Theory of (d,p) and (d,n) Reactions*

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The theory of angular distributions in (d,p) and (d,n) reactions is developed by means of standard Green's function techniques, thereby yielding a straightforward derivation of a formula originally deduced by Butler. To minimize formal complications the particles are assumed spinless, and the nucleus replaced by a center of force. It is shown that Butler's theory involves an approximation equivalent to Born approximation, which elucidates the agreement between the Butler and Born approximation derivations of the stripping formula. A discussion relating to the success of the theory is appended.

I. INTRODUCTION

THE theory of (d,p) and (d,n) reactions given by Butler[1] has been the subject of a number of theoretical papers.[2–6] Butler's original deduction of the angular distribution in stripping involved fitting together at the nuclear radius the solutions interior and exterior to the nucleus; it is fair to call complicated the procedure by which Butler obtained the cross section from his solution. Succeeding theoretical studies have been of two kinds: (*a*) attempts to simplify and clarify Butler's calculation of the cross section, but retaining his basic idea of fitting together the interior and exterior solutions[4,6] and (*b*) assuming the Born approximation matrix element for the reaction after which Butler's formula is obtained by more or less direct integration.[2,3] Since Butler's calculation does not seem equivalent to the Born approximation, it is somewhat surprising that the Born approximation gives Butler's result.[3] Austern[5] has attempted to explain this agreement.

In subsequent sections we shall rederive Butler's result by means of standard Green's function techniques, thereby automatically and obviously satisfying the boundary conditions at infinity and at the nuclear radius. To minimize formal complications we consider the following idealization of the stripping problem: A deuteron, spinless, composed of spinless neutron and proton, impinges on a fixed center of force which is the initial nucleus.[3] At infinity the solution Ψ must be of the form

$$\Psi=\psi_D+\Phi, \tag{1}$$

where ψ_D is the incident plane wave of deuterons on the initial nucleus, and Φ is everywhere outgoing. In the problem at hand this means: let the energy have a positive imaginary part; then Φ is everywhere outgoing if it remains bounded as r_N or r_P or both approach infinity. We have been careful to obtain the cross section by a mathematical procedure which corresponds evidently to the experimental situation. To amplify this remark, denote the wave function of the final nucleus in a (d,p) reaction, in which the neutron is captured into a bound state, by $\varphi(\mathbf{r}_N)$. Then the probability of finding the proton at $\mathbf{r}_P$, with the neutron bound in its final state, is $|\int d\mathbf{r}_N\varphi^*(\mathbf{r}_N)\Psi(\mathbf{r}_N,\mathbf{r}_P)|^2$. The experiment measures the flux at infinity of protons whose energy corresponds to leaving the neutron in state $\varphi(\mathbf{r}_N)$, which flux per unit solid angle is $(\hbar k/M)|A(\mathbf{n})|^2$, where the scattering amplitude $A(\mathbf{n})$ in the direction $\mathbf{n}$ is given by

$$A(\mathbf{n})\frac{e^{ikr_P}}{r_P}=\lim_{r_P\to\infty}\int d\mathbf{r}_N\varphi^*\Psi, \tag{2}$$

and r_P approaches infinity along $\mathbf{n}$. We always employ the definition Eq. (2) of $A(\mathbf{n})$ to evaluate the cross section.

Using Eq. (2) in the integral equation for the problem leads in a very straightforward way to the Born approximation matrix element, for which no satisfactory justification has been given previously. In so doing we illuminate the reason for the agreement between the two seemingly different methods (a) and (b) above. Our integral equation is the same as that obtained by Austern,[5] but his not using the definition (2) for the scattering amplitude caused him to overlook the fact that not all the terms in his equation yield protons at infinity, in (d,p) reactions. This statement will be further amplified below. Finally we append some discussion concerning the success of the theory.

