is then to be expected and

this was found, in comparisons against the similar nuclei Hg²⁰⁵, Tl²⁰⁶, and Pb²⁰⁹. Now Brysk reports⁶ calculations, based on jj -coupled pairs of single-particle states, that the destructive interference in RaE is to be expected from a $T+P$ combination (in the conventional definition⁵ of these forms). This contrasts in the expected way with the much shorter-lived Tl²⁰⁶, where constructive interference is found instead. RaE has 83 protons and 127 neutrons, each being one more than enough to fill magic shells. Tl²⁰⁶ has 81 protons and 125 neutrons.

Some evidence for the inclusion of the P interaction, independent of the RaE case, has also been reported. Nordheim^{7a} finds that once-forbidden transitions with $\Delta I=0$ (no spin change) are distinctly more rapid than $\Delta I=1$ transitions. Considering the lack of such a difference in allowed transitions, the extra contributions to the $\Delta I=0$, once-forbidden transitions are best ascribed to the P interaction, which makes its largest contribution to just such transitions.

The fact that the rates of muon decay and capture indicate the same interaction strength as does β decay and capture has led to the hypothesis of a Universal Fermi Interaction:^{7b} the same coupling among *any* four fermions. It thus becomes interesting to examine the STP form of interaction in μ decay.

One available datum on muon processes which is sensitive to the form of interaction is the spectrum of the electrons from μ decay. Michel⁸ has shown that, whatever specific form of Fermi coupling is assumed, the theoretical result for the spectrum can be written

⁶ H. Brysk, *Report of the Indiana Conference on Nuclear Spectroscopy and the Shell Model* (Indiana University, Bloomington, 1953); p. 27. Brysk also finds a larger P contribution than reported by Ahrens, Feenberg, and Primakoff, *Phys. Rev.* **87**, 663 (1952). M. Rudermann, *Phys. Rev.* **89**, 1227 (1953), presents other grounds for expecting a sufficiently large P contribution.

^{7a} L. Nordheim, *Report of the Indiana Conference on Nuclear Spectroscopy and the Shell Model* (Indiana University, Bloomington, 1953), p. 21.

^{7b} O. Klein, *Nature* **161**, 897 (1948). See also references 14 and 15. (Footnote added in proof.)

⁸ L. Michel, *Phys. Rev.* **86**, 814 (1952).

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¹ H. M. Mahmoud and E. J. Konopinski, *Phys. Rev.* **88**, 1266 (1952).

² J. S. Allen and W. K. Jentschke, *Phys. Rev.* **89**, 902 (1953); B. M. Rustad and S. L. Ruby, *Phys. Rev.* **89**, 880 (1953).

³ M. E. Rose, *Phys. Rev.* **90**, 1123 (1953).

⁴ A. Petschek and R. Marshak, *Phys. Rev.* **85**, 698 (1952).

⁵ E. J. Konopinski and L. M. Langer, *Annual Reviews* **2**, 261 (1952).

(valid except for electron energies as low as $\frac{1}{2}$ Mev):

$$d\lambda/\lambda dW = 4(W^2/W_0^4)[3(W_0 - W) + \frac{2}{3}\rho(4W - 3W_0)], \quad (1)$$

where W is the electron energy and W_0 its end-point value. λ is the decay constant and ρ is a parameter which depends on the coupling used. The spectrum (1) has a vanishing intensity at the end-point for $\rho=0$, and larger finite intensities for larger ρ .

The experimental measurements of the μ -decay spectrum disagree with each other. Sagane, Gardner, and Hubbard⁹ reported $\rho \approx 0$. Lagarrigue and Peyrou¹⁰ give $\rho = 0.19 \pm 0.12$ as the result of their own measurements and $\rho = 0.075 \pm 0.20$ as the value deduced from Leighton, Anderson, and Seriff's data.¹¹ Finally, Bramson, Seifert, and Havens¹² find $\rho = 0.41 \pm 0.13$.

