momentum integration remains in (A.2) and thus no spin-spin combinations can be formed. A similar phenomenon occurs in the second-order perturbation shift.

Upon evaluation of the first-order polarization operators, $\mathcal{G}^{(r)}$ may be written as

$$\mathcal{G}_{\mu\nu}{}^{(r)}(p) = -\delta_{\mu\nu}\frac{\alpha}{4\pi}\left[4\lambda^2\int_0^\infty \frac{ds}{s}\exp(-sm^2) + \int_0^1 du\frac{f_1(u)}{m_1{}^2+u(1-u)p^2} + \int_0^1 du\frac{f_2(u)}{m_2{}^2+u(1-u)p^2}\right], \quad \text{(A.3)}$$

the first term having an extra dipole infinity. These terms lead to a second-order interaction. Considering the constant term first, one obtains an interaction of

$$I(p,p') = \text{const}\times 4\alpha^2\lambda^2\gamma_1\gamma_2, \quad \text{(A.4)}$$

which leads to an energy shift of

$$\Delta E \sim \int \bar{\psi}(p)\gamma_1\gamma_2\psi(p')dpdp', \quad \text{(A.5)}$$

which has no spin-spin part. The evaluation of the energy shifts from the second and third terms would be analogous to the evaluation of $\Delta E^{(1)c}$ except that interaction is already second order and the low-frequency pole of D_+ has been replaced by a high-frequency pole. Thus these terms give no contribution to the desired order.

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A New Modification of Classical Electromagnetic Theory*

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A fundamental particle is treated as a unit charge whose rest mass and space time coordinates are variables of its motion. Classical electrodynamics, in its action at a distance formulation, is obtained from an action principle which is simpler than the usual one. In this new action principle the rest mass of a particle is varied as well as the coordinates. The rest masses of interacting particles, although not assumed constant *a priori*, become constants as a consequence of the equations of motion. Modifications of the old action principle can yield purely electromagnetic rest masses which are, however, the same for all particles. Similar modifications of the new action principle give purely electromagnetic rest masses to all charged fundamental particles. In this new modification of electrodynamics, particles interacting at small distances no longer have constant rest masses.

1. INTRODUCTION

Of the many fields which play an important role in quantum physics, the one whose classical counterpart is most familiar is the electromagnetic field. It has been known for a long time that classical electrodynamics can be formulated in two equivalent forms, as a field theory (Faraday-Maxwell-Lorentz) or as a theory of action at a distance between charged particles.[1] In the case of electrodynamics the two formulations are of the same order of simplicity. Other fields (such as meson fields) could also be described classically in an equivalent action at a distance formulation but, in general, the two descriptions would not be equally simple.

In modern physics it is the field-theoretic point of view which has been stressed. Ignoring quantum mechanical considerations such as statistics, each type of free fundamental particle (photon, electron, meson, nucleon, etc.) is described by a set of field variables whose behavior is characterized by a different Lagrangian function. Interaction is characterized by additional Lagrangians which are functions of the field variables of two or more different fundamental particles. Even if this kind of description gave good results, it can hardly be regarded as satisfactory at a time when the number of fundamental particles is of order 20 and still increasing.

It may be claimed, with only some measure of truth perhaps, that all simple field theories modeled on electrodynamics have been examined exhaustively, and that not one of them shows any indications of explaining all processes involving fundamental particles. It therefore seems worthwhile to investigate systematically all simple modifications of electrodynamics in its action at a distance formulation. The present field theories may well turn out to be asymptotic approximations of an even more complicated and nonlocal field theory which corresponds to a simple equivalent action at a

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[1] K. Schwarzschild, Nach. Akad. Wiss. Göttingen Math. physik. Kl. IIa **1903**, 128, 132, 245 (1903); H. Tetrode, Z. Physik **10**, 317 (1922); A. D. Fokker, Z. Physik **58**, 386 (1929); Physica **9**, 33 (1929); **12**, 145 (1932); J. A. Wheeler and R. P. Feynman, Revs. Modern Phys. **17**, 157 (1945); **21**, 425 (1949).

distance theory. Work in this direction has been done by Landé,[2] Groenewold,[3] Bopp,[4] Peierls, McManus,[5] and particularly by Feynman.[6]

In this paper we shall discuss a new and speculative kind of modification of classical electrodynamics in its action at a distance formulation. The principal idea is to give up the concept of a constant rest mass associated with a fundamental particle, and instead to treat the rest mass as a variable of the motion (like the coordinates). Some of the reasons for considering this possibility are given in the remainder of this section.

