The Variation of the Elastic Constants of Crystalline Aluminum with Temperature between 63°K and 773°K*

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(Received April 28, 1953)

The composite piezoelectric oscillator is employed to measure the adiabatic elastic moduli of crystalline aluminum over the temperature interval 63°K to 773°K. The data permit a valid extrapolation to 0°K. The Debye characteristic temperature of aluminum at 0°K, computed with these data, is 439°K. Various novel procedures designed to facilitate the use of the method are described.

I. EXPERIMENTAL METHOD

THE composite piezoelectric oscillator method is employed in the present research to measure the elastic constants of crystalline aluminum as a function of temperature between 63°K and 773°K. Details of the method have been given elsewhere,[1,2] therefore it will suffice here briefly to review its essential features and to describe certain novel procedures that facilitate its use.

The specimen, in the form of a right circular cylinder a few mm in diameter and a few cm long, is cemented coaxially to a suitably cut cylinder of crystalline quartz

FIG. 1. Arrangement for observing resonance of the piezoelectric oscillator. The circuit in the dotted enclosure is the electrical equivalent of the oscillator.

[1] L. Balamuth, Phys. Rev. 45, 715 (1934); 46, 933 (1934).
[2] F. C. Rose, Phys. Rev. 49, 50 (1936).

of identical cross section. Aluminum electrodes are deposited by evaporation in proper positions on the quartz, and a sinusoidally varying potential difference is established between them. In consequence, a stationary state of forced vibration is established in the composite system. The vibrations are either torsional or longitudinal, dependent upon the orientation of the quartz cylinder axis in the crystal lattice and the associated electrode configuration.

When the frequency of the applied voltage is nearly a frequency of free (longitudinal or torsional) vibration of the composite oscillator, the electrical characteristics of the oscillator closely approximate those of the circuit shown in the dotted enclosure of Fig. 1, namely, a fixed capacity shunted by an inductance, capacity, and resistance in series. The actual frequency of free vibration is identical with the resonant frequency of the series branch alone[3] and can therefore be deduced from the observed electrical behavior of the oscillator. The *fundamental* frequency of free vibration, hereafter denoted f_0, is employed throughout the present experiments.

It is desired to obtain the fundamental frequency of free vibration of the specimen cylinder *alone*. This is related to f_0 by the formula[1]

$$M_1 \frac{\tan(\pi f_0/f_1)}{(\pi f_0/f_1)} + M_2 \frac{\tan(\pi f_0/f_2)}{(\pi f_0/f_2)} = 0, \qquad (1)$$

where M_1, M_2 are the masses of the specimen and quartz cylinders, respectively, f_1 is the fundamental frequency of free vibration of the specimen cylinder alone, and f_2 is the observed value of f_0 when $M_1=0$, i.e., when the specimen cylinder is detached. Thus f_1 can be computed when f_0 and f_2 are known.

When the vibrations are longitudinal,

$$f_1 = (E^{\frac{1}{2}}/2\rho^{\frac{1}{2}}L)(1 - \pi^2\sigma^2 r^2/4L^2), \qquad (2)$$

and when the vibrations are torsional

$$f_1 = (G^{\frac{1}{2}}/2\rho^{\frac{1}{2}}L), \qquad (3)$$

where E, G, σ are respectively the adiabatic Young's modulus, torsion modulus, and Poisson's ratio, of the specimen material for the direction of the cylinder axis,

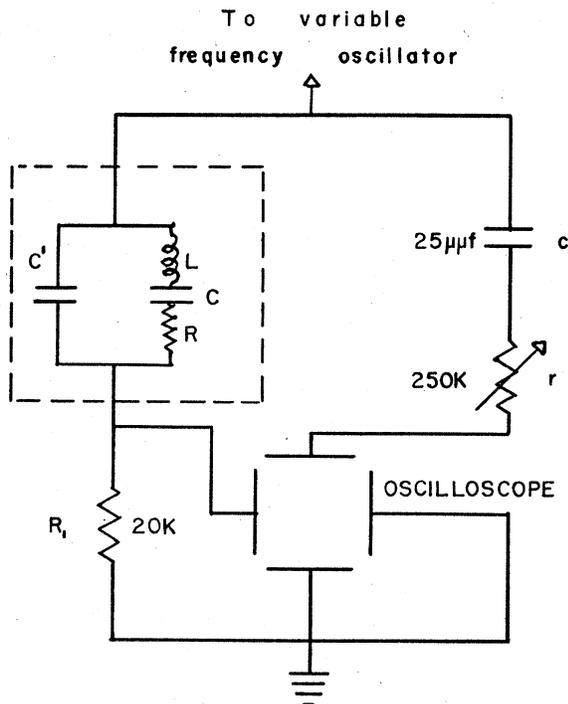[3] W. T. Cooke, Phys. Rev. 50, 1158 (1936).

