The Magnetic Moment of the Helium Atom in the Metastable Triplet State*†

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The atomic beam magnetic resonance method has been used to measure the ratio of the g value of the two-electron system in helium in the metastable $1s2s$, $^3S_1$ state to the g value of the one-electron system in atomic hydrogen in the ground $1s$, $^2S_{\frac{1}{2}}$ state. The metastable helium and the atomic hydrogen were alternately produced in a dc discharge tube. The helium beam was detected by measurement of the current of electrons ejected when the metastable atoms strike a wolfram wire. The hydrogen beam was detected by a Pirani gauge.

The resonance frequencies for the transitions $m=\pm 1\leftrightarrow 0$ of helium and $F, m=1, 0\leftrightarrow 1, -1$ of hydrogen were measured in the same magnetic field. Field values near 520 and 567 gauss were used. By the use of the Breit-Rabi formula for the hydrogen transition frequency, these measurements may be combined with experimental values for $g_p/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ and $\Delta\nu$ (hydrogen) to give the result $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})=1-(11\pm 16)\times 10^{-6}$. This result may be combined with the experimental value $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})_{\mathrm{exp}}=2(1.001128\pm 12\times 10^{-6})$ to give $g_J(\mathrm{He}, {}^3S_1)=2(1.001117\pm 20\times 10^{-6})$ or with the theoretical value $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})_{\mathrm{theoret}}=2(1.0011276)$ to give $g_J(\mathrm{He}, {}^3S_1)=2(1.001117\pm 16\times 10^{-6})$.

These results are in good agreement with the theoretical values,

$$[g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})]_{\mathrm{theoret}}=1-23\times 10^{-6}$$

and

$$g_J(\mathrm{He}, {}^3S_1)_{\mathrm{theoret}}=2(1.001104),$$

calculated to order α^2 by Perl and Hughes in the accompanying paper. This agreement tends to substantiate the arithmetic additivity of the anomalous magnetic moments of the two electrons and the nonradiative relativistic bound-state correction to the magnetic moment for the two electrons in helium. The mutual radiative correction arising from the Breit interaction is small compared with the experimental uncertainty.

1. INTRODUCTION

THE magnetic moment of a single electron in an atom has been studied intensively in recent years because its value provides an important test for the modern quantum-electrodynamic theory of the electron. The Dirac theory predicts that the spin magnetic moment of the free electron is one Bohr magneton and the spin gyromagnetic ratio g_s is 2. Measurements of the Zeeman effect of alkali-like atoms by the atomic beam magnetic resonance method[1] have given evidence that the actual electron spin magnetic moment differs from one Bohr magneton. The difference, which is called the anomalous magnetic moment of the electron, has been ascribed to the virtual radiative processes predicted by quantum electrodynamics, and has been calculated for the free electron to second order in the fine structure constant.[2] The theoretical value, expressed as the electron spin gyromagnetic ratio, is

$$g_s(\mathrm{free})=2(1+\alpha/2\pi-2.973\alpha^2/\pi^2)=2(1.0011454).$$

In this formula, α is the fine structure constant given by

$$\alpha=e^2/\hbar c=1/137.0365.^3$$

No direct experimental measurement of the spin g value of a free electron has yet been made, but a precise determination of the g value of the electron in hydrogen has been carried out. The conventional Dirac theory of the hydrogen atom predicts that the g value of the electron bound in hydrogen will be less than that of the free electron. For the $1s$, $^2S_{\frac{1}{2}}$ ground state this theory gives[4]

$$g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})=g_s(\mathrm{free})(1-\alpha^2/3)=g_s(\mathrm{free})(1-17.8\times 10^{-6}).$$

By using the quantum-electrodynamic value given above for g_s(free), it is found that $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})=2(1.0011276)$. The experimental determination of the ratio of the g value of the electron in hydrogen in the $^2S_{\frac{1}{2}}$ ground state to the g value of the proton,[5] when combined with the measurement of the ratio of the g value of the proton to the orbital g value of the free electron,[6] yields for the g value of the electron bound in hydrogen $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})=2(1.001128\pm 12\times 10^{-6})$.[7] This is in excellent agreement with the theoretical value.

The motivation for the experiment described in the present paper was to compare the spin magnetic moment of a two-electron system with that of a one-electron system in order to provide a test for the quantum-electrodynamic theory of the two-electron system;

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1 P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948).

2 J. Schwinger, Phys. Rev. **73**, 416 (1948); R. Karplus and N. Kroll, Phys. Rev. **77**, 536 (1950).

3 Dayhoff, Triebwasser, and Lamb, Phys. Rev. **89**, 106 (1953).

4 G. Breit, Nature **122**, 649 (1928); H. Margenau, Phys. Rev. **57**, 383 (1940).

5 Koenig, Prodell, and Kusch, Phys. Rev. **88**, 191 (1951).

6 J. H. Gardner and E. M. Purcell, Phys. Rev. **76**, 1262 (1949); J. H. Gardner, Phys. Rev. **83**, 996 (1951).

7 The value $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ is not explicitly given in reference 5, but it is implicit in the data, and can be calculated from the experimental value quoted for g_s(free) of $2(1.001146\pm 12\times 10^{-6})$ together with the theoretical relation $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})/g_s(\mathrm{free})=(1-\alpha^2/3)=(1-17.8\times 10^{-6})$. For the value of α see reference 3.

in particular, in order to test the additivity of the magnetic moments of the two electrons.

The two-electron system chosen was the lowest-energy triplet state of the He^4 atom, which is the metastable $1s2s, {}^3S_1$ state. The one-electron system chosen was the $1s, {}^2S_{\frac{1}{2}}$ ground state of atomic hydrogen. The ratio $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ may be obtained from measurements by the atomic beam magnetic resonance method of the transition frequencies between the Zeeman levels of each of these systems in the same magnetic field.

An alternate experiment considered was the determination of the ratio $g_J(\mathrm{He}, {}^3S_1)/g_p$, using the atomic beam method for helium and the nuclear resonance absorption method for the proton g value. This result could be combined with the known value for the ratio $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})/g_p$ to give $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$. The requirement that the two resonances involved in a determination take place in the same magnetic field, however, makes the former experiment employing two atomic beams seem simpler than the latter experiment employing an atomic beam and a proton sample.

Another possible experiment would be the determination of the ratio of $g_J(\mathrm{He}, {}^3S_1)$ to g_l, the orbital gyromagnetic ration of the free electron, using a helium atomic beam, and measuring the cyclotron frequency of the electron. This result could be combined with known values of g_p/g_l and $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})/g_p$ to give $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$. Once again the requirement that the two resonances be observed in the same magnetic field seems to make the chosen method simpler.

In the accompanying theoretical paper $g_J(\mathrm{He}, {}^3S_1)$ is calculated to order α^2. The relativistic bound-state correction to the $g_J(\mathrm{He}, {}^3S_1)$ is derived from Breit's generalization of the Dirac equation, and the quantum-electrodynamic radiative correction to the g_s for the free electron is used.

The primary new experimental problem was the production and detection of a beam of helium atoms in the metastable ${}^3S_1$ state. It has been reported[8] that a glow discharge in helium is an adequate source of the metastable atoms, and that they can be detected by means of the electrons which are ejected when the metastable atoms strike a metal surface.

A preliminary experimental value for the ratio $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ has also been reported.[9]

2. THEORY OF THE EXPERIMENT[10]

The part of the Hamiltonian for the helium atom which represents the interaction of the electrons with a constant magnetic field H_z applied in the z direction is given in the Pauli approximation by: $\mathcal{H}=\mu_0 g_L L_z H_z + \mu_0 g_S S_z H_z$ in which g_L and g_S are the orbital and spin gyromagnetic ratios for the two electrons, L_z and S_z are the operators for the z components of the total orbital and spin angular momenta, respectively, and μ_0 is the Bohr magneton. There are magnetic interaction terms in the more complete Hamiltonian which are due to the virtual radiative effects of quantum electrodynamics. These terms are incorporated into the Pauli approximation by allowing the value of g_S to differ from 2.

In addition there are magnetic interaction terms due to the relativistic bound-state effects predicted by Breit's generalization of the Dirac equation. These terms are incorporated into the Pauli approximation by allowing the value of g_S to differ from 2 and the value of g_L to differ from 1. The term in the Hamiltonian which is quadratic in the external field H_z is negligible at the magnetic field strengths used in the present experiment, which were near 540 gauss.

The lowest-energy triplet state of helium can be described by Russell-Saunders coupling with $L=0$ and $S=1$, that is, as a ${}^3S_1$ state, for the purpose of computing the interaction energy in an external magnetic field to order $\alpha^2\mu_0 H_z$. The magnetic energy is given by the diagonal matrix element of $\mathcal{H}$: $W(\mathrm{He}, {}^3S_1)=(J, m|\mathcal{H}||J, m)=\mu_0 g_S H_z m$, in which J is the quantum number for the total angular momentum, and m is the quantum number for the component of the total angular momentum in the z direction and takes on the values $0, \pm 1$. Off-diagonal matrix elements of $\mathcal{H}$ do not contribute to the magnetic energy to order $\alpha^2\mu_0 H_z$ because of the validity of the Russell-Saunders coupling approximation.