We know that the rest mass of fundamental particles is not, in fact, conserved. When electron-positron pairs annihilate, when neutrons or mesons decay, the total rest mass of the system changes. Usually such phenomena enter the theoretical description only with the process of quantization and the introduction of creation and annihilation operators. There seems to be no good reason why such changes of rest mass should not be incorporated in the classical theory before quantization.

From the mathematical point of view the requirement that the rest mass of a particle be constant is somewhat artificial and is quite independent of the requirement of relativistic invariance. The relativistic equation of motion of a particle is

$$dp_\mu/ds=F_\mu, \tag{1}$$

where

$$p_\mu=mdx_\mu/ds \tag{2}$$

is the momentum 4-vector of the particle, and F_μ is the 4-force which, in an action at a distance theory, is determined by the motion of the other particles. The requirement that the rest mass m be constant is equivalent to demanding that the 4-force be always perpendicular to the world line of the particle:

$$F_\mu p^\mu=0. \tag{3}$$

In electrodynamics the condition (3) is satisfied in a natural manner. But this is not always so. For example, in the theory of a scalar potential V, giving rise to an inverse square law of attraction, the condition (3) can be satisfied only by subtracting out the tangential component from the simplest definition $\partial V/\partial x^\mu$ of the 4-force.

In Sec. 3 it will be shown that classical electrodynamics can be obtained from an action principle which is simpler than the usual one. In this new action principle the rest mass of a particle is varied as well as the coordinates. The constancy of the rest mass then follows as a consequence of the equations of motion. Simple modifications of this action principle yield theories in which the rest mass is no longer a constant of the motion.

Modifications of the usual action principle are possible which yield a purely electromagnetic rest mass for a fundamental particle. But this electromagnetic rest mass is unique, being determined by the particular modification used. The new action principle permits a new class of simple modifications in which the rest masses of all charged fundamental particles are of purely electromagnetic origin.

The ultimate object of the point of view adopted here is to explain nature in terms of a single law of interaction between fundamental particles of one kind only. Such a particle is characterized mainly by its charge e, a fundamental constant. At distances large compared to 10^{-13} cm, particles interact electromagnetically and their rest masses are automatically conserved. At small distances particles interchange not only momentum (mutual accelaration) but also rest mass. In the classical theory the rest mass is treated as a continuous variable, but it is hoped that quantization will introduce discreteness. In a quantized theory it is also expected that exchange of rest mass will be accompanied by exchange of spin. Neutral particles have little room in this type of theory; they are the agents of rest-mass exchange between charged particles and play a role analogous to electromagnetic interaction.

2. UNITS AND NOTATION

We choose units such that the velocity of light $c=1$ and the magnitude of the electronic charge $|e|=1$. The charge of a fundamental particle is then dimensionless and always

$$e=\pm 1. \tag{4}$$

Length and time are measured in cm, and mass in cm^{-1}, the electronic mass being

$$m_0=3.6\times 10^{12}\ \text{cm}^{-1}. \tag{5}$$

Greek suffixes range over 0, 1, 2, 3 and the coordinates $x^\mu\equiv(t, \mathbf{r})$ are real. Proper time is denoted by s and the Minkowskian metric given by

$$ds^2=(dx^0)^2-(dx^1)^2-(dx^2)^2-(dx^3)^2. \tag{6}$$

The usual conventions apply to summation dummies and to the raising and lowering of suffixes.