ρ is its density, and r, L are the radius and length of the cylinder.

Under the circumstances of the present measurements, E and G are related to the adiabatic elastic moduli of the specimen material by the formulas[4]

$$1/E = S_{11} - 2S\Gamma, \tag{4}$$

$$1/G = S_{44} + 4S\Gamma - 2S^2(\Gamma - 4\Gamma^2 + 3\chi)/(S_{11} - 2S\Gamma), \tag{5}$$

$$S = S_{11} - S_{12} - \tfrac{1}{2}S_{44}, \tag{6}$$

where

$$\Gamma = b^2 c^2 + c^2 a^2 + a^2 b^2,$$

$$\chi = a^2 b^2 c^2,$$

and a, b, and c are the direction cosines of the cylinder axis with respect to the principal axes of the face-centered cubic lattice of aluminum.

The elastic moduli are related to the elastic constants and compressibility, κ, by the formulas

$$
\begin{aligned}
C_{11} &= (S_{11} + S_{12})/(S_{11} - S_{12})(S_{11} + 2S_{12}),\\
C_{12} &= -S_{12}/(S_{11} - S_{12})(S_{11} + 2S_{12}),\\
C_{44} &= 1/S_{44},\\
\kappa &= 3(S_{11} + 2S_{12}) = 3/(C_{11} + 2C_{12}),
\end{aligned}
\tag{7}
$$

and the isothermal to the adiabatic quantities by the formulas

$$
\begin{aligned}
(S_{11})_{\mathrm{ad}} - (S_{11})_{\mathrm{is}} &= (S_{12})_{\mathrm{ad}} - (S_{12})_{\mathrm{is}} = -T\alpha^2/\rho C_p,\\
(C_{11})_{\mathrm{ad}} - (C_{11})_{\mathrm{is}} &= (C_{12})_{\mathrm{ad}} - (C_{12})_{\mathrm{is}}\\
&= T\alpha^2/(\rho C_p)(S_{11} + 2S_{12})^2,
\end{aligned}
\tag{8}
$$

where T is the absolute temperature, α the coefficient of linear expansion, and C_p the specific heat at constant pressure. The adiabatic and isothermal S_{44} and C_{44} are the same.

It should be noted that the term in σ, in Eq. (2) is small of the order 10^{-3} and so may be computed with sufficient accuracy with the approximate values of the elastic constants obtained when this term is set equal to zero.

It is customary, in the present method, to compute the basic quantity f_0 from the observed variation with frequency of the amplitude of the current which flows through the oscillator.[5] The latter is proportional to the rectified voltage across a resistance (R_1 of Fig. 1) connected in series with the oscillator, and this voltage is measured with an electronic voltmeter. Such means must be employed when the internal friction of the specimen is large. If, however, the internal friction is sufficiently small that the admittance $2\pi f_0 C'$ of the capacity C' (Fig. 1) is less than half the admittance $1/R$ of the series branch at the same frequency, then the far simpler scheme shown in Fig. 1 can be used.