The relative energies of the hyperfine states of hydrogen in its ground $1s, {}^2S_{\frac{1}{2}}$ state in a constant magnetic field H_z applied in the z direction are given by the Breit-Rabi formula,[11,12]

$$W_{F=I\pm\frac{1}{2}, m} = -\frac{h\Delta\nu}{2(2I+1)} + g_I\mu_0 H_z \pm \frac{h\Delta\nu}{2}\left(1+\frac{4mx}{2I+1}+x^2\right)^{\frac{1}{2}},$$

in which $\Delta\nu$ is the hyperfine structure separation in cps, h is Planck's constant, I is the proton spin in units of $\hbar$ and equals $\frac{1}{2}$, g_I is the gyromagnetic ratio for the proton, F is the quantum number for total atomic angular momentum, m is the quantum number for the z component of total atomic angular momentum, and μ_0 is the Bohr magneton. The quantity x is proportional to the magnetic field intensity H_z and is defined by $x=(g_J-g_I)\mu_0 H_z/h\Delta\nu$ in which g_J is the electronic gyromagnetic ratio in the ground state of hydrogen.

In the experiment to be described in this paper the resonance frequency for the transitions between the states $m=\pm 1$ and the state $m=0$ of helium is meas-

[8] V. Hughes and G. Tucker, Phys. Rev. **82**, 322 (1951).
[9] Tucker, Hughes, Rhoderick, and Weinreich, Phys. Rev. **86**, 618 (1952).
[10] See W. Perl and V. Hughes, following paper [Phys. Rev. **91**, 842 (1953)].

[11] G. Breit and I. I. Rabi, Phys. Rev. **38**, 2082 (1931).
[12] J. E. Nafe and E. B. Nelson, Phys. Rev. **73**, 718 (1948).

used in a constant magnetic field, and then the resonance frequency for the transition between the states $F=+1$, $m=0$ and $F=+1$, $m=-1$ of hydrogen is measured in the same magnetic field. The ratio $g_J(\text{He}, {}^3S_1)/g_J(\text{H}, {}^2S_{\frac{1}{2}})$ can be computed from these two observations if use is made of values of $\Delta\nu$[12,13] and of $g_I/g_J(\text{H}, {}^2S_{\frac{1}{2}})$[5] determined from other atomic beam experiments. This ratio, when combined with the known value given above for $g_J(\text{H}, {}^2S_{\frac{1}{2}})$ gives $g_J(\text{He}, {}^3S_1)$.

3. PRODUCTION AND DETECTION OF A BEAM OF HELIUM ATOMS IN THE LOWEST-ENERGY TRIPLET STATE

The production and detection of a beam of helium atoms in the lowest-energy triplet state ($1s2s$, ${}^3S_1$) was the first experimental problem. The lifetime of an isolated atom in this metastable state is long, since an electric dipole transition to the ground singlet state ($1s^2$, ${}^1S_0$) is forbidden by the selection rule on parity change. The transition is possible by a double quantum emission process with an estimated lifetime of 10^5 sec.[14] This metastable state will not be quenched by

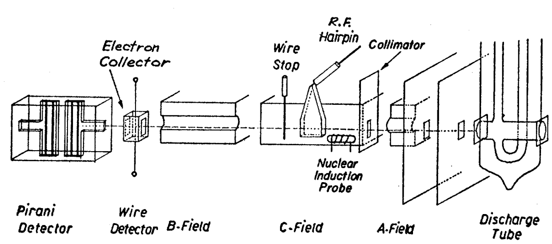


FIG. 1. Diagram of the atomic beam apparatus.

magnetic or electric fields present in the atomic beams apparatus. Hence the lifetime of a metastable atom in the beam will be long compared to the time of about 2×10^{-4} sec required to traverse the apparatus.

From optical spectroscopy it is known that helium atoms in the $1s2s$, ${}^3S_1$ state are present in a glow discharge in helium. The helium atoms in this metastable state are produced by inelastic exchange collisions between electrons and helium atoms in the ground state. This is a resonant process, with an experimental maximum cross section of 4.8×10^{-18} cm^2 for an electron with a kinetic energy approximately equal to the 19.77-ev difference between the energies of the metastable state and the ground state.[15] The atoms are lost from the metastable state principally upon diffusion to the walls of the discharge tube. In the appendix the equilibrium density of metastable atoms in the plasma region of a discharge is calculated roughly for a typical case to be 3.1×10^{11} metastable atoms per cm^3, or one metastable atom for every 2.3×10^4 atoms in the ground state.

The detection of metastable helium atoms is known to be possible by electron emission on striking a metal surface. When a metastable atom strikes the surface, it may make an inelastic collision of the second kind, in which it may be considered that an electron from the metal goes into the K shell of the atom, and the L-shell electron is ejected and carries off the excess energy.[16] The yield of this process has been measured to be 0.24 electron per atom.[17]

Search for the Beam of Metastable Atoms

The major equipment for the present experiment was the atomic beam apparatus described by Nafe and Nelson.[12] The beam source was a narrow slit in the wall of a discharge tube. A movable wire which could be used to block the beam was introduced into the "C" field region. For the detection of the metastable atoms, a movable unheated wolfram wire was introduced together with an electron collector plate of nickel attached to the grid of an FP54 electrometer tube. The detector wire was biased negatively with respect to ground,[18] and the collector plate was connected to ground through 10^{11} ohms. The over-all sensitivity of the electrometer tube and its associated circuit[19] was 10^{-16} ampere per mm. A sketch of the essential elements in the apparatus is given in Fig. 1. The apparatus will be described in detail in the next section.

Preliminary studies of the optical spectrum from helium in the discharge tube with a Hilger constant deviation spectrograph showed the 3889A line from the transition $1s3p$, ${}^3P\rightarrow1s2s$, ${}^3S$, and thus indicated the presence of the ${}^3S_1$, metastable atoms. The line intensity increased markedly with increase of the tube current in the range up to about 200 ma. There was a small dependence of line intensity on pressure with a maximum at a pressure near 0.5 mm of Hg.

In addition to helium atoms in the metastable ${}^3S_1$ state, other components of the discharge which may emerge from the source slit as constituents of the beam are: (1) helium atoms in the ground state; (2) helium atoms in the metastable $1s2s$, ${}^1S_0$ state; (3) helium atoms in nonmetastable excited states; (4) electrons and helium ions; (5) helium molecules; (6) photons; (7) possible impurities. The problem of distinguishing the helium atoms in the metastable ${}^3S_1$ state from these other constituents of the beam must be considered.

The helium atoms in the ground state will not be detected because they do not have the energy to cause the ejection of electrons at the detector. Atoms in the

[13] A. G. Prodell and P. Kusch, Phys. Rev. **88**, 184 (1952).
[14] G. Breit and E. Teller, Astrophys. J. **91**, 215 (1940).
[15] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. (London) **A132**, 605 (1931); H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, London, 1952), p. 160.
[16] A. Cobas and W. E. Lamb, Jr., Phys. Rev. **65**, 327 (1944).
[17] R. Dorrestein, Physica **9**, 433 (1942); **9**, 447 (1942). H. D. Hagstrum, Phys. Rev. **89**, 244 (1953).
[18] The bias ordinarily used was about 15 v. It was found that increase of the bias above this value did not increase the collected electron current appreciably.
[19] F. C. Armistead, Rev. Sci. Instr. **20**, 747 (1949).

metastable $^1S_0$ state can eject electrons. Atoms in other excited states have lifetimes of the order of 10^{-8} to 10^{-7} sec, and so will not last long enough to traverse the apparatus. Electrons and ions will be removed from the beam by the magnetic fields. The He_2 molecule is known to exist, and may be present in the discharge tube. It may have enough energy to eject electrons, but is probably short-lived; moreover, the ground state of this molecule is a $^3\Pi$ state, and so it would be distinguished from the $^3S_1$ metastable atoms because of its different magnetic moment. There is a high intensity of photons in the range of 500 to 600A present in the beam. These photons are energetic enough to eject photoelectrons from the detector wire. It is unlikely that impurities could cause confusion. The helium used was welding grade AA helium, supposed to be 99.99 percent pure; moreover, a search has been made for metastable atomic or molecular constituents from discharges in many of the gases more likely to be present as impurities, including argon, neon, oxygen, nitrogen, and water vapor. The results have indicated that traces of these gases could not give appreciable intensities.

Thus the only constituents of the beam from the helium discharge which can reach the detector and eject electrons are $^3S_1$ metastable atoms, $^1S_0$ metastable atoms, and photons. Since the wavelengths of the photons are small compared to the width of the collimator slit, diffraction effects will be negligible so the atom and photon beams will have the same relative intensity distributions at the detector. Only the $^3S_1$ metastable atoms, however, can be deflected by the inhomogeneous magnetic fields, and so they can be distinguished from the photons and the $^1S_0$ metastable atoms.