The rest mass of a particle is denoted by m. The electromagnetic potentials are $A_\mu\equiv(\phi, -\mathbf{A})$, and the electromagnetic field tensor is

$$F_{\mu\nu}=\partial A_\mu/\partial x^\nu-\partial A_\nu/\partial x^\mu. \tag{7}$$

When systems of particles are considered, the different particles are labeled by bracketed latin suffixes. Thus $x_{(a)}{}^\mu$, $e_{(a)}$, $m_{(a)}$, $\cdots$, are the coordinates, charge, rest mass, $\cdots$, of particle a, $A_{(a)\mu}$, $F_{(a)\mu\nu}$ describe the electromagnetic field at particle a due to the other

[2] A. Landé, Phys. Rev. **56**, 482 (1939); **76**, 1176 (1949); **77**, 814 (1950); J. Franklin Inst. **231**, 63 (1941).
[3] H. J. Groenewold, Physica **6**, 115 (1939).
[4] F. Bopp, Ann. Physik **42**, 573 (1943).
[5] H. McManus, Proc. Roy. Soc. (London) **A195**, 323 (1948); see also J. Irving, Proc. Phys. Soc. (London) **A62**, 780 (1949).
[6] R. P. Feynman, Phys. Rev. **74**, 939 (1948); a review article on the work of Bopp, Feynman, and others has recently been written by H. Hönl, Ergeb. exakt. Naturwiss. **26**, 291 (1952).

charges. The summation convention does not apply to these bracketed Latin suffixes.

3. MOTION OF A SINGLE PARTICLE

Before going on to the general problem of several particles interacting at a distance, we shall discuss the simpler case of a single charged particle in a known electromagnetic field. The field is given by electromagnetic potentials A_μ which are functions of the coordinates x^μ only.

The equations of motion of the particle can be obtained from the well-known action principle

$$\delta\int_{P_1}^{P_2}[m(dx_\mu dx^\mu)^{\frac{1}{2}}+eA_\mu dx^\mu]=0, \tag{8}$$

where the variations are assumed to vanish at the end points:

$$\delta x^\mu=0 \quad \text{at} \quad P_1, P_2. \tag{9}$$

This leads to the Lorentz equations

$$m\frac{d^2x_\mu}{ds^2}=e\frac{dx^\nu}{ds}F_{\nu\mu}. \tag{10}$$

Along the world line of the particle we now introduce a preferred (i.e., nongeneral) parameter u defined by

$$du=ds/m. \tag{11}$$

Then the momentum (2) of the particle is

$$p^\mu=dx^\mu/du, \tag{12}$$

and the rest mass m is given by

$$m^2=p^\mu p_\mu=\frac{dx^\mu}{du}\frac{dx_\mu}{du}. \tag{13}$$

In terms of the parameter u, the action principle (8) may be written in the form

$$\delta\int_{P_1}^{P_2}[p^\mu p_\mu+ep^\mu A_\mu]du=0, \tag{14}$$

where the variation is not only subject to the end conditions (9), but also to the auxiliary condition

$$\delta(p^\mu p_\mu)=0, \tag{15}$$

which expresses the fact that the constant rest mass m is not to be varied. Because of the auxiliary condition, the reformulation of the variational principle (8) in Eq. (14) is clumsy; it is given here only for comparison with the new action principle below.

For particles with constant rest mass m the parameters s and u are only trivially different. This is no longer the case when we consider a particle with variable rest mass. The motion of such a particle can be described by assigning any time-like world line parametrized arbitrarily by the monotonic parameter u. Analytically, this is equivalent to assigning the four functions

$$x^\mu=x^\mu(u). \tag{16}$$

The momentum of the particle and the variable rest mass can be obtained by differentiation, as in Eqs. (12) and (13). The only restriction on (16) is that the momenta be always time-like:

$$\frac{dx^\mu}{du}\frac{dx_\mu}{du}>0. \tag{17}$$

We now consider a new action principle for the motion of a particle with variable rest mass:

$$\delta\int_{u1}^{u2}L(x,p)du=0, \tag{18}$$

$$L\equiv p^\mu p_\mu+2ep^\mu A_\mu. \tag{19}$$

Here p^μ is not an independent variable but an abbreviation for dx^μ/du. In this variational principle the rest mass of the particle is varied as well as the coordinates x^μ; thus no auxiliary condition such as (15) is prescribed. On the other hand we add to the end conditions (9) the new restriction that the parameter u for the varied motion take on the same initial and final values u_1, u_2, as for the original motion.[7] We shall express this by writing