An application of the *STP* coupling to the problem is handicapped by two sources of ambiguity. In the first place, the relative sizes of the *S*, *T*, and *P* components is known only roughly. According to Blatt,¹³ G_S^2/G_T^2 may have any value from ~ 0.3 to ~ 1.0 , where $G_{S,T}$ are the Fermi coupling constants for the *S* and *T* interactions. The pseudoscalar coupling constant, G_P , is known only to have about the same order of magnitude as the others. Moreover, the relative signs of the *G*'s are important and unknown. One indication exists that $G_P = G_T$, in sign and magnitude; that is the work of Brysk mentioned above.

In the second place, the significance of the *STP* form for μ decay depends on the correspondence assumed between the particles participating in μ decay and those of β decay. Three different orderings are possible,¹⁴ known as "simple charge exchange," "charge retention" and "antisymmetrical charge exchange." The last of these differs from the others in that the two neutral particles produced in the μ decay are presumed *like* neutrinos, i.e., one is not an antineutrino.

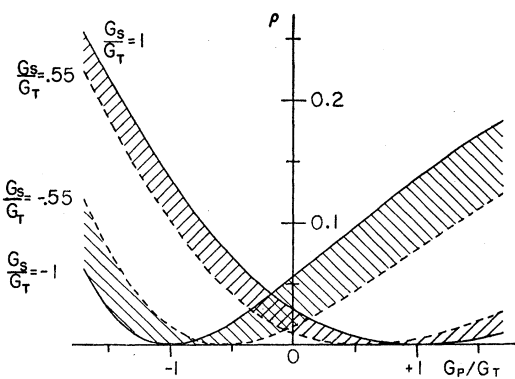


FIG. 1. ρ versus G_P/G_T consistent with $G_S^2/G_T^2 = 0.3$ to 1.

⁹ Sagane, Gardner, and Hubbard, Phys. Rev. **82**, 557 (1951).

¹⁰ A. Lagarrigue and C. Peyrou, J. phys. et radium **12**, 848 (1951).

¹¹ Leighton, Anderson, and Seriff, Phys. Rev. **75**, 1432 (1949).

¹² Bramson, Seifert, and Havens, Phys. Rev. **88**, 304 (1952).

¹³ J. M. Blatt, Phys. Rev. **89**, 83 (1953).

¹⁴ J. Tiomno and J. A. Wheeler, Revs. Modern Phys. **21**, 144 (1949).

Without some reduction of the ambiguities, no useful predictions of the μ spectrum can be made from the *STP* law. Any value of ρ in the range obtained in the various experiments can be reproduced. The point of interest is that the values predicted for ρ are surprisingly restricted, even with the uncertainties about the *G* values, when one presumes that two like neutrinos are emitted in the μ decay. We confine our attention to this case, not only for the sake of making relatively unique predictions, but more important, because a proper interpretation of the universal interaction seems to indicate that the two neutrinos must be alike. The argument for this is presented in the next section; here we investigate its consequences for the μ spectrum.

When two like neutrinos are emitted, the *STP* interaction gives

$$\rho = \frac{3}{2} \frac{(G_S - G_P)^2}{2(G_S - G_P)^2 + (G_S + 6G_T + G_P)^2}. \quad (2)$$

We thus have, with $G_S^2/G_T^2 = 0.3$ to 1.0:

- $\rho = 0$ for laws of the form $G_S(S+P) + G_T T$,
- $\rho = \frac{1}{2}$ for $G_S S + G_T(T - 3P)$,
- $0.01 < \rho < 0.06$ if $G_P = 0$,
- $\rho < 0.14$ if $G_P/G_T = \pm 1$.

These are the cases in which G_P is restricted to more plausible values. ρ is shown as a function of $G_P^2/G_T^2 \lesssim 3$ in Fig. 1. To get as high a value as $\rho \approx 0.4$, the largest reported by the experimenters, one would need $G_P^2/G_T^2 \gtrsim 6$, which is highly implausible.

2. THE SIGNIFICANCE OF A UNIVERSAL INTERFERMIONIC COUPLING

The attractive hypothesis that a Fermi type of interaction exists among any four fermions may have a serious drawback. Besides accounting for the several known processes, it may lead to the expectation of processes which contradict experience—yet are consistent with conservation principles. Yang and Tiomno¹⁵ tried to forbid the unwanted processes by defining intrinsic "generalized parity" contributions by pairs of fermions, in such a way that a "generalized parity" balance is violated in the unwanted processes. They were only partially successful in this. They could not forbid processes like $N + N \rightarrow \bar{N} + \bar{N}$ ($\bar{N} \equiv$ antineutron) or $P + \mu \rightarrow P + e$, on their interpretation of the universal interaction, without postulating special, "ad hoc", rules against these.

