To determine the scattering amplitude in closed form we must express G_1 in terms of G, Eq. (9), enabling us to employ the orthonormality of the set $\varphi(\lambda)$. From Eq. (40) the integral equation for G_1 is

$$G_1 = G + G(V_N - V_{NP})G_1. \tag{41}$$

Substituting Eq. (41) in Eq. (39), the scattering amplitude $A(\mathbf{n})$ is seen to be

$$\begin{aligned} A(\mathbf{n}) = &-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_N d\mathbf{r}_P e^{-i\mathbf{k}\cdot\mathbf{r}_P}\varphi^*(\mathbf{r}_N,\lambda_f)V_N\psi_D \\ &-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_N d\mathbf{r}_P d\mathbf{r}_N' d\mathbf{r}_P' e^{-i\mathbf{k}\cdot\mathbf{r}_P} \\ &\times\varphi^*(r_N,\lambda_f)[V_N(r_N)-V_{NP}(\mathbf{r}_P-\mathbf{r}_N)] \\ &\times G_1(\mathbf{r}_N,\mathbf{r}_N';\mathbf{r}_P,\mathbf{r}_P')V_N(r_N')\psi_D(\mathbf{r}_N',\mathbf{r}_P'). \end{aligned} \tag{42}$$

Equation (42) can be approximated by ignoring the term in $G_1V_N\psi_D = \psi_D - \Psi$, Eq. (39), which vanishes in the Born approximation $\Psi = \psi_D$. In first approximation therefore $A(\mathbf{n})$ of Eq. (42) is identical with the Born approximation to Eq. (19) with $V_P = 0$. Other equally reasonable ways of estimating Eq. (42) lead to the same conclusion.

The above discussion demonstrates that a variety of different approaches can lead to angular distributions resembling Butler's. This helps to make understandable the success of this theory in accounting for observed angular distributions. As a corollary, the success of Butler's theory with presently available data does not strongly support his particular model.

We consider the physics of the (d,p) reaction still somewhat obscure, and until this is elucidated we see no good reason why Butler's original formula Eq. (32) should be superior to, say, the Born approximation in Eq. (19) or to Eq. (42) including the second correction term.

We add that it seems possible to carry through the calculations of this paper including spin without making the approximation that the nucleus is a center of force. By this means we would arrive at the selection rules,[1] but would not otherwise add enough to the simpler theory we have presented to warrant the extra formal complications. The chief desideratum of a more careful discussion would be to arrive at an improved estimate of the magnitude of the cross section,[1,2,4,6] but this we are not yet prepared to do.

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The Correction for Finite Angular Resolution in Directional Correlation Measurements*

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The correction of a measured directional correlation function for the finite angular resolution of the radiation detectors (scintillation counters) has been investigated experimentally in the case of the Ni^{60} $\gamma-\gamma$ cascade. The results show that the correction factors depend upon the pulse-height selection. The significance of this result for precision measurements is discussed.

INTRODUCTION

RECENT developments in the measurement of directional correlations, such as the investigation of mixed multipole transitions and the influence of external fields, have revealed the need for higher precision. Because the obtainable accuracy is in most cases limited by statistical errors, one tries to increase the number of measured events (coincidences) by using radiation detectors with high counting efficiency and large solid angles. The measured data then have to be corrected for all deviations from an ideal arrangement,[1] especially for the finite angular resolution of the radiation detectors (*solid angle correction*).

The present paper is confined to the discussion of this correction and its experimental investigation. The work originated from a precision measurement of the directional correlation of the Ni^{60} $\gamma-\gamma$ cascade.[2] We found there that the measured directional correlation function depended strongly on the settings of the pulse-height discriminators in the counting system. In order to explain this result, we assumed tentatively that the effective solid angle depends on the pulse-height selection and started measuring directly the angular resolution curve of the radiation detectors. Once the effective angular resolution curves were known, the calculation of the correction (for our very small source) was straightforward. It showed that the discrepancy actually was due to different solid angles. This result proves that a solid angle correction without experimental de-

* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
[1] H. Frauenfelder, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1953), Vol. 2, p. 145.