$$\delta x^\mu=0,\quad \delta u=0,\quad \text{for } u=u_1, u_2. \tag{20}$$

The Euler-Lagrange equations of the new variational principle (18) are

$$\frac{d}{du}\frac{\partial L}{\partial p^\mu}-\frac{\partial L}{\partial x^\mu}=0, \tag{21}$$

or, by (19),

$$dp_\mu/du=ep^\nu F_{\nu\mu}. \tag{22}$$

From these equations of motion and the skew-symmetry of $F_{\mu\nu}$ it follows that

$$p^\mu\frac{dp_\mu}{du}=\frac{1}{2}\frac{d}{du}(p^\mu p_\mu)=0,$$

and, by integration, that

$$m^2=p^\mu p_\mu=\text{constant}. \tag{23}$$

Using this result, it is now easily seen that Eqs. (22) are identical with the Lorentz equations of motion (10).

Thus we regain the equations of motion of classical electrodynamics. Although the constancy of the rest mass is not assumed *a priori*, it now follows as a consequence of the equations of motion. The Lagrangian (19) is rational in the derivatives $p^\mu=dx^\mu/du$, and

[7] The new end condition suggests the possibility of a 5-dimendional representation with u as a fifth coordinate. For a 5-dimensional theory with some similarity to the present one, see H. C. Corben, Nuovo cimento 9, 235 (1952).

therefore the new action principle (18) is simpler than the old action principle (8).

The Euler-Lagrange equations (21) have, quite generally, the Jacobian integral

$$p^\mu \partial L/\partial p^\mu - L = \text{constant}. \tag{24}$$

If the Lagrangian L has the special form (19), this equation reduces to (23). Classical electrodynamics may be modified by retaining the action principle (18), (20), but with more general Lagrangians. For example, in Eq. (19) the potentials A_μ may be given as functions of the coordinates x^μ and of the momenta p^μ of the particle. Generalizations of this type are considered in Sec. 5. In these cases, Eq. (24) remains as an integral of the motion, but it no longer reduces to (23), so that the rest mass of a particle changes during its motion.

4. SYSTEMS OF PARTICLES

The physical laws governing the motion of several charged particles in electromagnetic interaction can be summarized in the Fokker[1] action principle:

$$\delta\Big[\sum_a m_{(a)}\int (dx_{(a)}{}^\mu dx_{(a)\mu})^{\frac{1}{2}}$$

$$+\sum_{a<b}\sum e_{(a)}e_{(b)}\int\int \delta(\xi_{(ab)}{}^\nu \xi_{(ab)\nu})dx_{(a)}{}^\mu dx_{(b)\mu}\Big]=0. \tag{25}$$

The δ in the double integral is the Dirac delta function, and $\xi_{(ab)}{}^\mu$ is a vector joining two points on the world lines of the particles a and b:

$$\xi_{(ab)}{}^\mu = x_{(a)}{}^\mu - x_{(b)}{}^\mu. \tag{26}$$

If the motion of particle a is varied, the action principle (25) reduces to (8):

$$\delta\int [m_{(a)}(dx_{(a)}{}^\mu dx_{(a)\mu})^{\frac{1}{2}} + e_{(a)}{}_{(a)}A_\mu dx_{(a)}{}^\mu]=0, \tag{27}$$

where

$$A_{(a)\mu} = \sum_{\substack{b\\(b\neq a)}} e_{(b)}\int \delta(\xi_{(ab)}{}^\nu \xi_{(ab)\nu})dx_{(b)\mu} \tag{28}$$

are electromagnetic potentials at particle a generated by the remaining particles of the system. The electromagnetic field tensor, derived from these potentials, satisfies Maxwell's equations.

Thus the action at a distance theory characterized by the Fokker action principle is essentially equivalent to classical electromagnetic field theory. There is, however, one important difference. The potentials (28) are not the usual retarded Lienard-Wiechert potentials, but are half the retarded plus half the advanced potentials. In their absorber theory of radiation,[1] Wheeler and Feynman have shown that in a universe with large numbers of charges such a theory not only reproduces the usual interaction by only retarded potentials, but that it also gives the correct relativistic radiation-reaction forces on a charged particle. We shall accept this general scheme and give no further discussion of possible difficulties associated with the advanced potentials.