To establish the presence of $^3S_1$ metastable atoms in the beam, it was therefore desired to demonstrate by means of a Stern-Gerlach experiment that there were indeed particles in the beam which were deflected by the inhomogeneous magnetic fields and whose magnetic moments were of the right order of magnitude. With the homogeneous magnetic "C" field present to remove ions and electrons from the beam, but without the inhomogeneous magnetic "A" and "B" fields, the intensity distribution at the detector was that to be expected from the dimensions of the apparatus. With the inhomogeneous "A" field present as well as the "C" field, the total beam intensity was markedly less, and the relative intensity distribution at the detector was slightly different. These effects may be seen in Fig. 2. The decrease in total intensity was due to the influence of the stray magnetic flux from the "A" field on the discharge. This effect was reduced by magnetic shielding around the tube. The smallness of the effect of the inhomogeneous field on the intensity distribution indicates that the $^3S_1$ metastable atoms constitute only a small portion of the total observed beam. Since the

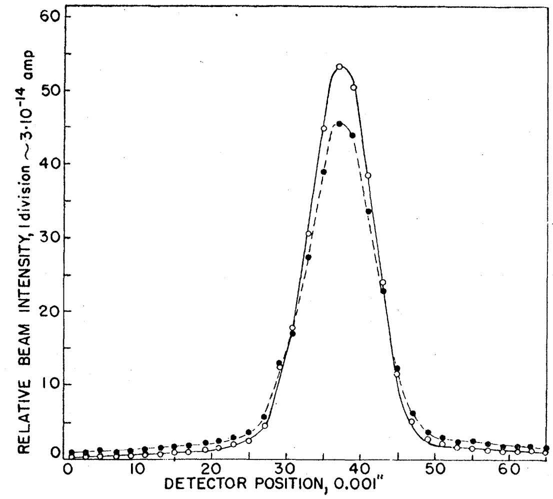


FIG. 2. Intensity distribution of the beam from the helium discharge. The solid curve is taken with the homogeneous "C" field present. The dashed curve is taken with the inhomogeneous "A" field present as well.

number of $^1S_0$ metastable atoms in the beam is not expected to be large compared to the number of $^3S_1$ metastable atoms, the major portion of the observed beam must be due to photons.

The difference between the intensity distributions with the "A" field on and off, and the difference between these distributions after they have been normalized to the same integrated intensity are shown in Fig. 3. Clearly the effect of the presence of the "A" field is to decrease the relative beam intensity at the center of the distribution and to increase it on the sides. This is the effect to be expected if particles with spin 1

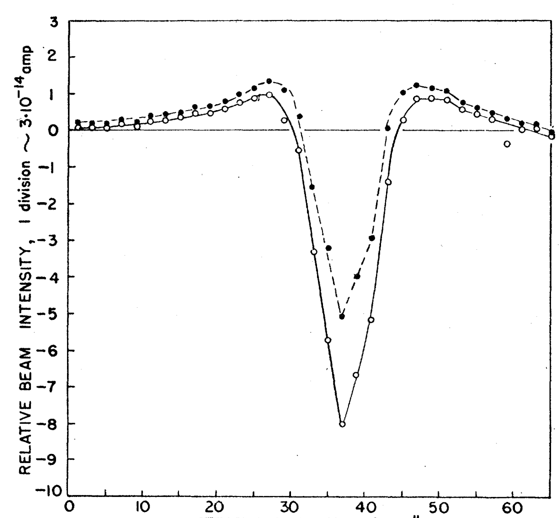


FIG. 3. Effect of the inhomogeneous "A" field on the intensity distribution of the beam. The solid curve is the difference between the dashed and solid curves of Fig. 2. The dashed curve is the difference between the dashed and solid curves of Fig. 2 after they have been normalized to the same integrated intensity.

are deflected. Atoms with angular momentum component $m=+1$ will be deflected in one direction, those with component $m=-1$ will be deflected in the other direction, and those with component $m=0$ will not be deflected.

The corresponding effect with the "B" field present instead of the "A" field is shown in Figs. 4 and 5. The large increase in total intensity resulting from the "B" field is ascribed to the effect of the stray magnetic flux at the detector in modifying the trajectories of the ejected electrons so that more of them strike the collector plate.

That the changes in relative intensity distribution resulting from the presence of the inhomogeneous fields are due to the deflection of helium $^3S_1$ metastable atoms is further verified by the agreement indicated in Fig. 5 between the observed deflection and the deflection calculated for helium atoms with a magnetic moment of two Bohr magnetons. The over-all measurement was sufficiently accurate to establish that the magnetic moment of the deflected helium atoms was within 0.4 Bohr magnetons of the expected value. The change in relative beam intensity distribution corresponds to a current due to $^3S_1$ metastable atoms of 10^{-12} amp. The portion of the observed beam which is due to photons corresponds to about 7×10^{-12} amp.

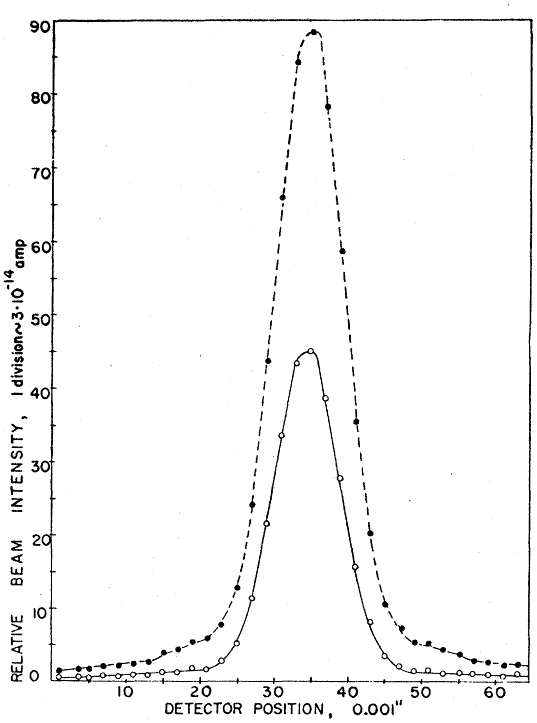


FIG. 4. Intensity distribution of the beam from the helium discharge. The solid curve is taken with the homogeneous "C" field present. The dashed curve is taken with the inhomogeneous "B" field present as well. The large increase in intensity is due to the effect on the detector of stray flux from the "B" field.

With both "A" and "B" fields present, particles having a magnetic moment should be focused on the detector. As a further test for the presence of $^3S_1$ metastable atoms, this focused beam was sought. To decrease the background of undeflected beam components at the detector, a wire stop was interposed in the direct path from source to detector. Deflected particles could pass around the wire stop and be focused on the detector. $^1S_0$ metastable atoms would be blocked from the detector by the wire stop. A large number of photons were diffracted around the wire stop. To differentiate between the focused beam and these diffracted photons, the difference between the intensity distributions with the "A" field present and absent was studied. This difference curve, which is shown in Fig. 6, is that to be expected from the dimensions of the apparatus. The maximum intensity of the focused beam corresponded to an electron current of 5×10^{-14} amp, which was about 1/100 of the total current observed when no wire stop was interposed.

4. APPARATUS FOR THE RESONANCE EXPERIMENT

As stated previously the apparatus used was that described by Nafe and Nelson. Several modifications were made. The old pumps were replaced with faster metal diffusion pumps, which maintained a pressure in the detector chamber when there was no gas in the discharge tube of about 3×10^{-7} mm of Hg as read on an ionization gauge. When there was gas in the discharge tube, the pressure in the detector chamber was about 1×10^{-6} mm of Hg. The old Pirani gauge was replaced with a faster and more sensitive one of the type described by Prodell and Kusch.[13] They have also described the associated circuit. A nuclear induction probe[20] was introduced into the "C" field region for measurement of the magnetic field strength and was intended as an aid in setting the field strength to the desired values. The inclusions are shown in Fig. 1.

The dimensions along the direction of the beam were the same as those used by Nafe and Nelson.[21] The source and collimator slits were 0.075 mm wide, the detector wire was 0.125 mm in diameter, and the Pirani slit was 0.025 mm wide. The wire stop was 1.5 mm in diameter. For the preliminary experiments described above, the beam height was limited to 9 mm by the height of the source slit, and by horizontal edges placed on the detector side of the "B" magnet. For the later resonance experiment the beam height was reduced to 5 mm by horizontal edges placed on the collimator, and on the detector side of the "B" magnet.

The Discharge Tube

The discharge tube was similar to that described by Kellogg, Rabi, and Zacharias.[22] At first a directly

[20] R. V. Pound and W. D. Knight, Rev. Sci. Instr. **21**, 219 (1950).
[21] See Fig. 3, p. 722 of reference 12.
[22] Kellogg, Rabi, and Zacharias, Phys. Rev. **50**, 472 (1936).