[2] Steffen, Lawson, Frauenfelder, and Jentschke (to be published).

termination of the angular resolution of the counters for the radiation to be investigated may lead to serious errors.

We give in the present paper first the main formulas for the solid angle correction and then proceed to the description and discussion of our experimental results.

FORMULAS FOR THE SOLID ANGLE CORRECTION

The solid angle correction has been treated in various approximations by Walter,[3,4] Frankel,[5,6] Church and Kraushaar,[7] and Rose.[8] The papers of these authors provide a sufficient theoretical background for the treatment of almost all experimental arrangements. We use in slightly modified form the results of Frankel and Rose. The complete derivation of the formulas may be found in their papers.

We assume that we are measuring the directional correlation $W(\theta)$ of successive γ rays, γ_1 and γ_2. The experimental arrangement (Fig. 1) shall consist of a point source in the center of the counting system, and of two circular (cylindrically symmetric) scintillation counters I and II.

The axes of the two counters subtend the angle ϑ at the source, the directions of emission of the two γ rays include the angle θ. The efficiency of a counter for a γ ray emitted at an angle α with respect to the counter axis shall be denoted by $\epsilon(\alpha)$. The efficiency $\epsilon(\alpha)$ is clearly also a function of the source-to-counter distance. This distance has to be kept the same for the measurement of the directional correlation and of the angular resolution.

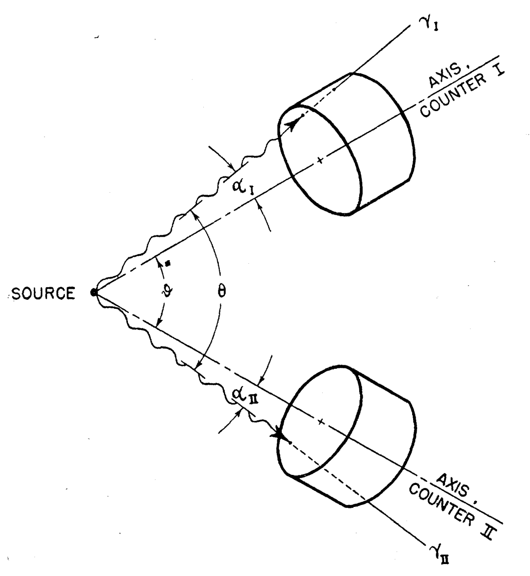

FIG. 1. Geometry of the detecting system for the measurement of the directional correlation between γ_1 and γ_2.

[3] Walter, Huber, and Zünti, Helv. Phys. Acta **23**, 697 (1950).
[4] M. Walter, formula given in reference 1, page 148.
[5] S. Frankel, Phys. Rev. **83**, 673 (1951).
[6] S. Frankel (to be published).
[7] E. L. Church and J. J. Kraushaar, Phys. Rev. **88**, 419 (1952).
[8] M. E. Rose (to be published).

The theoretical directional correlation function $W(\theta)$ may be written as an expansion in Legendre polynomials

$$W(\theta)=1+\sum_{k=1}^{k_m} A_{2k}P_{2k}(\cos\theta). \tag{1}$$

It was first shown by Frankel[5] that the experimentally measured ("smeared out") directional correlation $\langle W(\vartheta)\rangle_{\text{Av}}$, where ϑ denotes the angle between the counter axes, then can be written as

$$\langle W(\vartheta)\rangle_{\text{Av}}=Q_0+\sum_{k=1}^{k_m} Q_{2k}A_{2k}P_{2k}(\cos\vartheta). \tag{2}$$

The coefficients A_{2k} of the theoretical function simply are multiplied by appropriate attenuation factors Q_{2k}. This procedure is valid, however, only for centered point sources and circular detectors. As has been shown by Walter,[3,4] a mixing of the different A_{2k} occurs if these conditions are not met.