We now replace the Fokker action principle (25) by a new action principle for charged particles with variable rest masses:

$$\delta\Big[\sum_a\int p_{(a)}{}^\mu p_{(a)\mu}du_{(a)} + \sum_a \sum_{\substack{b\\(b\neq a)}} e_{(a)}e_{(b)}$$

$$\times\int\int \delta(\xi_{(ab)}{}^\nu \xi_{(ab)\nu})p_{(a)}{}^\mu p_{(b)\mu}du_{(a)}du_{(b)}\Big]=0, \tag{29}$$

$$p_{(a)}{}^\mu \equiv dx_{(a)}{}^\mu/du_{(a)}.$$

Note that the double sum in (29) contains two identical interaction terms for each pair of distinct particles, whereas the double sum in (25) contains only one interaction term for each such pair. For the variation of the motion of particle a, this action principle reduces to

$$\delta\int_{u(a)1}^{u(a)2}[p_{(a)}{}^\mu p_{(a)\mu} + 2e_{(a)}p_{(a)}{}^\mu A_{(a)\mu}]du_{(a)}=0, \tag{30}$$

$$A_{(a)\mu} = \sum_{\substack{b\\(b\neq a)}} e_{(b)}\int \delta(\xi_{(ab)}{}^\nu \xi_{(ab)\nu})p_{(b)\mu}du_{(b)}. \tag{31}$$

The action principle (30) is identical with (18), (19). It is again assumed that the u length,

$$\int_{u(a)1}^{u(a)2} du_{(a)},$$

of particle a is the same for the original and the varied motion. The electromagnetic potentials in (31) are identical with those of (28). The contribution $A_{(ab)\mu}$ of particle b to the potentials $A_{(a)\mu}$ at $x_{(a)}{}^\mu$ is easily obtained explicitly:

$$A_{(ab)\mu} = \tfrac{1}{2}e_{(b)}\frac{p_{(b-)\mu}}{\xi_{(ab-)\nu}p_{(b-)}{}^\nu} + \tfrac{1}{2}e_{(b)}\frac{p_{(b+)\mu}}{-\xi_{(ab+)\nu}p_{(b+)}{}^\nu}. \tag{32}$$

Here the $-$ and $+$ signs in the suffixes refer to the two points on the world line b which are respectively retarded and advanced relative to $x_{(a)}{}^\mu$ so that

$$\xi_{(ab-)\nu}\xi_{(ab-)}{}^\nu = \xi_{(ab+)\nu}\xi_{(ab+)}{}^\nu = 0. \tag{33}$$

This is illustrated in Fig. 1. The first term in (32) is one-half the usual retarded Lienard-Wiechert potential and the second term is the corresponding advanced potential. $A_{(ab)\mu}$ is homogeneous of degree zero in $p_{(b)\mu}$ so that only the direction of the world line b enters into this expression and not, for example, the rest mass $m_{(b)}$.

It follows from our previous discussion that the new action principle (29) gives the same physical results as the Fokker action principle and thus reproduces classical electromagnetism. In particular, the rest mass of each particle is a constant of the motion.

5. ELECTROMAGNETIC SELF-ENERGY

The only null vector $\xi_{(aa)}{}^{\mu}$ from a point $x_{(a)}{}^{\mu}$ to a time like world line passing through the point is the zero vector,

$$\xi_{(aa\pm)}{}^{\mu}=0. \tag{34}$$

Thus, the self-field of a particle, given by (32) with $b=a$, is infinite. This is the usual Coulomb infinity of a point charge. It can be avoided if the δ-function in the Fokker action principle is replaced by some other approximating function f. As Feynman has shown,[6] the interaction term of a particle with itself is then finite and for a particle with moderate acceleration, reduces to the inertial term $\int mds$. Thus the mass of a particle can be ascribed to an electromagnetic origin. However, the electromagnetic rest mass m is uniquely determined by the *structure* function f. If this function is chosen to fit the mass of electrons, then the masses of other fundamental particles, such as protons, will not be purely electromagnetic. This difficulty can be overcome by replacing the δ function in the new action principle (29) by a function not only of $\xi_{(ab)}{}^{\mu}$ but also of the momenta $p_{(a)}{}^{\mu}$, $p_{(b)}{}^{\mu}$.