The basis of this scheme is the fact that the current through the piezoelectric oscillator is in phase with the applied voltage at a frequency greater than f_0 by only one or two cps in 50 kc/sec.[6] Accordingly, the frequency at which this phase relation exists may be equated to f_0 with negligible error. The voltage across a resistance R_1, connected in series with the oscillator, is impressed on one pair of plates of a cathode-ray oscilloscope, and the total voltage across this series arrangement is impressed on the other pair of plates. The resonant frequency f_0 is then nearly enough the frequency at which the trace on the oscilloscope screen is a straight line. The circuit elements r and c of Fig. 1 permit compensation of unequal phase shifts in the oscilloscope amplifiers at the frequency of measurement. The resistance r is adjusted so that the trace is linear when the piezoelectric oscillator is short circuited. The advantage of the method lies in the fact that the observed phase relationships are unaffected by the inevitable change in the applied voltage incident to the large variation of the impedance of the piezoelectric oscillator as the frequency is varied in the neighborhood of resonance.

The computation of f_1 demands a precise solution of the transcendental Eq. (1), and this is a laborious procedure even after a five-figure table of values of $\tan(\pi x)/x$ has been constructed. A facile approximate solution of readily ascertainable accuracy is available when the respective lengths of the quartz and specimen cylinders are so adjusted that f_0, f_1, and f_2 do not differ by more than about 1000 cps in 50 kc/sec. For then f_1, regarded as a function of f_0 and f_2 in accordance with Eq. (1), can be expanded in a Taylor's series about the point $f_0 = f_1 = f_2$ and an appropriate number of terms employed in the computation of f_1. The result of the expansion is[7]

$$
\begin{aligned}
f_1 &= f_2 + \Sigma_n(nD), \quad 1D = \delta(\epsilon + 1), \quad 2D = 0,\\
3D &= \pi^2 \delta^3 \epsilon (1 - \epsilon^2)/3f_2^2,\\
4D &= 2\pi^2 \delta^4 \epsilon^3 (\epsilon + 1)/3f_2^3,\\
\delta &= (f_0 - f_2), \quad \epsilon = M_2/M_1,
\end{aligned}
\tag{9}
$$

and terms with n greater than 4 are invariably negligible in practice.

The vernier condenser which controls the frequency of the voltage applied to the composite oscillator is frequency calibrated at the time of measurement with the aid of a standard quartz crystal controlled constant frequency generator. The latter drives a multivibrator at an integral subharmonic of the controlled frequency. The voltage output of the multivibrator, rich in harmonic content, is applied to one pair of plates of an oscilloscope and voltage of variable frequency to the other. Calibration is effected at intervals of approxi-

[4] W. F. Brown, Phys. Rev. **58**, 998 (1940); R. F. S. Hearmon, Revs. Modern Phys. **18**, 414 (1936).

[5] S. Siegel and S. L. Quimby, Phys. Rev. **49**, 663 (1936); J. Zacharias, Phys. Rev. **44**, 116 (1933).

[6] W. G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), Sec. 275.

[7] The author is greatly indebted to Dr. J. Nafe and Mr. R. Scheib for these formulas.

mately 30 cycles in 50 kilocycles by observation of the resultant oscilloscope traces.

II. PREPARATION OF THE SPECIMEN

Quantitative spectroscopic analysis of the specimen material reveals the presence of Si, Fe, and Ga in amounts of the order 0.01 percent; Mn and Ti in amounts of the order 0.01 percent or less; and traces of Cu, Mg, and Ca. The specification of purity by the Aluminum Company of America, who supplied the material, is 99.93 percent. Single crystals, in the form of rods of circular cross section 4.57 mm in diameter and 6 cm long, are grown in graphite crucibles, in vacuum, by the Bridgman method. A section of small diameter is produced at one end of the specimen by reducing the diameter of the crucible bore to 2.5 mm over a 6 mm length at the bottom. This permits the use of Laue x-ray transmission photographs to determine the orientation of the specimen cylinder axis in the crystal lattice in the manner described by Quimby and Siegel.[8] The crystals as grown are etched[9] and examined, flawed crystals rejected, and the remainder annealed at 450°C

TABLE I. Orientation data for the five aluminum crystals.

No.	a	b	c	Γ	χ
1	0.5390	0.6536	−0.5313	0.3267	0.03503
2	0.4335	0.7554	−0.4913	0.2903	0.02588
3	0.7403	−0.6302	−0.2342	0.2695	0.01194
4	0.7342	−0.1100	−0.6699	0.2539	0.00293
5	−0.8715	0.0352	0.4891	0.1829	0.00022

for a half-hour,[10] after which the axis orientation measurements are made. Sections of the desired length are cut from the rod with a fine jeweler's saw and the ends lapped smooth and true with 600 carborundum in light machine oil. The crystal is supported in a V-block during these operations.