II. THE INTEGRAL EQUATION

We fix our attention on (d,p) reactions, i.e., we seek outgoing protons whose energy corresponds to leaving the neutron bound to the center of force. The Hamiltonian is

$$H=T_N+T_P+V_P+V_N+V_{NP}, \tag{3}$$

where T represents kinetic energy, V_N and V_P are the interaction of neutron and proton, respectively, with the fixed center of force, and V_{NP} is the neutron proton

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[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).
[2] A. B. Bhatia *et al.*, Phil. Mag. **43**, 485 (1952).
[3] P. B. Daitch and J. A. French, Phys. Rev. **87**, 900 (1952).
[4] R. Huby, Proc. Roy. Soc. (London) **A215**, 385 (1952).
[5] N. Austern, Phys. Rev. **89**, 318 (1953).
[6] F. Friedman and W. Tobocman (to be published).

interaction.[7] The solution satisfies

$$(H-E)\Psi=0, \tag{4}$$

with Ψ of the form given in Eq. (1) and

$$\psi_D=e^{i\mathbf{K}\cdot(\mathbf{r}_P+\mathbf{r}_N)/2}w(\mathbf{r}_P-\mathbf{r}_N). \tag{5}$$

w is the ground state of the deuteron, and ψ_D satisfies

$$(T_N+T_P+V_{NP}-E)\psi_D=0. \tag{6}$$

Using an obvious symbolic notation, the solution satisfies the integral equation:

$$\Psi=\psi_0-G(V_P+V_{NP})\Psi, \tag{7}$$

where

$$(T_N+T_P+V_N-E)\psi_0=0, \tag{8}$$

$$(T_N+T_P+V_N-E)G=1=\delta(\mathbf{r}_P-\mathbf{r}_P')\delta(\mathbf{r}_N-\mathbf{r}_N'). \tag{9}$$

The solution to Eq. (7) satisfies the boundary conditions at the nuclear radius, and satisfies the boundary condition at infinity if G is the outgoing Green's function, which is [8]

$$G(\mathbf{r}_P,\mathbf{r}_P';\mathbf{r}_N,\mathbf{r}_N') =\sum\nolimits_\lambda g(E-\lambda)\varphi(\mathbf{r}_N,\lambda)\varphi^*(\mathbf{r}_N',\lambda). \tag{10}$$

In Eq. (10) the sum over λ includes an integration in the continuum $\lambda>0$. $\varphi(\lambda)$ are the complete set of eigenfunctions of the neutrons in the field of the initial nucleus:

$$(T_N+V_N-\lambda)\varphi(\lambda)=0. \tag{11}$$

$g(E-\lambda)$ is the outgoing free space Green's function for the proton,[9] i.e.,

$$(T_P-E+\lambda)g(E-\lambda)=\delta(\mathbf{r}_P-\mathbf{r}_P'), \tag{12}$$

$$g(\mathbf{r}_P-\mathbf{r}_P')=\frac{1}{4\pi}\frac{2M}{\hbar^2}\frac{\exp[i(E-\lambda)^{\frac{1}{2}}|r_P-r_P'|]}{|\mathbf{r}_P-\mathbf{r}_P'|}. \tag{13}$$

In Eq. (13), M is the mass of proton or neutron and $\mathrm{Re}(E-\lambda)^{\frac{1}{2}}>0$ when E is imaginary.

[7] The discussion and notation of this section parallels that in E. Gerjuoy, University of Pittsburgh Precision Scattering Project Report No. 3 (unpublished).

[8] It is apparent that G, given in Eq. (10), satisfies Eq. (9) and is outgoing in the protons, in the sense which has been explained in the previous section. It is possible to prove that G is also outgoing in the neutrons, despite the fact that it seems to contain, through $\varphi(\mathbf{r}_N)$, both incoming and outgoing spherical waves in $\mathbf{r}_N$. It must be granted that some mathematical questions concerning the proof are not altogether settled, but its essential correctness seems established. The proof is contained in a report in preparation by B. Friedman and E. Gerjuoy, on the subject of many-particle scattering problems. Related problems are discussed in B. Friedman and E. Gerjuoy, Research Report No. CX-4 (unpublished) and in Harry E. Moses, Research Report No. CX-5 (unpublished) both issued by New York University, Washington Square College of Arts and Science Mathematics Research Group.