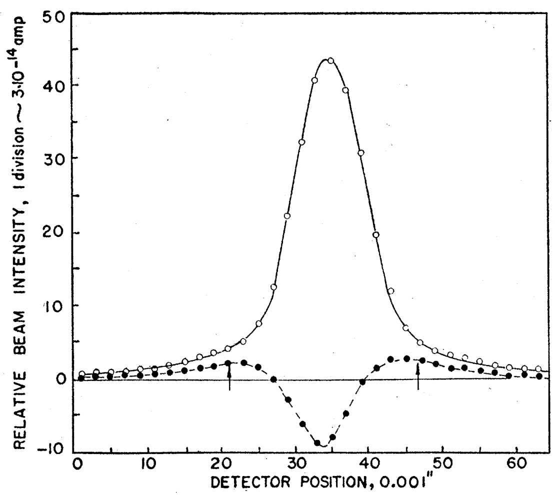


FIG. 5. Effect of the inhomogeneous "B" field on the intensity distribution of the beam. The solid curve is the difference between the dashed and solid curves of FIG. 4. The dashed curve is the difference between the dashed and solid curves of Fig. 4 after they have been normalized to the same integrated intensity. The arrows indicate the positions at which the maxima of the dashed curve would be expected to occur if helium atoms with a magnetic moment of two Bohr magnetons are being deflected by the "B" field.

heated filamentary wolfram cathode was used, but this cathode sputtered badly and deteriorated rapidly. In an attempt to reduce sputtering, hollow cup-shaped aluminum electrodes were finally used and the cathode was water-cooled. The inside diameter of the tube varied from 5 mm to 12 mm, and the length of the current path was about one meter. The source slit edges were waxed on in the manner described by Nafe and Nelson.[12] Gas was supplied to the tube from two supply systems of the type described by Prodell and Kusch.[13] Either or both of the supply systems could be connected to the tube through stopcocks. The discharge was excited by a dc power supply capable of supplying 1 ampere at 5000 v. The power supply was current regulated, and had an ac internal impedance of about 50 000 ohms. The helium discharge could be initiated and maintained with pressures in the range from 0.1 to 0.5 mm of Hg. Within this range the beam intensity did not vary greatly. The intensity increased with tube current in the range from 20 ma to 300 ma, but the cathode sputtered excessively at currents above 100 ma. As the cathode material sputtered onto the surrounding wall of the discharge tube, higher pressures were required to maintain the discharge. At these higher pressures, the rate of flow through the slit was greater, and the pressure outside the slit was increased to the point where the beam was attenuated. For this reason, a tube current of 100 ma was ordinarily used. This required a voltage of about 2500 v.

The Magnets

The "A," "B," and "C" field magnets were those used by Nafe and Nelson. The "A" and "B" magnets had a ratio of field gradient to field strength of 1.25 at the position of the beam. "A" and "B" field strengths of 10 000 gauss were used. The resulting field gradients were such that either magnet produced a deflection of 0.7 mm at the detector for a $^3S_1$ metastable helium atom moving with a speed equal to the most probable speed in the source. "C" field strengths from 450 to 600 gauss were used. The "C" magnet poles were clamped tightly against brass spacers. A mechanical test of the spacing indicated uniformity over most of the gap to within 0.0002 in.

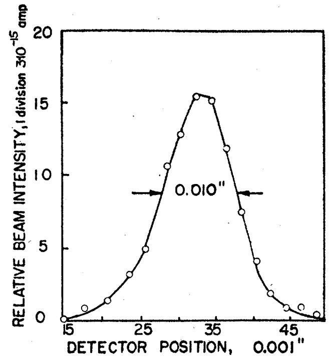
The currents in the "A," "B," and "C" magnet coils were ordinarily near 54, 78, and 25 amperes, respectively. The "A" and "B" currents were monitored manually to within 40 parts in 10^6, and the "C" current to within 6 parts in 10^6.

Preliminary studies of the "C" field were made with a nuclear induction probe.[20] This was a cylindrical coil $\frac{1}{4}$ in. in diameter and about $\frac{1}{2}$ in. long, containing a water sample to which a small amount of copper sulfate had been added. It was found that stray flux from the "A" and "B" magnets contributed about 15 percent to the total "C" field. In general this stray flux improved the homogeneity of the "C" field. It was possible approximately to maximize the homogeneity by small variations in the relative strengths of the "A" and "B" fields. The residual inhomogeneity was critically dependent upon the process of magnetization. Turning one or more of the magnets off and on again had a pronounced effect on the inhomogeneity. For optimum adjustment the field was homogeneous to within a few parts in a thousand throughout most of the "C" field region.

THE RADIO-FREQUENCY SYSTEM

A block diagram of the radio-frequency system is shown in Fig. 7. For the "C" field strengths used the helium transition frequency was between 1450 and 1600 Mc/sec and the hydrogen transition frequency was between 1000 and 1150 Mc/sec. The radio-frequency currents were supplied by two grounded-grid coaxial line oscillators which were the transmitter sections of Radar Jammers T-85/APT-5 which use 3C22 lighthouse tubes. These oscillators are continuously variable in frequency from about 300 to 1600 Mc/sec, supply an output

FIG. 6. The refocused beam. This curve is the difference between the distributions of beam intensity with the "A," "B," and "C" fields present and with only the "B" and "C" fields present, taken with the wire stop interposed into the direct path from source to detector.



power of about 7 w when operated from a 300-v "B" supply, and have a short-time drift of less than a few kc/sec in one second when the "B" supply is regulated. A coaxial switch was used so that either of the oscillators could be connected to the remainder of the system.

The current to the rf loop, or "hairpin," was controlled by a coaxial line triple-stub tuner, a variable resistive attentuator, and a fixed coaxial attenuator. The current was monitored with a type 1N23 silicon crystal detector in a section of slotted line. The current was led into the vacuum chamber through a coaxial metal-to-glass seal. The "hairpin" was of the type described by Prodell and Kusch,[13] and had an extension along the beam of 1.5 cm. The "hairpin" was surrounded by a copper box with narrow entrance and exit slits for the beam. This box served as a shield to reduce stray rf leakage outside of the transition region.

The frequency was measured by mixing a part of the output from the oscillator with the output from a crystal harmonic generator[23] which was driven by a signal from a frequency standard. A beat frequency in the range from 10 to 20 Mc/sec was measured with a

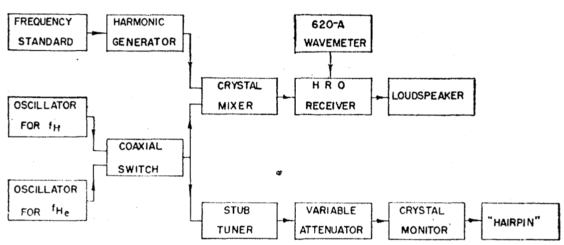


FIG. 7. The radio-frequency system.

National HRO receiver. The frequency standard[24] employed a 50-kc/sec crystal oscillator and frequency multipliers, and gave output signals at 40, 80, and 240 Mc/sec. The crystal was kept in a constant temperature oven. The standard could be adjusted to agree with calibrating signals from WWV to within better than 1 part in 10^7. The crystal harmonic generator provided frequencies which were multiples of 240 Mc/sec with 40-Mc/sec and 80-Mc/sec sidebands. The receiver was calibrated to within 1 part in 20 000 with a General Radio 620A heterodyne frequency meter. The over-all precision of the frequency measurement was better than one part in 10^6.

5. EXPERIMENTAL PROCEDURE

Ideal Case

The theoretical line shape for the hydrogen resonance in a perfectly homogeneous magnetic field has been calculated by Torrey.[25] It is symmetrical and has its maximum at the Bohr frequency. When the rf amplitude is adjusted to maximize the line intensity at its peak, the natural width of the line is given by $1.072/\tau$, in which τ is the time spent in the transition region by an atom with a velocity equal to the most probable velocity in the source. For a 1.5-cm "hairpin" and a source at room temperature the natural width of the hydrogen line is 160 kc/sec.

The theoretical line shape for the helium resonance is calculated in Appendix II. It is symmetrical with the Bohr frequency at the center of symmetry. When the rf amplitude is adjusted to its optimum value, the natural width of the line is 90 kc/sec. For certain conditions the helium line may have two peaks with a minimum at the center of symmetry. (See Appendix II.)

If the hydrogen, helium, and photon beams were all perfectly steady, if the "C" field were perfectly homogeneous and steady, and if the detectors were free from noise or drift, a sufficient procedure would be to run with one gas and then with the other and measure the rf frequency for which a minimum is observed in the detector signal for each gas. From these two frequencies $g_J(\text{He}, {}^3S_1)/g_J(\text{H}, {}^2S_{\frac{1}{2}})$ could be calculated.

Practical Problems

The experiment was complicated in practice by beam and detector noise and drift, and by inhomogeneity and drift of the "C" field.