Caianiello¹⁶ reports that all unwanted processes can be successfully eliminated by adopting interactions in a special way. His procedure amounts to admitting coupling constants which are not just numbers, but quantities which transform in certain ways during

¹⁵ C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950).

¹⁶ E. R. Caianiello, Nuovo cimento **8**, 534 and 749 (1951); **9**, 336 (1952).

changes of coordinates. This seems to us undesirably artificial and also unnecessary.

A thesis of this paper is that former interpretations of the Universal interaction have been unnecessarily, and even unjustifiably, broad. We shall use what seems to us a better justified interpretation which makes the processes like $N+N\rightarrow\bar{N}+\bar{N}$ and $P+\mu\rightarrow P+e$ unformulable *ab initio*. A further concrete consequence will be the conclusion that the neutral particles ejected in μ decay are *like* neutrinos (i.e., one is not an anti-neutrino).

Consider the electron capture process:



When formulating this alone, it is clearly only a matter of nomenclature as to whether one considers all these to be normal particles, or whether one or more is treated as an antiparticle (hole). This follows from the well known "charge-conjugacy" of the Dirac equations. We shall adopt the convention of calling the particles symbolized in (3) normal particles, so that (3) entails the annihilation of two normal particles and the creation of two others.

When one next formulates a *second* process like the μ capture:



then it has physical consequences to treat the particles symbolized here as all normal. It implies that the neutrino of this process can be captured by a neutron to produce an electron, reversing (3), as well as the inverse of (4). It may be that the light neutral particle ejected in μ capture is an antineutrino ($\bar{\nu}$) instead, capable of being captured by neutrons to give muons, but not electrons; by protons to give positrons but not muons. It will be convenient to write all processes in terms of normal particles only, and since the creation of $\bar{\nu}$ is equivalent to the absorption of ν , we write the μ capture with antineutrino ejection as:



This is an *alternative* to (4), as a representation of the process.

Our way of writing (4) and (5) implies that the negative muon, μ^{-} , is the normal particle, while μ^{+} is the antimuon. In relation to further processes, it may instead be more consistent to treat μ^{+} as normal and μ^{-} as the antimuon. Hence, further alternatives to (4) and (5) are

$$\mu^{+} \text{ normal} \begin{cases} P\rightarrow\mu^{+}+N+\nu, & (6) \\ P+\nu\rightarrow N+\mu^{+}. & (7) \end{cases}$$

The corresponding alternatives, normal forms for μ

decay are:

$$\mu^{-} \text{ normal} \begin{cases} \mu^{-}\rightarrow e^{-}+\nu+\nu & (8) \\ \mu^{-}+\nu\rightarrow e^{-}+\nu & (9) \\ \mu^{-}+\nu+\nu\rightarrow e^{-}, & (10) \end{cases}$$

$$\mu^{+} \text{ normal} \begin{cases} \mu^{+}+e^{-}\rightarrow\nu+\nu & (11) \\ \mu^{+}+e^{-}+\nu\rightarrow\nu & (12) \\ \mu^{+}+e^{-}+\nu+\nu\rightarrow 0. & (13) \end{cases}$$

It is now apparent that the chief distinction between the various alternatives is that the corresponding formulations must variously provide for annihilations of one, two, three, or four normal fermions.

The reaction (3) must first be provided for; an interaction energy operator must be introduced which annihilates two normal fermions and creates two others. Then (3) is to serve as a prototype of the universal interaction among fermions; the same form of operator is assumed to act on *any* fermions. All resulting processes consistent with conservation principles are expected to occur. In each case, the operator destroys just two fermions in occupied states (i.e., normal fermions) of the fields. With such a "universalization," μ capture and μ decay are predicted to occur, and moreover in the forms (4), (9) or (7), (11). The *two*-particle annihilation processes follow unavoidably from (3), unless arbitrary changes of the interaction form are introduced at each application.