Observations on five crystals are here reported. Values of a, b, c, Γ, and χ for these crystals appear in Table I.

III. PREPARATION OF THE COMPOSITE OSCILLATOR

Three different quartz to specimen adhesives are employed for observations in various temperature intervals, as follows.

Below 213°K: A thin film of Dow Corning stopcock grease is applied to the interface and the two cylinders are held together under a compressive force of 180 g, in vacuum, for ten minutes. This cement solidifies at

223°K. Above this temperature the adhesive action persists, but the equivalent electrical resistance of the oscillator is greatly increased.

213°K to 300°K: The interface is coated with liquid phenylsalicylate (salol) which is supercooled from its melting point at 316°K, and the oscillator is assembled at atmospheric pressure under 180 g compressive force. Liquid extruded at the interface is touched with a minute crystal of solid salol, whereupon the entire film solidifies and forms a strong adhesive bond between the two elements.

273° to 773°K: Two parts of powdered sodium silicofluoride and one part of barium sulfate are ground together in a mortar, and a quick hardening cement is prepared by introducing the powder into a drop or two of water glass. This is applied rapidly and thinly to the interface and the assembled oscillator is maintained under 1000 g compressive force for 12 hours. The resultant strong bond decomposes only after several hours at 773°K.

The composite oscillator is suspended vertically in an appropriate glass or fused quartz envelope by light spring clips which engage fine opposing slots ground in the surface of the quartz cylinder at its midpoint, and which effect electrical contact with the aluminum electrodes.

Apart from the considerations heretofore noted in connection with the solution of Eq. (1), it is desirable that f_0, f_1, and f_2 should not differ by more than 1000 cps in order to minimize the effect on f_0 of two factors ignored in the analysis. The first is the adhesive, and the second is the mounting clips, which do not lie at a displacement node of vibration of the quartz cylinder unless $f_0 = f_2$. Eleven different lengths are required to maintain this condition over the entire temperature range; furthermore, a definite program of measurement of the Young's or torsion modulus over specified high- or low-temperature intervals must be followed, in order that the specimen be progressively *shortened*.

IV. PRODUCTION AND MEASUREMENT OF TEMPERATURE

63°K to 77°K: The oscillator, enclosed in a glass envelope, is immersed in liquid nitrogen contained in a closed Dewar vessel connected at the top to an exhaust pump. The vapor pressure above the liquid is measured with a mercury manometer, and this is varied by varying the rate of removal of the vapor. The data of Henning and Otto[11] relate the vapor pressure and temperature. In this, as in the several arrangements described below, at atmosphere of helium at 3-mm pressure is maintained in the envelope to establish thermal contact between the oscillator and its environment. A plug of glass wool separates the portion of the envelope containing the oscillator from the remainder and so prevents the the transfer of heat by convection. The estimated

[8] S. L. Quimby and S. Siegel, Phys. Rev. 54, 293 (1938). The accuracy of this determination is five minutes of arc.
[9] C. S. Barrett, *Structure of Metals* (McGraw-Hill Book Company, Inc., New York, 1943), p. 175.
[10] Mr. Donald Blattner, of this laboratory, has observed that strain induced changes of the Young's modulus of Al crystals disappear after a few hours at room temperature.
[11] F. Henning and J. Otto, Physik. Z. 37, 633 (1936).

accuracy of the temperature determination in this interval is 0.25°K.

143°K to 293°K: The oscillator, in its envelope, is immersed in a well-stirred bath of composition 14.5 percent chloroform, 25.3 percent methylene chloride, 33.4 percent ethyl bromide, 10.4 percent transdichlorethylene, and 16.4 percent trichlorethylene.[12] This bath is contained in an unsilvered Dewar vessel mounted in a second Dewar filled with liquid nitrogen. The insulating space of the inner Dewar can be exhausted or filled with helium at will, and the temperature of the bath can be raised by an electric heater immersed therein. Suitable adjustment of the current in the heater and the pressure of helium in the Dewar permits stabilization of the bath temperature at any desired value in the interval.