In order to be specific we shall discuss a simple structure function of the type proposed by Landé[2] and Groenewold.[3] The δ-function in (29) is replaced by

$$\delta(\xi_{(ab)}{}^{\nu}\xi_{(ab)\nu}-\lambda^2), \tag{35}$$

where λ is small and may, for the moment, be considered constant. The potentials, obtained with this structure function, are still given by (32). However, the retarded or advanced events are no longer connected by null vectors satisfying (33), but by time-like vectors of the small magnitude λ:

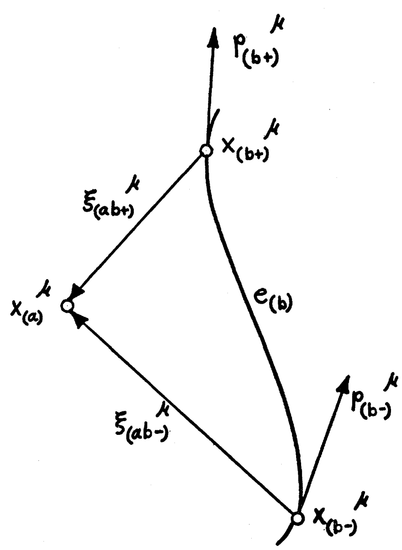

Fig. 1. Retarded and advanced potentials.

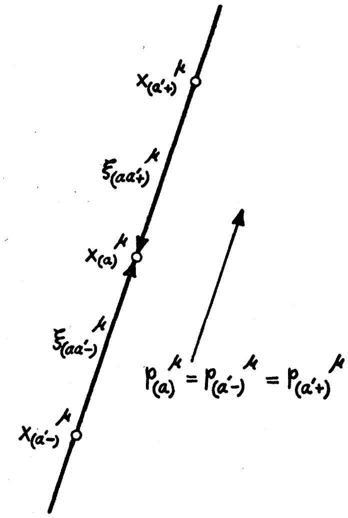

Fig. 2. Self-action for moderate acceleration.

$$\xi_{(ab-)\nu}\xi_{(ab-)}{}^{\nu}=\xi_{(ab+)\nu}\xi_{(ab+)}{}^{\nu}=\lambda^2. \tag{36}$$

The potentials of a particle at a point on its own world line are now finite. They can easily be computed approximately if we neglect changes of the momentum of the particle during short time intervals of the order of λ. Then, as seen from Fig. 2,

$$\xi_{(aa'-)}{}^{\mu}=-\xi_{(aa'+)}{}^{\mu}=\lambda\frac{p_{(a)}{}^{\mu}}{(p_{(a)}{}^{\nu}p_{(a)\nu})^{\frac{1}{2}}}, \tag{37}$$

and

$$p_{(a'+)}{}^{\mu}=p_{(a'-)}{}^{\mu}=p_{(a)}{}^{\mu}. \tag{38}$$

The self-field of a particle, Eq. (32), is now given by

$$A_{(aa)\mu}=e_{(a)}\frac{p_{(a)\mu}}{\lambda(p_{(a)}{}^{\nu}p_{(a)\nu})^{\frac{1}{2}}}. \tag{39}$$

The self-action term of a particle is defined by

$$e_{(a)}\int A_{(aa)\mu}p_{(a)}{}^{\mu}du_{(a)}$$
$$=\int\int\delta(\xi_{(aa')}{}^{\nu}\xi_{(aa')\nu}-\lambda^2)p_{(a)}{}^{\mu}p_{(a')\mu}du_{(a)}du_{(a')}. \tag{40}$$

This becomes identical with the inertial term

$$\int p_{(a)}{}^{\mu}p_{(a)\mu}du_{(a)}, \tag{41}$$

if we put

$$\lambda^2=1/p_{(a)}{}^{\nu}p_{(a)\nu} \tag{42}$$

in the self-action term or, more generally,

$$\lambda^2=1/p_{(a)}{}^{\nu}p_{(b)\nu} \tag{43}$$

in the interaction term of two particles. In the derivation, the form (32), (36) of the modified electromagnetic potentials was used. This form is no longer rigorous when λ is not a constant, but it remains valid in the approximation of moderate accelerations made above.