Now it seems conspicuously unnecessary, and would require new justification, to introduce *additional* terms, which would repeat provisions for the μ -capture and μ -decay processes in the other ways, as represented by (5), (6), (8), (10), (12), and (13). New interaction operators would indeed be needed, capable of annihilating one, three or four normal particles instead of just two. It can be construed as inconsistent with a "universality" of the interaction, thus to fail to retain the prototype annihilation-creation properties, as one does the magnitude, and the covariant properties, of the interaction. Yet, one, three, and four normal particle annihilations have been included in previous interpretations: the process written $N+N\rightarrow\bar{N}+\bar{N}$, above, is an annihilation of four *normal* particles, $4N\rightarrow 0$; the process $P+\mu\rightarrow P+e$ may be the one-particle annihilation, $P\rightarrow P+\mu^{+}+e$, if μ^{+} should be regarded as normal.

As a physical argument, our objection to the broadening of "universality" by previous interpreters seems clear. When, for example, $a+b\rightarrow c+d$ occurs, we object to the implication that $b\rightarrow c+d+a$ also occurs. The last process may be $\bar{a}+b\rightarrow c+d$ and thus it is implied that a particle b can annihilate either against a , or its antiparticle \bar{a} , to produce $c+d$. We object to this non-unique behavior of a which is to accompany the same behavior of b, c, d in the cases compared.

On the formal side, the situation is more complex. The contact interaction which gives $a+b\rightarrow c+d$ must

be proportional to the annihilation operators ψ_a, ψ_b and also the creation operators ψ_c^*, ψ_d^* . The interaction energy density will be the sum of invariant contractions of like covariant operations, Ω ;

$$G\psi_c^*\Omega\psi_a\cdot\psi_d^*\Omega\psi_b. \quad (14)$$

When each participating fermion is in a positive energy state, $\psi \sim \exp(-iEt/\hbar)$ and (14) is proportional to

$$\exp[i(E_c - E_a + E_d - E_b)t/\hbar].$$

One thus can get a permanent transition, rather than virtual oscillations, only if $E_c - E_a + E_d - E_b = 0$; hence the interpretation $a + b \rightarrow c + d$. If, for example, a is in a negative-energy state, $E_a = -\bar{E}_a$ then $E_b = E_c + \bar{E}_a + E_d$ as expected for $b \rightarrow c + d + \bar{a}$. Now, to formulate $b \rightarrow c + d + a$ one must substitute the creation of a for its annihilation in (14):

$$G\psi_c^*\Omega W\psi_a^*\cdot\psi_d^*\Omega\psi_b. \quad (15)$$

Here W is whatever operator is needed to preserve invariance. As is plausible, it can be shown that¹⁷ $WK = C$, where K is the operation of complex conjugation and C is the well-known charge-conjugation operator; W is a symmetric, unitary matrix which is $W = c\beta\alpha_2$ in a Dirac representation and merely a phase factor $W = c$ in a Majorana representation. Thus, it is shown that, indeed a new interaction operator ($G\Omega C \cdot \Omega$) has to be added to the prototype ($G\Omega \cdot \Omega$) in order to have $b \rightarrow c + d + a$ as well as $a + b \rightarrow c + d$. The interaction (15) owes its interpretation as giving $b \rightarrow c + d + a$ to its being viewed as an operation ($G\Omega C \cdot \Omega$), acting on a *positive* energy state ψ_a (and interpreted as *creating* it, because of the complex conjugation due to operator C). Now, it is well known that the *negative* energy state, as used in (14) to give $b \rightarrow c + d + \bar{a}$, is just¹⁸ $C\psi_a$ when ψ_a is a positive energy state. Hence, the expression (15) with ψ_a a positive energy state is formally identical with (14) when ψ_a is a negative energy state. Thus it is only a forced reinterpretation which allows (15) to describe $b \rightarrow c + d + a$ instead of $b \rightarrow c + d + \bar{a}$. It seems that one cannot¹⁹ formulate $b \rightarrow c + d + a$ at all, together with $b \rightarrow c + d + \bar{a}$, without this forced reinterpretation

¹⁷ As an alternative, a factor γ_5 may need to be included, depending on the properties of the spinor ψ_a under space inversion. This would yield the "odd" type of interaction introduced by Yang and Tiomno as an alternative to the conventional "even" type, (14). This point has no bearing on our conclusions.

