Nucleon Polarization in Pion Proton Scattering*

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The polarization of the recoiling nucleons after the scattering of a pion by a nucleon is calculated. It is found that the recoiling nucleons are polarized in a direction perpendicular to the scattering plane and that the intensity ratio for spin parallel or antiparallel to this direction in several cases is quite large. Simple formulas are given for computing the polarization as a function of the scattering angle in terms of the phase shifts.

WHEN a pion is scattered by hydrogen, the recoiling proton will be polarized with the spin oriented preferentially in a direction perpendicular to the plane in which the scattering takes place. The purpose of this paper is to calculate the amount of polarization to be expected.

In this discussion, we will take as a basis the analysis of the scattering process in terms of phase shifts,[1] and in particular, we will assume that only the phase shifts of the s and p waves will be important. We consider first the scattering of positive pions. The primary pion wave $\exp(ikz)$ may be scattered by a proton with spin up or with spin down. Let α and β be the proton spin wave functions corresponding to the two cases. If initially the proton has spin up (α state), the scattered wave contains a superposition of states with spin up and with spin down. We can write, therefore, this scattered wave as follows:

$$S_{\alpha\alpha}\alpha+S_{\alpha\beta}\beta, \tag{1}$$

where $S_{\alpha\alpha}$ is the scattering amplitude for the wave with spin up and $S_{\alpha\beta}$ for the wave with spin down. Similarly, if the spin was initially down (β state), the scattered wave will be

$$S_{\beta\alpha}\alpha+S_{\beta\beta}\beta. \tag{2}$$

By straightforward application of the scattering theory,[2] one can express the scattering amplitudes S as follows:

$$S_{\alpha\alpha}=S_{\beta\beta}=f(r)\{e_3+(2e_{33}+e_{31})\cos\theta\}, \tag{3}$$

$$S_{\alpha\beta}=f(r)(e_{31}-e_{33})\sin\theta e^{i\varphi}, \tag{4}$$

$$S_{\beta\alpha}=-f(r)(e_{31}-e_{33})\sin\theta e^{-i\varphi}. \tag{5}$$

The notation is the same as in A. The quantities e_3, e_{33}, e_{31} are expressed in terms of phase shifts by

$$e_3=e^{2i\alpha_3}-1,\quad e_{33}=e^{2i\alpha_{33}}-1,\quad e_{31}=e^{2i\alpha_{31}}-1. \tag{6}$$

α_3 is the phase shift of the s wave; α_{33} and α_{31} are the phase shifts of the p waves of the angular momentum $\frac{3}{2}$ and $\frac{1}{2}$, respectively. All these phase shifts belong to the isotopic spin $\frac{3}{2}$ because in the scattering of positive pions by protons this is the only isotopic spin state. The function $f(r)$ is

$$f(r)=\exp(ikr)/(2ikr). \tag{7}$$

θ and φ are the polar angles defining the direction of the scattered pion in the center-of-mass system. We shall consider a scattering process in which the pion is scattered in the x, z plane corresponding to $\varphi=0$. In this case, from (4) and (5) one has $S_{\alpha\beta}=-S_{\beta\alpha}$. If the spin of the proton before collision is down, the scattering wave given by (2) becomes then

$$-S_{\alpha\beta}\alpha+S_{\alpha\alpha}\beta. \tag{8}$$

If the protons against which the collision takes place are nonpolarized, there will be 50 percent probability that the initial spin is α and 50 percent probability that it is β. In the two cases the scattered waves shall be (1) and (8). It is clear from these formulas that the probability that the spin after the scattering is α is 50 percent; that is, the scattering will produce no polarization in the z direction as is otherwise evident for reasons of symmetry. Similarly, one would find that there is no polarization of the scattered proton in any direction parallel to the x, z plane.

One finds, however, a polarization in the direction y perpendicular to the scattering plane. The amount of polarization can be obtained immediately by analyzing the scattered waves (1) and (8) in terms of the spin eigenfunctions,

$$\gamma=(\alpha+i\beta)/\sqrt{2}\quad\text{and}\quad\delta=(\alpha-i\beta)/\sqrt{2}, \tag{9}$$

corresponding to spin parallel or antiparallel to the y direction. If the initial spin was α, the scattered wave (1) can be written

$$\frac{1}{\sqrt{2}}(S_{\alpha\alpha}-iS_{\alpha\beta})\gamma+\frac{1}{\sqrt{2}}(S_{\alpha\alpha}+iS_{\alpha\beta})\delta. \tag{10}$$

In this case the probabilities that the spin is parallel or antiparallel with respect to y after the scattering are, therefore, proportional to

$$|S_{\alpha\alpha}-iS_{\alpha\beta}|^2\quad\text{and}\quad|S_{\alpha\alpha}+iS_{\alpha\beta}|^2. \tag{11}$$

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[1] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. 86, 793 (1952); Anderson, Fermi, Martin, and Nagle, Phys. Rev. 91, 155 (1953), quoted as A.