For γ rays of nearly the same energies [i.e., $\epsilon(\alpha,\gamma_1)\cong\epsilon(\alpha,\gamma_2)$], the attenuation factors can be written as

$$Q_{2k}=J_{2k}{}^{I}J_{2k}{}^{II}, \tag{3}$$

with

$$J_{2k}=\int P_{2k}(\cos\alpha)\epsilon(\alpha)|\sin\alpha|d\alpha, \tag{4}$$

and where the integrals $J_{2k}{}^{I}$ and $J_{2k}{}^{II}$ are evaluated, respectively, for the first and second counter.

In the case of essentially different γ-ray energies, expression (3) has to be replaced by

$$Q_{2k}=\tfrac{1}{2}\{J_{2k}{}^{I}(\gamma_1)J_{2k}{}^{II}(\gamma_2)+J_{2k}{}^{I}(\gamma_2)J_{2k}{}^{II}(\gamma_1)\}. \tag{5}$$

The procedure of correcting a measured correlation is now straightforward.

(a) The angular efficiencies $\epsilon^{I}(\alpha)$ and $\epsilon^{II}(\alpha)$ of the counters have to be measured by means of a well-collimated beam of γ rays of the substance to be investigated. If γ_1 and γ_2 differ considerably in energy, two different γ sources with suitable energies have to be used to determine $\epsilon(\alpha,\gamma_1)$ and $\epsilon(\alpha,\gamma_2)$.

(b) The correction factors Q_{2k} are obtained by numerically carrying out the integrations (4), using the experimentally determined angular efficiencies.

(c) A least square fit of the experimentally measured correlation $\langle W(\vartheta)\rangle_{\text{Av}}$ using the function

$$\langle W(\vartheta)\rangle_{\text{Av}}=\text{const}\{1+A_2{}^*P_2(\cos\vartheta)+A_4{}^*P_4(\cos\vartheta)\} \tag{6}$$

determines the coefficients $A_2{}^*$ and $A_4{}^*$. Higher terms are usually not necessary. An exhaustive treatment of the method of obtaining a least square fit has been given by Rose.[8]

(d) The corrected coefficients are then given by

$$A_{2k}=(Q_0/Q_{2k})A_{2k}{}^*. \tag{7}$$

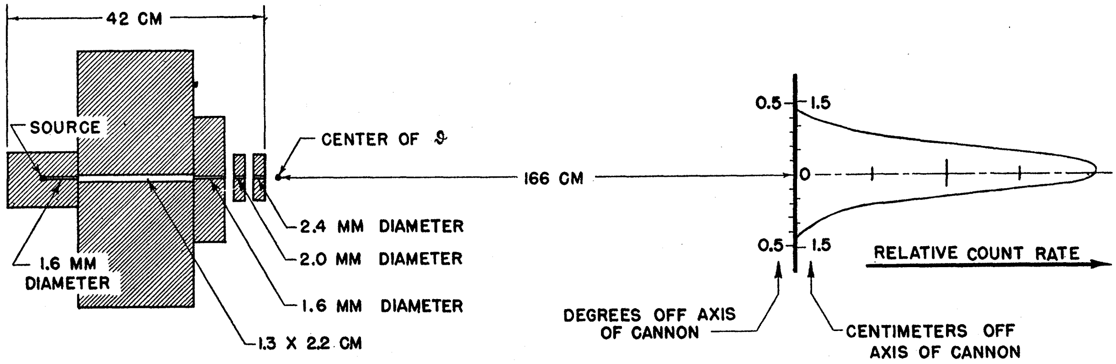

FIG. 2. Collimating system and shape of the γ-ray beam used for the determination of the angular resolution of the scintillation counters.

If the energies of the γ rays γ_1 and γ_2 are not very different from the energy of annihilation radiation, the method of Church and Kraushaar[7] can be used. The angular resolution $F(\alpha)$ of the counter system for a source emitting annihilation radiation has to be measured. Q_{2k} is then directly given by the expression (4) with $\epsilon(\alpha)$ replaced by $F(\alpha)$ and not by the product of two such integrals.

These methods hold only for centered point sources. The correction becomes much more complicated for extended sources. No general formula has yet been developed. But a useful estimate of the correction can be obtained by generalizing Walter's treatment.[4] Despite the fact that we do not use this generalization in the present work (because our source was extremely small), we will outline its derivation.