¹⁸ This is strictly true only for free particles. However, the number of particles is well-defined (diagonal) only in the field-free case.

¹⁹ This conclusion is unchanged when the charge-conjugate description of antiparticles is used in place of the hole theory. Proper use of the charge-conjugate formalism needs care to make it consistent; the unavoidable existence of negative energy solutions, or their equivalents, forces consistency with the hole theory. We make the exposition briefer, and avoid discussion of what constitutes a proper interpretation of the charge-conjugate formalism, by adhering to the hole theory. We make explicit here only the point that in the charge-conjugate formalism, the energy operator is $-i\hbar\partial/\partial t$ and the momentum operator $-(\hbar/i)\nabla$, i.e., the complex conjugates of the usual operators.

of a formally identical expression. This adds support to our view that one should not include 1, 3, and 4 normal particle annihilations when "universalizing" the Fermi interaction.

Our restriction to two normal-particle annihilations of course simplifies the task of eliminating unwanted processes. First, it is clear that it would always be difficult to eliminate the nonexistent²⁰ process, $P + \mu^- \rightarrow P + e^-$, if it is the negative muon that is a normal particle; certainly no procedure of the Yang-Tiomno type could do so. For the existence of $P + \mu^- \rightarrow N + \nu \leftarrow P + e^-$ implies the unwanted process immediately. On the other hand, if μ^+ is the normal muon, then $P + \mu^- \leftrightarrow N + \bar{\nu}$ [(7), rewritten in an appropriate non-normal form]. Now, the unwanted process cannot even be formulated, on our interpretation of the universal interaction; it would have to be the rejected one normal-particle annihilation, $P \rightarrow P + \mu^+ + e^-$ instead.

The conclusion that μ^- is the antimuon has concrete physical consequences. It would imply that energetic antineutrinos could not produce muons when captured by protons, in an experiment of the Reines-Cowan²¹ type. Of course, pile antineutrinos are not sufficiently energetic to test this. There is a chance, however, that the large cosmic background contains sufficiently energetic ones. Antineutrinos would have to be distinguished from neutrinos by comparisons of electron productions in hydrogen and deuterium. There is also the problem of distinguishing muon and electron production to be solved.

For the μ decay, the conclusion that μ^+ is the normal muon implies that the two neutral particles ejected are *like* neutrinos, as in (9). Hence, the chief ambiguity of the μ -decay theory is removed. The "simple charge exchange" and "charge retention" theories are eliminated in favor of the "antisymmetrical charge exchange." This conclusion was already employed in the previous section.

Another unwanted process, $\mu^- \rightarrow e^- + e^+ + e^-$ is immediately eliminated. If μ^+ is normal, this cannot be expressed as a two normal particle annihilation. If μ^+ were the antimuon, the process could not be forbidden by a procedure of the Yang-Tiomno type.

A further attractive feature of the proposed theory is that *all* unwanted processes can be forbidden and by a less explicit procedure than Yang and Tiomno's. In addition to the hypothesis that μ^+ is the normal muon, only the "conservation of heaviness" is needed to forbid all unwanted processes. The latter principle expresses the observation that the disappearance of a normal nucleon must result in the appearance of a neutron or a proton. Whatever deeper principle is the cause of this, aside from the Yang-Tiomno theory, its acceptance forbids all the types of unwanted processes which remain to be considered. All that can be formulated (consistently with charge conservation, two

²⁰ A. Lagarrigue and C. Peyrou, Compt. rend. **234**, 1873 (1952).

²¹ F. Reines and C. L. Cowan, Phys. Rev. **90**, 492 (1953).

normal-particle annihilation, μ^+ -normal, etc) violate the conservation principle for nucleons.