[9] In order that Eqs. (10) and (12) yield a convergent result in Eq. (7), Coulomb forces must be neglected or replaced by screened fields. We are also ignoring some formal difficulties connected with the fact that V_{NP} is a function of $\mathbf{r}_P-\mathbf{r}_N$ only, and does not approach zero along all radii of an infinite sphere in the six-dimensional $\mathbf{r}_N$, $\mathbf{r}_P$ space.

Equations (1) and (7) imply that the difference between ψ_0 and ψ_D is everywhere outgoing. Rewriting Eq. (6) as

$$(T_N+T_P+V_N-E)\psi_D=(V_N-V_{NP})\psi_D, \tag{14}$$

it follows that[7]

$$\psi_D=\psi_0+G(V_N-V_{NP})\psi_D, \tag{15}$$

so that Eq. (7) becomes

$$\Psi=\psi_D-G(V_N-V_{NP})\psi_D-G(V_P+V_{NP})\Psi. \tag{16}$$

Equation (16) is identical with Austern's integral equation.[5]

Since V_{NP} does not appear in Eq. (8), ψ_0 may be said to represent a combination of free space proton functions and of neutron functions $\varphi(\mathbf{r}_N,\lambda)$, which at infinity looks like an incoming plane wave of free deuterons, but in which the neutrons and protons propagate independently of each other. In order that a neutron be captured it is necessary that the proton remove the excess neutron energy. But Eq. (8) contains no neutron-proton coupling. Consequently it is to be expected that ψ_0 makes a vanishing contribution to the scattering amplitude, in (d,p) reactions.

This plausible argument can be made rigorous. Substitute Eq. (16) in Eq. (2). Using Eq. (5) it is seen that the term in ψ_D vanishes exponentially as $r_P\to\infty$, since both φ and w are bound states. The terms in G simplify with the aid of Eq. (10) and the orthonormality of the set $\varphi(\lambda)$. Letting $r_P\to\infty$, there results

$$A(\mathbf{n})=A_1(\mathbf{n})+A_2(\mathbf{n}), \tag{17}$$

$$A_1(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_Pd\mathbf{r}_Ne^{-i\mathbf{k}\cdot\mathbf{r}_P}\times\varphi^*(\mathbf{r}_N,\lambda_f)(V_N-V_{NP})\psi_D, \tag{18}$$

$$A_2(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_Pd\mathbf{r}_Ne^{-i\mathbf{k}\cdot\mathbf{r}_P}\times\varphi^*(\mathbf{r}_N,\lambda_f)(V_P+V_{NP})\Psi. \tag{19}$$

In Eqs. (18) and (19), λ_f is the energy of the neutron in its final bound state, $\mathbf{k}=k\mathbf{n}$, and

$$\frac{\hbar^2k^2}{2M}+\lambda_f=E=\frac{\hbar^2K^2}{4M}+\epsilon, \tag{20}$$

where, referring to Eq. (5), ϵ is the energy of the deuteron in its ground state.

It can be shown that[7]

$$A_1(\mathbf{n})=0. \tag{21}$$

The demonstration is trivial, involves merely elimination of V_{NP} and V_N by Eqs. (6) and (11), followed by an integration by parts. Equation (21) remains valid when the particles are not assumed spinless, and when w is not spherically symmetric, i.e., when V_{NP} is noncentral and spin-dependent.[7]

Equation (21) means that, as asserted previously, $\psi_0=\psi_D-G(V_N-V_{NP})\psi_D$ makes no contribution to the scattering amplitude. In Born approximation we replace Ψ by ψ_D in Eq. (19). In this approximation, by virtue of Eq. (18), we may replace V_{NP} by V_N, without additional error. This yields the starting point[2,3] for the Born approximation deductions of the (d,p) angular distribution. Our derivation may be compared with that of Austern.[5] It will be noted that the valid use[2,3] of the Born approximation matrix element depends on the plausible but not obvious circumstance that ψ_0 makes no contribution to the scattering amplitude.