The bath temperature is measured with a copper-constantan thermocouple.[13] Auxiliary experiments reveal that the temperatures of the specimen and bath agree within ±0.05°K. The estimated accuracy of the temperature determination in this interval is 0.1°K.

The bath cannot be used in the temperature interval 77°K to 143°K. Probably the most satisfactory method for the production of stable temperatures in this interval is that devised by Nix and MacNair.[14]

273°K to 373°K: The oscillator is immersed in a well-stirred water bath contained in a Dewar vessel and heated electrically. The temperature is measured with a copper-constantan thermocouple.

373°K to 773°K: The envelope is a clear fused quartz tube inserted in a tubular wire wound electric furnace. A metallic tube between the quartz tube and furnace core serves to reduce the longitudinal temperature gradient. The temperature is stabilized with the proportional controller of Lazarus and Lawson[15] and is measured with a Chromel-Alumel thermocouple in juxtaposition with the specimen. The estimated accuracy is 0.2°K.

V. REDUCTION OF OBSERVATIONS

The lengths of the specimens are measured at room temperature on a temperature corrected Geneva Society comparator, with an accuracy of 0.03 percent. The adopted value of the density of aluminum at 25°C is 2.6971 g/cm³.[16] Lengths and densities at other temperatures are computed with the data on thermal expansion of aluminum obtained by Nix and MacNair[17] and Hidnert.[18]

Values of E and G for each specimen are computed in the Eqs. (2) and (3) and plotted as a function of temperature on a scale commensurate with the precision of measurement. It is then found that segments of the curve, for a single specimen, corresponding to different temperature intervals, are displaced parallel to the modulus axis by the equivalent of some 100 cps or less in 50 kc/sec frequency. This is the residual effect of the clip mount and adhesive heretofore noted. The curve which passes through 0°C is left unaltered and the remainder shifted so that the sections at the termini of the temperature intervals are superimposed. It is then found that the resultant curve is without significant discontinuity in slope, which illustrates a characteristic of the method, viz., changes in the moduli can be measured with greater accuracy than the absolute values. The curve through 0°C is chosen as standard because of the proximity of this temperature to the ice point at which the thermocouple is calibrated.

It is desired next to obtain, by the method of least squares,[19] the values of the quantities S_{11}, S_{44}, and S, corresponding to a given temperature, which will reproduce with minimum error the observed values of $1/E$ and $1/G$ for the five crystals in accordance with Eqs. (4) and (5). The value of S_{12} is then to be computed with Eq. (6). The problem is complicated by two factors: first, the statistical weights[20] to be assigned the measurements of $1/E$ and $1/G$, respectively, are not the same; second, the quantity S occurs in Eq. (5) in a manner not easily disentangled for the purpose of a least-squares solution. The problem is simplified by the fact that the value of the third term in the right-hand member of Eq. (5) is less than one percent of the remainder. The procedure adopted is as follows:

1. The five $1/E$ equations [Eq. (4)] are solved by least squares for S_{11} and S.

2. This value of S is inserted in the third terms only of the five $1/G$ equations [Eqs. (5)], and these equations are solved by least squares for the quantity S. This value of S is now inserted in the third term of Eqs. (5) and the equations resolved for S_{44} and S.

3. The results of steps 1 and 2 are employed to compute the respective statistical weights of the observation of $1/E$ and $1/G$.

4. The ten Eqs. (4) and (5) are solved by least squares for the quantity S, with these statistical weights.

5. The five Eqs. (4) are solved for S_{11} with the value of S obtained in step 4, and similarly the five Eqs. (5) are solved for S_{44}.

6. The quantity S_{12} is computed with Eq. (6).

The three adiabatic elastic moduli corresponding to twenty-six temperatures in the interval of observation are evaluated in the manner described above. The adia-

[12] C. W. Kanolt, Sci. Papers Bur. Standards, No. 520, 619 (1926).