[2] See, for example, C. L. Critchfield and D. C. Dodder, Phys. Rev. 76, 602 (1949).

Similarly, if the initial spin was β, the scattered wave (8) can be analyzed in terms of γ and δ and one finds that also for this case the probabilities for the resultant spin to be parallel or antiparallel to the y direction are proportional to the expressions (11). Observe that these two probabilities in general will be different because $S_{\alpha\alpha}$ and $S_{\alpha\beta}$ are complex quantities.

Substituting in (11) the expressions (3) and (4) with $\varphi=0$, one finds that the probabilities that the scattered proton has spin parallel or antiparallel to y are proportional to

$$I_{\pm}\sim|e_3+(2e_{33}+e_{31})\cos\theta\mp i(e_{31}-e_{33})\sin\theta|^2, \quad (12)$$

where the upper sign corresponds to spin parallel and the lower sign corresponds to spin antiparallel to y. The degree of polarization will, therefore, depend on the scattering angle.

For example, the scattering of 120-Mev pions on protons has been interpreted in A in terms of a set of phase shifts,

$$\begin{matrix} \alpha_3=-15.2°, & \alpha_1=9.0°, & \alpha_{33}=29.6°, \\ \alpha_{31}=3.9°, & \alpha_{13}=1.8°, & \alpha_{11}=-2.8°. \end{matrix} \quad (13)$$

Substituting these phase angles in (12), one obtains

$$I_{\pm}\sim 1-1.56\cos\theta+3.56\cos^2\theta \pm(0.66-0.34\cos\theta)\sin\theta. \quad (14)$$

For example, for $\theta=90°$, this formula gives $I_+=1.66$ and $I_-=0.34$. The ratio of the intensities polarized in opposite directions is therefore almost 5.

The phase shifts (13) are not the only set that is compatible with the experiments. A second set can be obtained by changing the signs of all the phase shifts. Such a change does not affect the cross sections except at very small scattering angles where the interference with the Coulomb scattering becomes appreciable. On the other hand, changing the sign of the phase shifts has the effect of inverting the polarization direction, as one can see immediately from (12) and (6). Therefore, if it were possible to observe the polarization of the recoil protons, one could immediately decide which is the appropriate sign of the phase shifts.

In addition to the indeterminacy of the signs, there is another set of angles that was given in A which represents the experimental data with about the same accuracy as the angles (13). This is the set of the Yang phase angles which are

$$\begin{matrix} \alpha_3=-15.4°, & \alpha_1=9.1°, & \alpha_{33}=12.9°, \\ \alpha_{31}=38.6°, & \alpha_{13}=-1.4°, & \alpha_{11}=\ 3.8°. \end{matrix} \quad (15)$$

Also for this case one can compute the intensities I_+ and I_- of the scattered protons polarized parallel and antiparallel to the y direction, and one finds that they are

$$I_{\pm}=1-1.56\cos\theta+3.49\cos^2\theta \mp(0.82-1.40\cos\theta)\sin\theta. \quad (16)$$

The polarization for the Yang solution differs appreciably from the one corresponding to the phase angles (13), so that again observation of the polarization might permit a discrimination between these two sets of phase shifts.

We give now similar numerical results for the polarization produced in the scattering of negative pions by protons. In this case we must distinguish between the elastic scattering in which the recoiling nucleon is a proton and the charge exchange scattering in which the recoiling nucleon is a neutron. In the former case one obtains the polarization from a formula like (12) in which, however, e_3, e_{33}, and e_{31} are replaced by

$$(e_3+2e_1)/3, \quad (e_{33}+2e_{13})/3, \quad \text{and} \quad (e_{31}+2e_{11})/3.$$

For the case of the exchange scattering again the polarization of the recoiling neutron is obtained from a formula similar to (12) with the substitution of

$$\sqrt{2}(e_3-e_1)/3, \quad \sqrt{2}(e_{33}-e_{13})/3, \quad \text{and} \quad \sqrt{2}(e_{31}-e_{11})/3,$$

in place of e_3, e_{33}, and e_{31}. Assuming the phase shift angles (13), one finds at 120 Mev the following polarization formulas: For elastic scattering,

$$I_{\pm}=1+0.65\cos\theta+2.42\cos^2\theta\pm(0.28+0.21\cos\theta)\sin\theta.$$

For the recoil neutron in the exchange scattering the corresponding formulas are

$$I_{\pm}=1-2.30\cos\theta+3.06\cos^2\theta\pm(0.72-0.54\cos\theta)\sin\theta.$$

If one assumes the phase angles (15) instead of (13), the polarization for elastic scattering is

$$I_{\pm}=1+0.67\cos\theta+2.47\cos^2\theta\mp(0.20-0.62\cos\theta)\sin\theta.$$

For the exchange scattering, one finds

$$I_{\pm}=1-2.22\cos\theta+2.87\cos^2\theta\mp(0.92-1.37\cos\theta)\sin\theta.$$

In many cases the polarization effect according to these formulas are very large and their observation, if possible, would offer an interesting method for improving our knowledge of the scattering of pions by nucleons.