III. BUTLER'S THEORY

Equation (16) remains valid whatever the forms of V_P, V_N and V_{NP}, subject to the remark in reference 9. The special assumptions made by Butler[1] may, for (d,p) reactions, be summarized as follows:[4] (a) $V_P=0$; (b) within the nucleus, $r_N<a$, neglect V_{NP} in the Schrödinger equation for Ψ; (c) exterior to the nucleus, $r_N>a$, assume $V_N=0$; (d) for $r_N>a$, Ψ is of the form given in Eq. (1), where Φ is composed of free particles only, i.e., Φ satisfies, when $r_N>a$,

$$(T_N+T_P-E)\Phi=0. \tag{22}$$

We shall see that assumptions (c) and (d) are not entirely consistent. Assumptions (a), (b), and (c) mean the Schrödinger equation for the problem is

$$r_N<a: \qquad (T_N+T_P+V_N-E)\Psi=0, \tag{23a}$$

$$r_N>a: \qquad (T_N+T_P+V_{NP}-E)\Psi=0. \tag{23b}$$

Equations (23a) and (23b) may be expressed in the form, valid for all $\mathbf{r}_N$, $\mathbf{r}_P$:

$$(T_N+T_P+V_N-E)\Psi=P_>(V_N-V_{NP})\Psi, \tag{24}$$

where we have introduced the projection operator $P_>$:

$$P_>=0, \quad r_N<a; \quad P_>=1, \quad r_N>a. \tag{25}$$

The integral equation equivalent to Eq. (24) is

$$\Psi=\psi_0+GP_>(V_N-V_{NP})\Psi=\psi_0-GP_>V_{NP}\Psi. \tag{26}$$

since $V_N=0$ for $r_N>a$. As in the preceding section Eq. (26) leads to the scattering amplitude:

$$A(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_P\int_{r_N>a} d\mathbf{r}_N e^{-i\mathbf{k}\cdot\mathbf{r}_P} \times\varphi^*(\mathbf{r}_N,\lambda_f)V_{NP}\Psi. \tag{27}$$

In Born approximation Ψ is replaced by ψ_D in Eq. (27). Since the right side of Eq. (24) can be interpreted as a source term, the Born approximation in the theory of this section, i.e., the Born approximation in Eq. (24), amounts to neglecting as a source of scattered proton waves at infinity the term $V_{NP}\Phi$ exterior to the nucleus. But this is exactly the approximation which is implied by Eq. (22), in which $V_{NP}\Phi$ is neglected for $r_N>a$. Consequently, the fact that Butler's solution[1] is equivalent to the Born approximation theory of this section is no longer surprising. In fact Eq. (22) implies, using Eqs. (1) and (6),

$$r_N>a: \quad (T_N+T_P-E)\Psi=(T_N+T_P-E)\psi_D = -V_{NP}\psi_D. \tag{28}$$

Equation (28) may be rewritten as

$$r_N>a: \quad (T_N+T_P+V_{NP}-E)\Psi = V_{NP}(\Psi-\psi_D)=V_{NP}\Phi. \tag{29}$$

Equation (29) is not identical with Eq. (23b).