[13] All the thermocouples used in these experiments are calibrated by comparison with a platinum resistance thermometer certificated by the U. S. Bureau of Standards.

[14] F. C. Nix and D. MacNair, Rev. Sci. Instr. 12, 66 (1941).

[15] D. Lazarus and A. W. Lawson, Rev. Sci. Instr. 18, 730 (1947).

[16] F. Foote and E. R. Jette, Phys. Rev. 58, 81 (1940).

[17] F. C. Nix and D. MacNair, Phys. Rev. 60, 597 (1941).

[18] P. Hidnert, Sci. Papers Bur. Standards 19, 697, S497 (1923–1924).

[19] W. E. Deming, Statistical Adjustment of Data (John Wiley and Sons, Inc., New York, 1938).

[20] L. D. Weld, Theory of Errors and Least Squares (Macmillan and Company, New York, 1916), p. 155.

TABLE II. Observed and computed moduli at three temperatures, showing deviations in percent.

Crystal	$E\times10^{-12}$ dyne/cm² Observed	Computed	Percent dev.	$G\times10^{-12}$ dyne/cm² Observed	Computed	Percent dev.	T°K
1	0.5536	0.5522	0.25	0.1700	0.1707	+0.41	
2	0.5357	0.5354	0.06	0.1759	0.1745	−0.80	
3	0.5236	0.5263	−0.52	0.1744	0.1762	1.02	773.2°K
4	0.5173	0.5196	−0.44	0.1766	0.1775	0.51	
5	0.4946	0.4915	0.63	0.1880	0.1857	−1.24	
1	0.7491	0.7472	0.25	0.2455	0.2463	0.32	
2	0.7346	0.7329	0.23	0.2529	0.2499	−1.20	
3	0.7206	0.7250	−0.61	0.2471	0.2515	1.75	293.2°K
4	0.7170	0.7192	−0.31	0.2516	0.2528	0.47	
5	0.6968	0.6938	0.43	0.2639	0.2601	−1.46	
1	0.8250	0.8230	0.24	0.2736	0.2746	0.36	
2	0.8109	0.8087	0.27	0.2822	0.2782	−1.43	
3	0.7956	0.8006	−0.63	0.2745	0.2799	1.93	63.2°K
4	0.7928	0.7947	−0.24	0.2800	0.2813	0.46	
5	0.7720	0.7691	0.38	0.2928	0.2886	−1.45	

batic elastic constants are then computed with Eq. (7) and the isothermal quantities with Eq. (8). Values of C_p reported by Maier and Anderson[21] and Kelley[22] are employed in the last-named calculation.

The concordance of the data is exhibited in Table II, in which the computed values of E and G for the five crystals are compared with those observed at three temperatures. The average deviation is 0.4 percent in the Young's modulus and 1.0 percent in the torsion modulus.

VI. RESULTS

The computed values of the adiabatic moduli and constants are tabulated as functions of the temperature in Table III and the corresponding isothermal quantities similarly in Table IV. The number of significant figures employed in these tables is greater than is warranted by the accuracy of the absolute values. This permits a valid determination of the respective temperature coefficients since, as previously remarked, changes in the moduli can be measured with greater accuracy than their absolute values. The estimated uncertainty in S_{11} varies from about 0.8 percent at lower temperatures to 1.2 percent at higher. Corresponding estimates of the uncertainty in S_{44} are 1.5 percent and 2.0 percent. The uncertainty in S_{12} is about 2 percent over the temperature range.

The values corresponding to $T=0°K$ in Table III are obtained by graphical extrapolation. It can readily be shown[23] that the temperature coefficients of the elastic moduli must be zero at absolute zero. In consequence, the accuracy of values extrapolated from 63°K is nearly as great as that of the data upon which the extrapolation is based.

The values of the adiabatic elastic moduli and the calculated isothermal compressibility at 293°K and

[21] C. G. Maier and C. T. Anderson, J. Chem. Phys. 2, 518 (1934).
[22] K. Kelley, Bull. Bur. Mines No. 371 (1934).
[23] J. K. Galt, Phys. Rev. 73, 1460 (1948).