Walter has carried out his calculations for the case of a linear source (length L, source-to-counter distance D), placed symmetrically on the axis of rotation of the counter system, with counters of constant efficiency over their whole area. For circular detectors with solid angles $\Omega=2\pi\omega$ (steradian), Walter's correction formula can be transformed into the following form:

$$\begin{aligned}\text{const}\times\langle W(\vartheta)\rangle_{\text{Av}}=1&+(1-3\omega)A_2P_2(\cos\vartheta)\\&+(1-10\omega)A_4P_4(\cos\vartheta)+\tfrac{1}{4}(L/D)^2[1-P_1(\cos\vartheta)]\\&\times\{A_2P_1(\cos\vartheta)+A_4[P_1(\cos\vartheta)+(7/3)P_3(\cos\vartheta)]\}.\quad(8)\end{aligned}$$

This expression, which is valid for $\omega\ll1$ and $(L/D)^2\ll1$, shows that the corrections for solid angle and for source length then occur independently. A comparison of (8) and (2) shows immediately that Walter's formula can be extended to account for variable counter efficiencies $\epsilon(\alpha)$ by replacing $(1-3\omega)$ by Q_2/Q_0, and $(1-10\omega)$ by Q_4/Q_0. In this form, Eq. (8) may be useful for approximately linear sources.

EXPERIMENTAL RESULTS AND DISCUSSION

As we have mentioned in the introduction, the experimental investigation of the solid angle correction originated through the results of our measurements of the Ni^{60} $\gamma-\gamma$ correlation.[2] In this work, the source consisted of a small piece of pile-irradiated metallic cobalt, with linear dimensions smaller than 0.05 cm. The condition of having a point source was therefore well fulfilled. NaI scintillation counters were used as radiation detectors. The NaI(Tl) crystals were cut in the form of right circular cylinders (diameter 3.5 cm, length 4.0 cm) and immersed in mineral oil in an aluminum pot. The pot was closed airtight by means of a glass plate. The glass was shaped so as to give good optical contact between the scintillation crystal and the surface of the photomultiplier (5819). The counters were laterally shielded against scattered γ rays by lead (minimum thickness 1.5 cm). The discrimination against counter-to-counter scattering at angles near $\vartheta=180°$ was achieved by two different methods: by using heavy front lead shields (*abs*) or by accepting only pulses corresponding to γ rays of at least 0.55 Mev (*disc*). The characteristics of the two methods are given in Table I.

It was found that the measured anisotropy A^* $(A^*=\langle W(180°)\rangle_{\text{Av}}/\langle W(90°)\rangle_{\text{Av}}-1)$ was markedly different in the two cases *abs* and *disc*. Additional experiments showed that the difference could not be attributed to scattered γ radiation. We then assumed that the two cases *abs* and *disc* had different effective solid angles and measured the angular resolution curves $\epsilon(\alpha)$.

The two γ-rays of Ni^{60} have nearly the same energy $[E(\gamma_1)=1.33$ Mev, $E(\gamma_2)=1.17$ Mev$]$, so that the application of the simple formula (3) instead of (5) is justified. Therefore we used a well-collimated beam of Ni^{60} γ rays to determine the angular efficiency curves $\epsilon^I(\alpha)$ and $\epsilon^{II}(\alpha)$. The shape of the beam was measured in turn with a small scintillation crystal, (0.2-cm width) at a distance of 166 cm from the center of the counter

TABLE I. Characteristics of the two methods for eliminating scattered γ rays.

Method	Front absorber	Discriminator level. Pulses accepted above
abs	0.2 cm Al, 0.7 cm Pb	0.15 Mev
disc (0.55 Mev)	0.2 cm Al	0.55 Mev