Noise

There were continual fluctuations in the intensities of the beams although a constant current power supply was used to excite the discharge. Typically, the hydrogen resonance gave a peak deflection of 2.5 cm on the galvanometer scale. The mean amplitude of the fluctuations in the hydrogen beam was about 5 percent of the amplitude of the resonance. The mean amplitude of the fluctuations in the helium beam and the photon beam together was about 15 percent of the amplitude of the helium resonance. In addition, there were occasional sudden changes in the photon beam of a magnitude several times the amplitude of the helium resonance, followed by fairly rapid drifting back toward the quiescent state.

To minimize the noise introduced into the helium resonance curves by the fluctuations in the photon background, the wire stop was interposed so as to block the direct path from the source to the detector, while allowing helium atoms with wide trajectories to pass on one side. When this was done, the noise was reduced to about 5 percent of the resonance amplitude.[26]

[23] The harmonic generator was a coaxial crystal cavity of the type described by R. G. Talpey and H. Goldberg, Proc. Inst. Radio Engrs. **35**, 965 (1947).

[24] The frequency standard was built and maintained for the Columbia Molecular Beams Laboratory by Richard Blume. A detailed description of the standard is to be published.

[25] H. Torrey, Phys. Rev. **59**, 293 (1941).

[26] An alternative scheme of selectively absorbing the photons was tried. Helium gas was introduced into the interchamber region in the hope that resonance radiation from the discharge would be absorbed by the gas whereas the atom beam would be relatively unattenuated. It was found, however, that when the

The wire stop could not be left in place for the hydrogen resonance, since the resultant decrease in total beam made the resonance hard to distinguish from noise in the detector.

The accuracy with which the frequency can be set for the resonance maximum is poor in the presence of noise because the derivative of resonance amplitude with respect to frequency vanishes at the maximum. An appropriate procedure, therefore, involves setting the rf at a series of equally spaced frequencies near the maximum of the resonance, and measuring the beam amplitude at each setting. A parabola can be fitted to these data by the method of least squares, and the center of symmetry of the parabola taken as the resonant frequency.

Drift of Beam and Detector

There was a steady slow drift of the beam intensities presumably due to a gradual change of pressure in the discharge tube. In addition the zero reading of the Pirani circuit drifted by an amount equal to the maximum amplitude of the hydrogen resonance in about 20 min. This drift had the effect of shifting the apparent position of the resonance minimum. The effect may be eliminated if the drift is linear by averaging resonance curves taken first with frequencies increasing and then with frequencies decreasing.

Drift of "C" Field

The "*C*" field strength drifted by about 30 parts in 10^6 per hour despite the monitoring of the magnet current. Hence resonances taken at different times correspond to different field strengths, so it was desirable to run hydrogen and helium beams simultaneously. An attempt was made to do this by mixing the two gases in the discharge tube. It was found, however, that about 30 percent (partial pressure) of hydrogen added to the helium reduced the yield of metastable helium atoms by more than 95 percent, and gave an atomic hydrogen beam scarcely above noise. The quenching of metastable helium atoms by hydrogen is believed to be due to the fact that in a collision between a metastable helium atom and a hydrogen atom the helium atom may transfer its energy to the hydrogen atom and make a transition to the ground state, so that the metastability is lost, whereas in a collision between a metastable and an unexcited helium atom, the metastability can only be transferred from one helium atom to the other.

Another way of producing the two beams simultaneously would be to run two discharge tubes, one giving a beam directly above the other. It was considered, however, that the loss in intensity resulting from reducing the individual beam heights would make the experiment more difficult. Moreover, the two beams would not have the same trajectories in the "*C*" field.

The drift in beam intensity was unusually large when a discharge was first initiated. For this reason a reliable resonance curve could not be taken for several minutes after initiation of the discharge. It was found possible, however, to maintain the discharge while the gas was being changed, by connecting the supply of the new gas before shutting off the old. When this was done the unusually large initial drifts were avoided and resonance curves could be taken with the new gas within about three minutes. The rapidity of this gas changing technique made it less important to run the two beams simultaneously.

It was also desirable to minimize the time spent in locating each resonance maximum. This requirement was to be balanced against the requirement that beam intensity measurements be taken at closely spaced frequencies so as to locate the maximum accurately, and the requirement that several readings be taken at each frequency so as to average out random fluctuations in intensity.

Inhomogeneity of "C" Field

Effect on line width.—The variation of field strength with position within the "hairpin" had the effect of broadening the resonance lines and making them asymmetrical. The narrowest helium lines which could usually be obtained were about 350 kc/sec wide, and the narrowest hydrogen lines about 400 kc/sec wide. The natural width for the helium lines is 90 kc/sec. Thus the inhomogeneity broadened these lines by about 260 kc/sec or 17 parts in 10^5. This indicates a variation of field of 17 parts in 10^5 within the transition region. The natural width for the hydrogen line is 160 kc/sec. Thus the inhomogeneity broadened these lines by about 240 kc/sec or 22 parts in 10^5. Taking into account the hfs of hydrogen this indicates a variation of field of 19 parts in 10^5 which is nearly the same as that indicated by the width of the helium lines.

The variation of field strength in the transition region and the resulting line widths could have been reduced by decreasing the area of this region. However, since the vertical variation of the field is probably large, as indicated below, it would have been necessary to reduce the height of the beam, thus sacrificing intensity.

Effect on use of wire stop.—With no wire stop the trajectories of both helium and hydrogen are symmetrically distributed about the undeflected path. The use of the wire stop for helium and not for hydrogen moves the mean trajectory for helium about 0.1 mm horizontally away from that for hydrogen. If the wire stop is not perfectly parallel to the collimator so that the upper part of the helium beam is blocked to a different extent from the lower part, then the use of the wire stop will also shift the mean trajectory for helium vertically. A tilt of the wire stop by as little as 0.1°, for example, would shift the mean trajectory

pressure of helium in the interchamber was raised to a value for which the light intensity was reduced by $\frac{1}{3}$, the metastable atom beam was reduced by more than 90 percent.

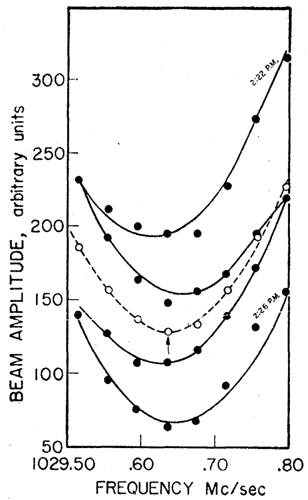


FIG. 8. A hydrogen resonance curve. The solid circles are measured points taken in the sequence indicated by the solid line. The open circles are averages of the four points taken at each frequency. The dashed curve is a parabola fitted to the open circles by the method of least squares. The arrow indicates the position of the minimum of this curve.

vertically about $\frac{1}{4}$ mm. Since the field in the transition region is inhomogeneous, a shift of the mean trajectory

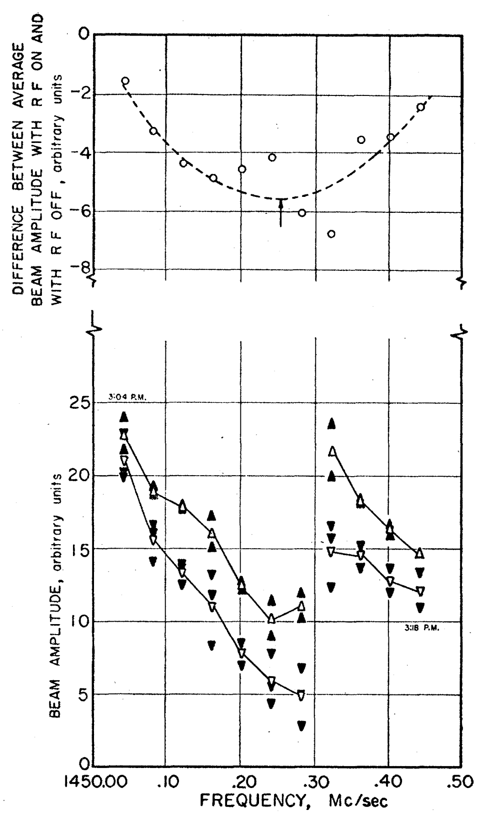


FIG. 9. A helium resonance curve. The solid triangles are measured points. Those pointing up are taken with the rf power on and those pointing down are taken with the rf power off. Five points are taken at a fixed frequency with the rf alternately on and off before moving to the next frequency setting. The hollow triangles are averages of the corresponding solid triangles. The open circles are differences between the hollow triangles. The dashed curve is a parabola fitted to the open circles by the method of least squares. The arrow indicates the position of the minimum of this curve. The large change in beam amplitude near 1450.30 Mc/sec is attributable to a sudden fluctuation in the photon background.

for helium will also shift the average value of the field over the helium trajectories. This effect should be in opposite directions when opposite edges of the wire stop are used.