This discussion suggests the action principle

$$\delta \sum_a \sum_b e_{(a)}e_{(b)} \int\int \delta\left(\xi_{(ab)}{}^{\nu}\xi_{(ab)\nu} - \frac{1}{p_{(a)}{}^{\nu}p_{(b)\nu}} \right) \times p_{(a)}{}^{\mu}p_{(b)\mu}du_{(a)}du_{(b)} = 0. \quad (44)$$

A diagonal term $b=a$ in the double sum is to be interpreted as

$$\int\int \delta\left(\xi_{(aa')}{}^{\nu}\xi_{(aa')\nu} - \frac{1}{p_{(a)}{}^{\nu}p_{(a')\nu}} \right) \times p_{(a)}{}^{\mu}p_{(a')\mu}du_{(a)}du_{(a')}, \quad (45)$$

where $u_{(a)}$ and $u_{(a')}$ are two points on the world line of particle a. Also $e_{(a)}e_{(a)}=1$, by (4).

We wish to show now that the modifying term λ, Eq. (43), is small. This cannot be done completely within the framework of the classical theory presented here. We shall use the fact that there exists no charged particle in nature with a rest mass smaller than that of the electron:

$$m \geqslant m_0. \quad (46)$$

Whether this can be derived from a quantized theory of the type considered here is a matter of speculation. We shall accept (46) as an empirical fact. In contrast to Euclidean geometry, the absolute value of the scalar product of two time-like vectors in Minkowski space is greater than or equal to the product of their magnitudes. Thus

$$|p_{(a)}{}^{\nu}p_{(b)\nu}| \geqslant m_{(a)}m_{(b)} \geqslant m_0^2. \quad (47)$$

It follows that λ, given by (43), satisfies the inequality

$$\lambda \leqslant 1/m_0 = 2.8 \times 10^{-13} \text{ cm}. \quad (48)$$

The physical consequences of the modification (44) of classical electrodynamics can be summarized as follows:

For moderate accelerations, where we can neglect the changes in the momenta of particles during time intervals of the order of 10^{-13} cm or 10^{-23} sec, the self action terms ($a=b$) in (44) reduce to the corresponding inertial terms (41). Thus all rest masses are of electromagnetic origin.

When particles are at distances from each other which are large compared to 10^{-13} cm, the difference between (36) and (33) is negligible for $a \neq b$. The particles interact electromagnetically and each rest mass is a constant, being an integral of the motion.

When particles are close together their interaction is of a new type and is accompanied by changes in the rest masses.

The action principle (44) is not unique. Different action principles can be obtained by starting from structure functions other than (35). This lack of uniqueness is a drawback which the present theory shares with those of Bopp and Feynman.

There is one simple modification of (44) which will be mentioned here. Wheeler and Feynman[6] have suggested that, instead of distinguishing between positive and negative charges, one can, equivalently, consider particles moving forwards or backwards in time. Thus a particle whose momentum vector points into the future null cone ($p^4 > 0$) can be interpreted as carrying a positive charge, a particle whose momentum vector points into the past null cone ($p^4 < 0$) as carrying a negative charge. With this convention the most natural action principle of the general form of (44) can be written:

$$\delta \sum_a \sum_b \int\int \delta\left(\xi_{(ab)}{}^{\nu}\xi_{(ab)\nu} - \frac{1}{p_{(a)}{}^{\nu}p_{(b)\nu}} \right) \times p_{(a)}{}^{\mu}p_{(b)\mu}du_{(a)}du_{(b)} = 0. \quad (49)$$

In the old notation this means that λ^2 is given by

$$\lambda^2 = e_{(a)}e_{(b)}/p_{(a)}{}^{\nu}p_{(b)\nu}, \quad (50)$$

rather than by (43). This change has no effect on the self action terms since $(e_{(a)})^2 = +1$. However, the interaction of unlike charges ($e_{(a)}e_{(b)} = -1$) is now propagated along space-like directions $\xi_{(ab\pm)}{}^{\mu}$. Thus the velocity of propagation exceeds the velocity of light, if only by very small amounts for particles that are not too close.