0°K here reported are compared with those obtained by Goens[24] at the same temperatures and Lazarus[25] at 293°K, in Table V. Bridgman's direct measurement of the compressibility[26] yielded $\kappa=1.36\times10^{-12}$ cm²/dyne.

Table VI shows the variation of the Debye characteristic temperature with temperature. Here θ_D is evaluated by a modification of the method of Hopf and Lechner.[27] It is a feature of this method that the function $(C+Z)^{-\frac{1}{2}}$ is represented approximately over the interval $0<Z<1$ by a polynomial in Z. Here C is a constant and Z is a root of the cubic equation whose roots are related to the three velocities of propagation of plane waves in the crystal. The three roots lie in the

TABLE III. The adiabatic elastic moduli and constants of aluminum. The number of significant figures does not indicate the accuracy of the absolute value.

T°K	Adiabatic moduli $\times10^{12}$ cm²/dyne S_{11}	$-S_{12}$	S_{44}	Adiabatic constants $\times10^{-12}$ dyne/cm² C_{11}	C_{12}	C_{44}
773.2	2.320	0.880	4.847	0.803	0.491	0.2063
733.2	2.228	0.842	4.687	0.831	0.505	0.2134
693.2	2.142	0.807	4.544	0.858	0.519	0.2201
653.2	2.064	0.776	4.415	0.885	0.533	0.2265
613.2	1.991	0.746	4.299	0.913	0.547	0.2326
573.2	1.924	0.720	4.192	0.941	0.563	0.2385
533.2	1.861	0.696	4.093	0.969	0.578	0.2443
493.2	1.804	0.673	3.999	0.999	0.595	0.2500
453.2	1.750	0.653	3.911	1.027	0.611	0.2557
413.2	1.700	0.633	3.826	1.055	0.627	0.2613
373.2	1.654	0.615	3.745	1.082	0.641	0.2670
353.2	1.632	0.607	3.706	1.094	0.648	0.2698
333.2	1.611	0.599	3.669	1.106	0.654	0.2727
313.2	1.591	0.591	3.630	1.118	0.660	2.2755
293.2	1.572	0.583	3.593	1.129	0.665	0.2783
273.2	1.554	0.575	3.557	1.140	0.670	0.2812
253.2	1.536	0.568	3.521	1.151	0.675	0.2840
233.2	1.519	0.561	3.487	1.161	0.680	0.2868
213.2	1.503	0.554	3.454	1.170	0.684	0.2895
193.2	1.488	0.548	3.422	1.179	0.688	0.2922
173.2	1.473	0.542	3.391	1.188	0.692	0.2949
153.2	1.459	0.536	3.362	1.197	0.696	0.2974
143.2	1.452	0.534	3.349	1.201	0.698	0.2986
133.2	1.446	0.531	3.335	1.206	0.700	0.2998
77.2	1.414	0.517	3.265	1.225	0.707	0.3063
70.2	1.411	0.516	3.260	1.226	0.707	0.3068
63.2	1.409	0.515	3.254	1.227	0.707	0.3073
0	1.404	0.513	3.236	1.230	0.708	0.3090

specified interval in the case of most ionic crystals. But for all the metals thus far examined two of the roots are negative and the third is greater than unity. The modification consists in representing the aforementioned function by a polynomial over the actual range of values assumed by Z.[28]

[24] E. Goens, Ann. Physik 38, 456 (1940).
[25] D. Lazarus, Phys. Rev. 76, 545 (1949).
[26] P. W. Bridgman, Proc. Am. Acad. Arts Sci. 70, 285 (1935); Revs. Modern Phys. 18, 1 (1946).
[27] L. Hopf and G. Lechner, Verhandl. deut. physik. Ges. 16, 643 (1914). A description of this method is given by M. Born, Encyklopedie der Mathematischen Wissenschaften (Teubner, Leipzig), Vol. 3, p. 649.
[28] A detailed description of this method for calculating θ_D is scheduled for publication in this journal.

Kok and Keesom[29] measured the temperature variation of the specific heat at constant volume of aluminum in the interval 1.1°K to 20°K. They found that a satisfactory separation of the electronic and lattice contributions to the specific heat led to the value

TABLE IV. The isothermal elastic moduli, constants, and compressibility of aluminum. The number of significant figures does not indicate the accuracy of the absolute value.