To study the effect of using the wire stop, the helium beam was alternately cut with opposite edges of the wire stop. Typically, the resonance maxima for the two edges differed by 90 kc or 6 parts in 10^5.

To study separately the transverse horizontal gradient of the field in the transition region, the hydrogen beam was shifted laterally by displacing the collimator and detector. A shift of the beam through about 8 times the mean deflection of the trajectories changed the frequency by 50 kc/sec or 5 parts in 10^5. This result implies a horizontal gradient of about 4 parts in 10^5 per mm, which would account for a difference between the helium resonance maxima taken on the two sides of the wire stop of 30 kc/sec. The remainder of the observed difference must be ascribed to the vertical gradient of the field together with the vertical shift of the mean trajectory resulting from a tilt in the wire stop.

An attempt to use the wire stop to block the direct photon beam from the detector wire without cutting the helium beam with the edge of the wire stop was made by shifting the detector wire from its usual position where it receives the focused beam to the side where it receives atoms that have undergone the transitions $m=0\rightarrow-1$ and $m=+1\rightarrow0$. Since these two components are well separated at the wire stop, it was possible to place the wire stop so that the photons and the latter component were nearly blocked and the former component was not appreciably blocked. In this way the effect of the tilt of the wire stop was eliminated. Since, however, atoms that have undergone a transition are not focused, but are deflected less the greater their velocity, the displaced wire will receive atoms with velocities in a limited range, and so with trajectories in a limited range. Therefore the average field for the detected transitions will differ from that for hydrogen, where this method cannot be used because of the loss in intensity.

Procedure Adopted

In the final procedure the gases were alternated while the discharge was maintained as discussed earlier in this section. The wire stop was not used. The two detectors were placed as close to the center of the undeflected beam as they could be without having the detector wire block the beam to the Pirani gauge. Thus the trajectories of the detected hydrogen and helium beams were made as close together in the "hairpin" as possible.

For hydrogen the beam amplitude was measured at frequencies spaced 40 kc/sec apart and covering a range of 280 kc/sec. The zero reading of the Pirani gauge drifted so rapidly that it was necessary to take alternately resonance curves with the frequency increasing and with the frequency decreasing, and then to average.

A typical set of experimental data is shown by the solid circles in Fig. 8. The hollow circles are averages. The drift of the zero is apparent in this figure.

For helium the beam amplitude was also measured at frequencies spaced 40 kc/sec apart. Noise and drift obscured the helium resonance curve so that it was necessary to cover 400 kc/sec to be sure of including the resonance maximum. The beam amplitude was measured several times at each frequency with the rf power alternately on and off to average out noise and irregular drifting. Data from a typical run are shown in Fig. 9. The solid triangles pointing up are points taken with rf power off. Those pointing down are taken with rf power on. The hollow triangles are averages of the corresponding solid triangles, and the circles are differences between the averages with power off and with power on. From the plot the drift in background is apparent. Between the times of the data taken just below and just above 1450.30 Mc/sec there was a large and sudden change in the photon beam which sent the detector off scale. Such large kicks were so frequent that a complete resonance curve could seldom be taken without at least one.

In taking each point of a resonance curve for either hydrogen or helium the rf oscillator was set at the desired frequency and then the beam intensity was measured. The oscillator was stable enough so that its drift in the few seconds between the setting of its frequency and the measurement of the corresponding beam intensity was not over 1 part in 10^6. Immediately before each setting the receiver was calibrated at the desired frequency with the General Radio wave meter. The wave meter was calibrated against its own crystal standard about every half-hour. The drift of the wave meter during this time was about 1 part in 10^6.

About four measurements of the beam intensity were usually made at each frequency before changing gases. About five changes of gas were made during a day's experiment.

6. TREATMENT OF THE DATA

In order to locate the center of symmetry of an experimental resonance curve, a parabola was fitted to the averages of the experimental points by the method of least squares. These parabolas are shown in Figs. 8 and 9 by dashed lines and the centers of symmetry are indicated by arrows.

In Fig. 10 these centers of symmetry are plotted against the mean of the times at which the experimental points involved were taken. Hydrogen resonances are represented by triangles and helium resonances by circles. The drift in magnetic field is apparent from this figure. A pair of straight lines corresponding to the same dependence of magnetic field on time was fitted to the data by the method of least squares. The fitted lines are shown in Fig. 10. From these lines simultaneous values of the resonance frequencies for hydrogen and helium are obtained.

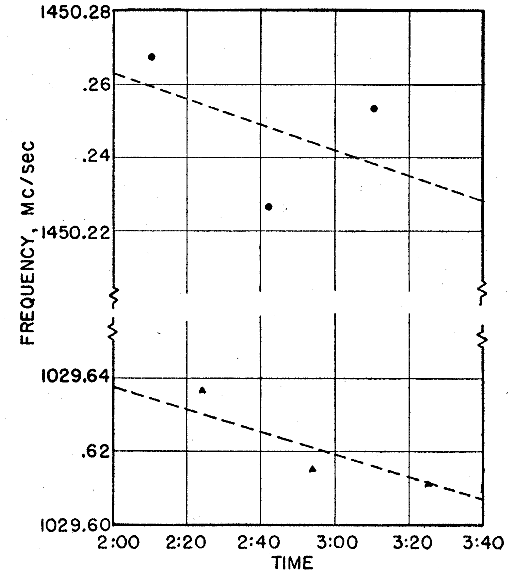


FIG. 10. The resonant frequencies of helium and hydrogen *vs* time. The solid circles are the observed helium frequencies, and the solid triangles are the observed hydrogen frequencies. The dashed straight lines correspond to the same dependence of magnetic field on time and are fitted to the data by the method of least squares.

The hydrogen frequency is given in terms of the parameter $x=[g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})-g_I]\mu_0 H_z/h\Delta\nu$ by the Breit-Rabi formula:

$$f_{\mathrm{H}}=\Delta\nu\left[-\frac{1}{2}+\frac{1}{2}\frac{g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})/g_I}{g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})/g_I-1}x+\tfrac{1}{2}(1+x^2)^{\frac{1}{2}}\right].$$

With this formula the value of x may be calculated from the hydrogen frequency. The helium frequency is given by $f_{\mathrm{He}}=g_J(\mathrm{He}, {}^3S_1)\mu_0 H_z/h$. This formula together with the expression for x gives

$$\frac{g_J(\mathrm{He}, {}^3S_1)}{g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})}=\frac{f_{\mathrm{He}}}{x\Delta\nu}\left[1-\frac{g_I}{g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})}\right].$$

With this formula, $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ may be calculated from the helium frequency and the value of x. In these calculations the values of $\Delta\nu=1420.4051\pm0.0002$ Mc/sec[13] and $g_I/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})=658.2171\pm0.0006$[5] were used. The uncertainties in these values will not affect the accuracy of the calculated g-value ratio.

The reliability of the ratio of g values calculated from a set of data such as that shown in Fig. 10 will depend upon the number of points taken, upon their average scatter about the corresponding straight lines and upon the average time interval between successive points. For the purpose of combining ratios of g values from different days, a relative weight was assigned to the data from each day.

TABLE I. Results of the measurements of the frequencies of the transitions $m=\pm1\leftrightarrow0$ of the $1s2s,\,^3S_1$ state of helium and $F, m=1, 0\leftrightarrow1, -1$ of the ground state of atomic hydrogen.[a]

Day	f_{He}(Mc/sec)	f_H(Mc/sec)	$\frac{g_J(\text{He},\,^3S_1)}{g_J(H,\,^2S_{\frac{1}{2}})}$	g_J(He, $^3S_1$)	Rel. weight
3/10	1450.440	1029.772	0.999987	2(1.001145)	0.043
3/14	1450.297	1029.672	0.999968	2(1.001096)	0.068
4/1	1450.276	1029.635	0.999984	2(1.001112)	0.055
4/2	1450.220	1029.610	0.999969	2(1.001097)	0.194
4/3	1450.246	1029.614	0.999973	2(1.001101)	0.130
4/4	1587.679	1148.480	0.999995	2(1.001123)	0.106
4/7	1587.628	1148.444	1.000006	2(1.001134)	0.404

[a] The values of f_{He} and f_H listed for each day are the results of interpolation to the same value of the magnetic field strength. For the first five days listed values of the magnetic field strength near 520 gauss were used, and for the last two days values near 567 gauss were used. The relative weight assigned to the data of each day is based on the number of individual measurements involved, and on their internal consistency.

7. RESULTS

In Table I are listed all the values of the ratio $g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})$ calculated from the data taken without using the wire stop, together with their relative weights. The weighted average of all these values is $1-11\times10^{-6}$ and the standard deviation is 16 parts in 10^6. It will be noticed, however, that the first five values, which were taken at magnetic fields near 520 gauss, all are lower than the last two, which were taken at fields near 567 gauss. For the low field values the weighted average is $1-27\times10^{-6}$ with a standard deviation of 2.0 parts in 10^6 and for the high field values the weighted average is $1+4\times10^{-6}$ with a standard deviation of 1.4 parts in 10^6. Since these averages differ by 31 parts in 10^6 or more than 10 times their standard deviations, it is evident that there are systematic differences between the measurements at the two field strengths.