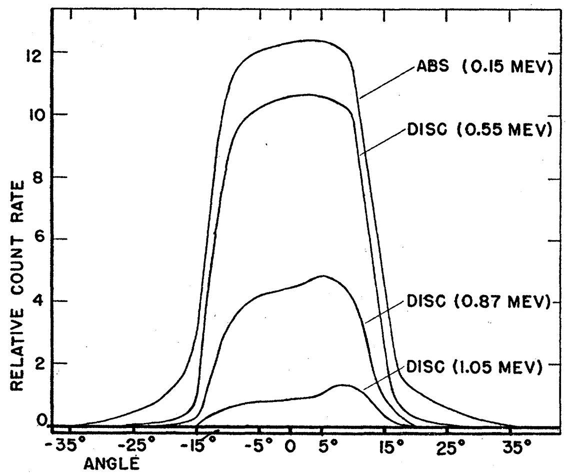

FIG. 3. The experimentally determined angular efficiency curves $\epsilon(\alpha)$ for discriminator levels of 0.15, 0.55, 0.87, and 1.05 Mev. For the measurement of the curve denoted with *abs* (0.15 Mev), a lead front absorber of 0.7-cm thickness was used.

system. Beam shape and collimating system are shown in Fig. 2. The γ-ray beam was found to have a total width of less than 0.8°, so that no correction of the measured angular resolution curves $\epsilon(\alpha)$ for its shape was necessary. Resolution curves were determined for the two different cases *abs* and *disc* of Table I, and for two arrangements with different distances between source and counters. (The two arrangements with different source-to-counter distances shall be characterized by the width α_0 of the efficiency curve at half-maximum.) In order to investigate further the effect of the discriminator level, we measured the resolution curve for two additional level settings, corresponding to energies of 0.87 and 1.05 Mev. Figure 3 represents the resulting curves for counter *I* and $\alpha_0 = 25°$.

The different shapes of the curves in Fig. 3 are caused by differences in fluorescence efficiency and light collection from different parts of the crystal and by low-energy γ rays, scattered into the crystal by the lead absorber around the counter.

The measured angular resolution curves, of which Fig. 3 represents a typical example, now show clearly that the correction factors Q_{2k} will depend upon the lead shielding around the counters, upon the pulse-height selection, and upon the light collection. We have calculated the factors Q_0, Q_2, and Q_4, by numerical integration for the various cases used in our investigation. The results are shown in Table II.

TABLE II. Solid angle correction factors, determined by numerical integration of the measured angular resolution curves.

	Method	(Q_2/Q_0)	(Q_4/Q_0)
14°	*abs* (0.15 Mev)	0.96	0.88
	disc (0.55 Mev)	0.97	0.91
25°	*abs* (0.15 Mev)	0.847	0.577
	disc (0.55 Mev)	0.908	0.721
25°	*disc* (0.87 Mev)	0.914	0.740
	disc (1.05 Mev)	0.912	0.729

In order to test the assumption of cylindrically symmetric counter efficiency, we rotated the crystal of counter I by 90° and remeasured the angular resolution for the case *abs* (0.15 Mev). The curve shape changed slightly, but the correction factors remained unaltered. It should, however, be noted from Fig. 3 that the curves are no longer symmetric at high discriminator levels. The formulas (2)–(5) are then only approximately valid.

The directional correlation of the Ni^{60} $\gamma-\gamma$ cascade has been measured for four of the six cases listed in Table II. The experimental data and the resulting anisotropy values corrected by means of the factors from Table II are given in Table III. All errors are mean statistical deviations; they do not take into account the errors introduced through the solid angle correction.

TABLE III. Measured and corrected values of the anisotropy of the Ni^{60} $\gamma-\gamma$ cascade.

	Method	A^*	A	Theoretical value
14°	*abs* (0.15 Mev)	0.159	0.166±0.002	
	disc (0.55 Mev)	0.163	0.168±0.003	
25°	*abs* (0.15 Mev)	0.139	0.167±0.001	
	disc (0.55 Mev)	0.150	0.167±0.001	
	Weighted average	···	0.167	0.1667

The main results of the present investigation are contained in Fig. 3 and in Tables II and III, and can be summarized as follows.

The precise correction of a measured directional correlation function for the effect of the finite solid angle of the detectors can best be achieved by using experimentally determined angular resolution curves. Other correction methods, using annihilation radiation or calculated efficiencies, are useful as approximations or for small solid angles. For precise measurements with large solid angles, however, they may lead to errors, unless it is certain that all underlying assumptions are fulfilled.

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