$T°K$	Isothermal moduli $\times10^{12}$ cm²/dyne		Isothermal constants $\times10^{-12}$ dyne/cm²		Isothermal compressibility
	S_{11}	$-S_{12}$	C_{11}	C_{12}	$\kappa\times10^{12}$
773.2	2.347	0.853	0.717	0.405	1.92
733.2	2.252	0.817	0.746	0.421	1.85
693.2	2.165	0.785	0.776	0.437	1.79
653.2	2.085	0.755	0.806	0.454	1.72
613.2	2.010	0.728	0.837	0.472	1.66
573.2	1.941	0.703	0.868	0.490	1.60
533.2	1.877	0.680	0.901	0.510	1.55
493.2	1.817	0.660	0.934	0.530	1.49
453.2	1.762	0.641	0.967	0.551	1.44
413.2	1.711	0.623	1.000	0.572	1.39
373.2	1.663	0.606	1.032	0.591	1.35
353.2	1.641	0.599	1.047	0.600	1.33
333.2	1.619	0.591	1.062	0.609	1.31
313.2	1.598	0.584	1.077	0.618	1.29
293.2	1.579	0.576	1.089	0.625	1.28
273.2	1.560	0.569	1.101	0.632	1.27
253.2	1.541	0.563	1.117	0.642	1.25
233.2	1.524	0.557	1.132	0.651	1.23
213.2	1.507	0.550	1.143	0.657	1.22
193.2	1.491	0.544	1.155	0.663	1.21
173.2	1.476	0.539	1.168	0.672	1.19
153.2	1.462	0.534	1.180	0.679	1.18
143.2	1.455	0.531	1.186	0.683	1.18
133.2	1.448	0.529	1.194	0.688	1.17
77.2	1.414	0.517	1.221	0.703	1.14
70.2	1.412	0.516	1.222	0.703	1.14
63.2	1.410	0.514	1.222	0.703	1.14
0	1.404	0.513	1.230	0.708	1.13

$\theta_D=419°K$, which is to be compared with the value 439°K here reported.

It is of interest to note that the temperature variation of C_{44} is represented over the entire temperature range with an accuracy better than one-half percent by the

[29] J. A. Kok and W. H. Keesom, Physica 4, 835 (1937).

TABLE V. Comparison of the present measurements (S) with those of Goens (G) and Lazarus (L). The units are 10^{-12} cm²/dyne.

$T°K$	S_{11}	$-S_{12}$	S_{44}	κ	Obs
293°	1.57	0.58	3.59	1.28	S
	1.59	0.58	3.52	1.35	G
	1.74	0.66	3.51	1.35	L
0°	1.40	0.51	3.24	1.13	S
	1.42	0.50	3.15	1.26	G

TABLE VI. The Debye characteristic temperature of aluminum.

$T°K$	0°	50°	100°	200°	300°	450°	600°	750°
θ_D	439°	438°	436°	427°	416°	398°	380°	358°

formula

$$C_{44}=Ae^{-B\alpha T}R^{-2},$$

where A and B are constants, α is the coefficient of thermal expansion, and R is the ratio of the volume at temperature T to that at an arbitrary temperature.[30] This formula is in accord with the finding established experimentally by Lazarus,[25] that the elastic constants must be explicit functions of both temperature and volume. But no such simple formulas have been discovered with which to represent the temperature variations of C_{11} and C_{12}.

In conclusion, the writer gratefully acknowledges his indebtedness to the Aluminum Company of America for the gift of the specimen material, to Dr. R. H. Gillette of the Linde Air Products Company, who suggested the formula for the high temperature adhesive, to Dr. T. I. Taylor and Mr. J. Dunbar of Columbia University for the spectroscopic analysis of the crystals, and to Dr. S. L. Quimby who followed the progress of the research with helpful counsel and encouragement.

[30] A similar formula, with α omitted, has been employed to describe the temperature variation of the Young's modulus of polycrystalline metals. See D. J. Mack, Metals Technol. 12, No. 8 (1945).