The Yang-Tiomno theory assigns definite (relative) phases to the transformations undergone by each spinor field in space inversions. The transformation is $\psi' = c\beta\psi$ with c an undetermined phase factor, $|c|^2 = 1$, in general. The indeterminacy cannot be removed²² as for proper transformations, in which continuity with the identity transformation can be demanded. Nor is it necessary that on repetition, $c\beta\psi' = \psi$, because of the well-known two-valuedness of spinors: in rotations by 2π , $\psi \rightarrow \psi' = -\psi$! If one limits oneself to $c\beta\psi' = \pm\psi$, then $c = \pm 1$ or $c = \pm i$. These four are the possibilities to which Yang and Tiomno restricted themselves in defining their four "types" of spinor fields; it is sufficient for most purposes to consider only relations among the c 's characterizing each spinor field, without specifying their values. The idea is to assign such values to c_N , c_P , c_e , c_μ , and c_ν that the interaction (14) exhibits invariance under inversion when applied to the wanted processes (3), (7), and (11), but not for any unwanted two-normal-particle annihilations (the *parity* conservation is contained in the requirement here).

The invariance of the interaction (14) in space inversions is easily seen to require

$$c_a c_b = c_c c_d. \quad (16)$$

Applied to (3), (7) and (11) this gives $c_P c_e = c_N c_\nu$ and $c_\mu c_e = c_\nu^2$. Following Yang and Tiomno, we can derive the condition $c_P = c_N$ from the pion-nucleon interaction. The result is that $c_P = c_N = c_H$ and $c_e = c_\mu = c_\nu = c_L$ are required by the wanted processes (the subscripts H , L stand for "heavy" and "light").

An elementary but tedious review of all the possible unwanted processes shows that, for most of them, (14) loses the required invariance if $c_H \neq c_L$. The prevention of processes of the type $N + N \rightarrow \nu + \nu$ seems to require the stronger conditions: $c_H^2 \neq c_L^2$. In the Yang and Tiomno scheme, $c^2 = \pm 1$ only, hence c_H and c_L are required to differ by 90° of phase. This is the Yang-Tiomno expression of the principle of nucleon conservation.

3. CONCLUDING REMARKS

It should be pointed out that the above discussion did not include the possibility that the neutrino and

antineutrino are indistinguishable, i.e., a Majorana rather than Dirac particle. We believe that this possibility is remote. The repeated failures to find the comparatively short-lived double β decay expected with Majorana neutrinos indicate this; moreover, the one case in which the existence of double β decay may have been proved conforms to expectations for a Dirac neutrino.²³

It is interesting that the sum of interactions (14) and (15) is proportional to $(1+C)\psi_a$, if equal magnitudes and phases are given to the two interactions. Now $(1+C)\psi$ is just the Majorana wave function. Hence, the inclusion of (15) together with (14) implies at least partial behavior by particle a as a Majorana particle. Since the Majorana formalism is certainly incorrect for charged particles and neutrons, and highly questionable also for neutrinos, this adds to the doubt that anything but the double-annihilation interaction (14) should be part of the Universal coupling.

The failure of the Majorana formalism to conform to anything in nature adds interest to Caianiello's²² argument that a Majorana field is non-covariant in inversions. To show this, Caianiello had to make a partial determination of the phase factor, c , of the spinor inversion transformation. He concluded that $c^2 = +1$, hence $c = \pm 1$ only, by requiring that the inversion be continuous with the identity transformation when the space inversion is defined as a continuous rotation through an arbitrary fifth dimension.

The conclusion that the inversion phase is $c = \pm 1$, only, throws doubt on the conclusion of the last section: that c_H and c_L should differ by a factor i . This depends on whether the strong requirement $c_H^2 \neq c_L^2$ is actually needed to forbid processes like $N + N \rightarrow \nu + \nu$ or whether $c_H \neq c_L$ is sufficient. The latter is adequate if one requires that a factor like $\psi_c^* \Omega \psi_a$ of (14) preserve the proper covariance under inversion or whether only the invariance of the product (14) should be demanded. It should further be pointed out that the same condition (16) must be obeyed²⁴ for the phases generated by time reversal or charge conjugation. On these, there is no limitation to real c as, perhaps, for the space inversion. Further, for our essential conclusions we have only the requirement of the "conservation of heaviness" regardless of what specific basis this is given.

²² See, however, the discussion of E. R. Caianiello's attempt [Phys. Rev. **86**, 564 (1952)] in the concluding section.

²³ H. Primakoff, Phys. Rev. **85**, 888 (1952).

²⁴ A. Gamba, Nuovo cimento **7**, 919 (1950).