The systematic differences are ascribed to the effect of different inhomogeneities at the two field strengths. Since the hydrogen and helium trajectories are symmetrically distributed about the undeflected path, the effect of inhomogeneities cannot be related to differences between trajectories for the two gases. The shape of a resonance line in an inhomogeneous field, however, will depend on the natural width of the line, especially when the broadening due to the inhomogeneity is of the same order as the natural width. Since the broadening of the hydrogen and helium lines is of the same order as their natural widths, it is believed that the difference in their natural widths caused a relative shift of their resonance maxima which varied with the inhomogeneity. This effect should be small compared to the difference between the natural widths, which is 80 kc/sec or about 6 parts in 10^5.

A difference in the relative distribution of rf power for helium and hydrogen in an inhomogeneous field could cause a relative shift in their resonance maxima. The distribution of rf power in the "hairpin" is difficult to determine experimentally; however, its effect should be negligible because the wavelengths are long compared to the dimensions of the "hairpin."

Before the systematic error introduced by the use of the wire stop was recognized, a series of sixteen determinations of the ratio of the g values was made with the wire stop interposed into the helium beam so as to reduce the photon background. These data gave the result

$$g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})=1+1\times10^{-6}.$$

When these data are corrected for the error due to the use of the wire stop by employing values for the "C" field gradient measured as described in the previous section, the result is $g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})=1-12\times10^{-6}$. This result compares well with the weighted average of the values taken without the use of the wire stop. The measurements using the wire stop were made in fields from 465 to 582 gauss, and no variation of the result with field strength was observed greater than the 31 parts in 10^6 seen in the data taken without the wire stop.

The weighted average of the data of Table I is taken as the final result. It is $g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})=1-(11\pm16)\times10^{-6}$, where, in consideration of the discussion given above, an uncertainty of 16×10^{-6} is stated. This includes the weighted averages at both the lower and higher fields.

8. DISCUSSION OF RESULTS

The accompanying theoretical paper discusses the contributions to $g_J(\text{He},\,^3S_1)$ to order α^2. The predicted value is

$$\begin{aligned}g_J(\text{He},\,^3S_1)&=2\left(1+\frac{\alpha}{2\pi}-2.973\frac{\alpha^2}{\pi^2}-\frac{1}{3}\frac{\langle T\rangle}{mc^2}-\frac{1}{6}\frac{\langle e^2/r_{12}\rangle}{mc^2}\right)\\&=2(1+1161.4\times10^{-6}-16.0\times10^{-6}\\&\qquad-38.7\times10^{-6}-2.3\times10^{-6})\\&=2(1.0011044).\end{aligned}$$

In this expression $\langle T\rangle$ is the expectation value for the kinetic energy of the electrons, and $\langle e^2/r_{12}\rangle$ is the expectation value for the electrostatic interaction between the two electrons. The term $(\alpha/2\pi-2.973\alpha^2/\pi^2)$ is the quantum-electroydnamic self-radiative correction to the spin g value of the free electron and its use here predicts the arithmetic additivity of the anomalous magnetic moments of the two electrons. The term $-\frac{1}{3}\langle T\rangle/mc^2$ is the nonradiative relativistic bound-state correction and is analogous to the relativistic bound-state correction to the g value of the electron in hydrogen. The term $-\frac{1}{6}\langle e^2/r_{12}\rangle/mc^2$ is the mutual radiative correction which arises from the Breit interaction between the two electrons and may be interpreted classically as a diamagnetic correction. The theoretical value for $g_J(\text{H},\,^2S_{\frac{1}{2}})$ is 2(1.0011276). Hence the theoretical value for $g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})$ is $1-23\times10^{-6}$.

The experimental value for $g_J(\text{He},\,^3S_1)/g_J(\text{H},\,^2S_{\frac{1}{2}})$ is $1-(11\pm16)\times10^{-6}$. This result may be combined with the experimental value $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})_{\mathrm{exp}} = 2(1.001128 \pm 12 \times 10^{-6})$ to give $g_J(\mathrm{He}, {}^3S_1) = 2(1.001117 \pm 20 \times 10^{-6})$. Instead of the experimental value for $g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$, the theoretical value may be used, giving $g_J(\mathrm{He}, {}^3S_1) = 2(1.001117 \pm 16 \times 10^{-6})$.

The experimental and theoretical results agree to within the stated uncertainty. This agreement tends to substantiate the arithmetic additivity of the anomalous magnetic moments of the electrons and the nonradiative relativistic bound-state correction for the two electrons in helium. The mutual radiative correction arising from the Breit interaction is small compared with the experimental uncertainty.

A measurement of the ratio $g_J(\mathrm{He}, {}^3S_1)/g_J(\mathrm{H}, {}^2S_{\frac{1}{2}})$ to an accuracy of $1/10^6$ would be highly desirable to provide a more incisive test of the theory to order α^2. Such an improved determination should be attainable through the use of a more homogeneous magnetic "C" field, together with some method for eliminating the photon background signal as, for example, the use of a beam periodically interrupted at the source and detector so as to block photons but allow slower atoms to pass.

We are indebted to Professors Rabi and Lamb for early encouragement to do this experiment and to Professor Rabi for helpful discussions during the course of the experiment. Mr. Robert Miller provided valuable help during the initial stages of the work. We wish to thank Mr. Karl Schumann for his superb glass-blowing. We are grateful to Mrs. Inge Hughes and Miss Helen Harwell for assistance in preparation of the manuscript.

APPENDIX I: DENSITY OF METASTABLE ${}^3S_1$ HELIUM ATOMS IN THE DISCHARGE TUBE

The equilibrium density of metastable ${}^3S_1$ helium atoms in the plasma region of the discharge will be estimated under the following assumptions: (1) helium atoms in the metastable $1s2s$, ${}^3S_1$ state are formed by inelastic collisions between electrons and helium atoms in the ground $1s^2$, ${}^1S_0$ state; (2) the metastable helium atoms are destroyed when they diffuse to the walls of the discharge tube.

The equilibrium density, n', of metastable helium atoms will be determined by equating the rate of formation of the atoms to the rate of their destruction:

$$nJ\sigma = n'/\tau.$$

τ is the lifetime of the metastable atoms in the discharge tube, n is the density of helium atoms in the ground state, J is the electron current density passing through the discharge tube in electrons per cm^2 per sec, and σ is the cross section for formation of a metastable helium atom by collision between an electron and an atom in the ground state.

The formation of the metastable atoms by electron excitation of ground state atoms to higher-energy triplet states followed by radiative decay to the metastable state, and also by electron capture by helium ions, is disregarded. The destruction of metastable atoms by collisions with other helium atoms, electrons, or impurities is also disregarded. A careful consideration of all factors which determine the density of metastable atoms in the discharge tube would be a major problem and only a rough estimate is sought here.

The excitation function for the formation of the metastable state of helium by an exchange inelastic collision between an electron and a helium atom in the ground state has been measured and calculated.[15] The experimental and theoretical excitation functions are shown in Fig. 11. The maximum cross section determined experimentally is about one-third of the maximum cross section predicted theoretically, and the maximum experimental cross section occurs at 0.5 ev above the threshold energy of 19.77 ev whereas the maximum theoretical cross section occurs at 3.2 ev above the threshold energy. The experimental excitation function will be used in this appendix.

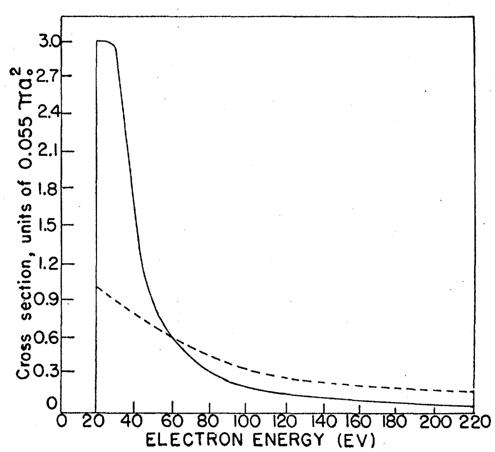


FIG. 11. Excitation function for the formation of the metastable $1s2s$, ${}^3S_1$ state of helium by collision of an electron with a helium atom in the ground state (reference 15). Solid curve: theoretical. Dashed curve: experimental. a_0 is the radius of the first Bohr orbit.

The velocity distribution of the electrons in the plasma region of the discharge will be approximately Maxwellian. The electron temperature, T_e, can be calculated.[27] For a pressure of 0.2 mm of Hg and a tube radius of 0.45 cm, T_e is about 300 000°K, so that the most probable value for the kinetic energy of the electrons is 4×10^{-11} ergs or 25 ev.

The determination of the rate of production of metastable atoms requires the evaluation of the integral $\int J(v)\sigma(v)dv$ taken over the Maxwellian velocity distribution of the electrons. An approximate graphical evaluation of the integral using the experimental cross section, $\sigma(v)$, and the electron temperature, gives the value 3.0 sec^{-1} for a tube current of 100 ma.

[27] A. v. Engel and M. Steenbeck, *Elektrische Gasentladungen* (Verlag. Julius Springer, Berlin, 1934), Vol. 2, p. 85.

The lifetime of a metastable atom is the time required for it to diffuse to the wall of the discharge tube. The mean time for diffusion to the wall is given by $\tau=\Lambda^2/D_m$, in which D_m is the diffusion coefficient for the metastable atoms and Λ is the mean distance to the wall, which is about 0.4 of the radius of the tube. From experimental data[28] D_m is estimated to be 2.6×10^3 cm²/sec for a pressure of 0.2 mm of Hg. Using this value of D_m, τ is found to be 1.4×10^{-5} sec for a tube radius of 0.45 cm.

Using these values of τ and $\int J(v)\sigma(v)dv$, the ratio n'/n is found to be 4.3×10^{-5}. For a pressure of 0.2 mm of Hg, $n=7.2\times10^{15}$ atoms/cm³, so that $n'=3.1\times10^{11}$ metastable atoms/cm³.

The beam intensity of metastable atoms will be given by

$$S=(n'aAv/4\pi R^2)\eta,$$

in which S is the electron current to the collector, a is the area of the source slit, A is the area of the detector wire, v is the average velocity of helium atoms in the discharge, R is the distance from the source to the detector, and η is the efficiency of the detection process. For the experimental arrangement discussed in section 3, $a=6.9\times10^{-3}$ cm², $A=1.1\times10^{-2}$ cm², $R=35$ cm, $v=1.27\times10^5$ cm/sec, $\eta=0.24$,[17] and the tube current was about 70 ma. These values give $S=3.3\times10^7$ electrons/sec or 5.3×10^{-12} amp. The observed metastable beam intensity given in section 3 is 10^{-12} amp.

APPENDIX II: NATURAL LINE SHAPES FOR THE HELIUM RESONANCE

In this appendix the transition probabilities for the transitions between the Zeeman levels of the $1s2s$, $^3S_1$ state of helium are calculated. Because the energy differences between the Zeeman levels $m=1$ and $m=0$ and between $m=0$ and $m=-1$ are equal, the calculation of the transition probabilities must involve all three levels. The transition probability from a state m to a state m' is given by the Majorana formula:[29]

$$P_{m,m'}=\left(\cos\frac{\alpha}{2}\right)^4(1+m)!(1+m')!(1-m)!(1-m')!$$

$$\times\left[\sum_{n=0}^{2}\frac{(-1)^n(\tan\frac{1}{2}\alpha)^{2n-m+m'}}{n!(n-m+m')!(1+m-n)!(1-m'-n)!}\right]^2.$$

The quantity α is defined by the relation:

$$\sin^2\frac{\alpha}{2}=\frac{1+\epsilon}{1+\delta^2}\sin^2\left(\frac{\tau}{2}(1+\delta^2)^{\frac{1}{2}}\right).$$

The parameters τ and δ are given by $\tau=\gamma H_{rf}t$ and $\delta=(\omega-\omega_0)/\gamma H_{rf}$ in which $\gamma=g_Je/2mc$, H_{rf} is the amplitude of the rotating magnetic field, t is the time spent in the transition region, ω is the angular frequency of the rotating field, and ω_0 is the angular Bohr frequency for the transition between adjacent Zeeman levels in an external field H_z. The quantity ϵ is small compared to unity when $H_{rf}\ll H_z$ and so it may be neglected.

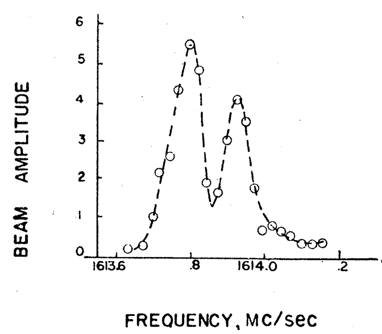


FIG. 13. An experimental helium resonance line having two peaks. The wire stop is used so that only transitions from the $m=+1$ component are observed.

Evaluation of the transition probabilities gives

$$P_{1,0}=P_{0,1}=P_{-1,0}=P_{0,-1}=2\cos^2(\alpha/2)\sin^2(\alpha/2),$$
$$P_{1,-1}=P_{-1,1}=\sin^4(\alpha/2),$$
$$P_{1,1}=P_{-1,-1}=\cos^4(\alpha/2),$$
$$P_{0,0}=[\cos^2(\alpha/2)-\sin^2(\alpha/2)]^2.$$

When no wire stop is used, all three Zeeman components of the beam are refocused on the detector in the absence of an rf magnetic field. The observed reduction in beam intensity produced by the rf magnetic field is due to transitions from each of the Zeeman levels to the two other Zeeman levels. Hence the reduction in beam intensity will be proportional to the quantity $P=\frac{1}{3}(1-P_{1,1})+\frac{1}{3}(1-P_{0,0})+\frac{1}{3}(1-P_{-1,-1})$.

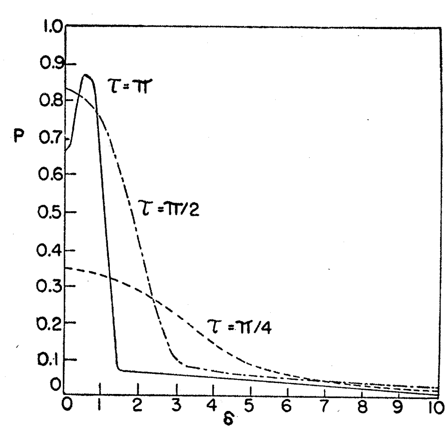


FIG. 12. Calculated natural line shapes for helium. $\delta=(\omega-\omega_0)/\gamma H_{rf}$. $\tau=\gamma H_{rf}t$.

[28] M. A. Biondi, Phys. Rev. **88**, 660 (1952); A. C. G. Mitchell and M. K. Zemansky, *Resonance Radiation and Excited Atoms* (Cambridge University Press, Cambridge, 1934), p. 249.

[29] E. Majorana, Nuovo cimento **9**, 43 (1932); F. Bitter and J. Brossel, Massachusetts Institute of Technology, Research Laboratory of Electronics Technical Report No. 176, Sept. 12, 1950 (unpublished); I. I. Rabi, Phys. Rev. **51**, 652 (1937). There is a misprint in Eq. (17) of this last paper. The factor in the denominator given as $(J-m-\nu)!$ should be $(J+m-\nu)!$

In the experiment the amplitude of the rf field was customarily adjusted to maximize the line intensity at the center when the wire stop was interposed so that only transitions from the $m=+1$ component were observed. This adjustment made $\tau \simeq \pi$. For $\tau=\pi$ the natural width of the line taken without the wire stop is about $1.2/t$. For a 1.5-cm "hairpin" and atoms moving with a velocity equal to the most probable velocity in a source at 300°K the natural width would be 90 kc/sec. The natural width will be slightly greater than this due to the velocity distribution in the beam.

Figure 12 shows the theoretical line shapes for three values of the rf field intensity for a beam of atoms all of which have the same velocity. These three lines are symmetrical about the Bohr resonance frequency. The line for $\tau=\pi$ has a minimum at its center.

Ordinarily the natural shape of a resonance line is obscured by broadening due to inhomogeneity of the magnetic field. A few times, however, when the wire stop was used unusually narrow lines with double peaks have been observed. One such line is shown in Fig. 13. The use of the wire stop allows only transitions from the state $m=+1$ to be observed. Figure 14 shows the theoretical line shapes for transitions from $m=+1$ for several values of the rf field intensity and for a beam of atoms all of which have the same velocity. The detailed shape of these lines would be modified by the distribution of velocities in the beam and by inhomogeneities of the field. The curve in Fig. 14 for which $\tau=7\pi/4$ resembles the experimental line shown in Fig. 13.

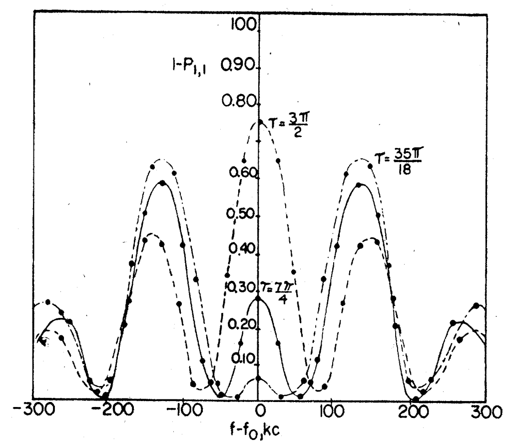


FIG. 14. Calculated natural line shapes for helium for transitions from $m=+1$ only. $\tau=\gamma H_{rf}t$.