It has been Eqs. (23a) and (28) whose solutions have been fitted at the nuclear radius,[1,4,6] not Eqs. (23a) and (23b). Recalling $V_N=0$ for $r_N>a$, Eqs. (23a) and (28) are equivalent to

$$(T_N+T_P+V_N-E)\Psi=-P_>V_{NP}\psi_D. \tag{30}$$

Equation (30) is an inhomogeneous differential equation, whose solution, satisfying the boundary conditions, is

$$\Psi=\psi_0-GP_>V_{NP}\psi_D. \tag{31}$$

Equation (31) is presumably the solution which is obtained by fitting the exterior and interior solutions at the nuclear radius. Comparing Eqs. (26), (27), and (31), it is at once seen that the scattering amplitude in Butler's theory is the same as the Born approximation in Eq. (27), namely

$$A(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_P\int_{r_N>a} d\mathbf{r}_N e^{-i\mathbf{k}\cdot\mathbf{r}_P} \times\varphi^*(\mathbf{r}_N,\lambda_f)V_{NP}\psi_D. \tag{32}$$

The Born approximation in Eq. (19) reduces of course to Eq. (32) if the additional assumptions involved in obtaining Eq. (27) are included in Eq. (19), namely, $V_P=0$ and V_{NP} neglected for $r_N<a$.

IV. EVALUATION OF SCATTERING AMPLITUDE

Equation (32) must be integrated over the region $r_N>a$, and cannot be replaced by an integral over all space, even though we seemingly derived Eq. (32) by neglecting V_{NP} for $r_N<a$. V_{NP} was neglected for $r_N<a$ in Eq. (23a) only, which combined with Eq. (28) led without further approximation to Eq. (32). In Eq. (6) for ψ_D no such assumption about V_{NP} is made. Including in Eq. (32) the region $r_N<a$ would amount to going back to the Born approximation of Sec. II, but with $V_P=0$.

As a consequence, it is not legitimate, in Eq. (32), to replace V_{NP} by V_N, since this substitution is justified only by Eqs. (18) and (21) in which the integrals are extended over all space. That Eq. (32) involves V_{NP}

rather than V_N is desirable, as it enables us to avoid such difficulties as those of Daitch and French[3] who obtain the angular distribution by neglecting the contribution from $r_N<a$ to the overlap integral between $\varphi^*(\mathbf{r}_N, \lambda_f)$ and a spherical Bessel function. Because $\varphi(\mathbf{r}_N, \lambda_f)$ is a bound state, decreasing exponentially for $r_N>a$, it is hard to justify their approximation.

We proceed now to evaluate $A(\mathbf{n})$ from Eq. (32), to satisfy ourselves that it leads without further assumptions to Butler's angular distribution. The evaluation is straightforward and doubtless can be done in a number of ways. We have found it convenient to introduce in Eq. (32) replacing $\mathbf{r}_P$, the new variable $\mathbf{r}=\mathbf{r}_P-\mathbf{r}_N$, and to make use of Eq. (5) and the fact that V_{NP} is a function of r only. Then

$$A(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}V_{NP}(r)w(r)e^{i\mathbf{r}\cdot(\mathbf{K}/2-\mathbf{k})}\times\int_{r_N>a}d\mathbf{r}_Ne^{i\mathbf{r}_N\cdot(\mathbf{K}-\mathbf{k})}\varphi^*(\mathbf{r}_N,\lambda_f). \quad (33)$$

In the integral over $\mathbf{r}$ in Eq. (33), we use

$$[-(\hbar^2/M)\Delta_r+V_{NP}(r)]w(r)=\epsilon w(r) \quad (34)$$

to eliminate V_{NP}, integrate by parts (justified because w is a bound state), and employ Eq. (20), obtaining

$$\int d\mathbf{r}V_{NP}we^{i\mathbf{r}\cdot(\mathbf{K}/2-\mathbf{k})}=\left[\lambda_f-\frac{\hbar^2}{2M}(\mathbf{K}-\mathbf{k})^2\right]\int d\mathbf{r}we^{i\mathbf{r}\cdot(\mathbf{K}/2-\mathbf{k})}. \quad (35)$$

From Eq. (11), for $r_N>a$, where $V_N=0$, we find

$$\left[\lambda_f-\frac{\hbar^2}{2M}(\mathbf{K}-\mathbf{k})^2\right]e^{i\mathbf{r}_N\cdot(\mathbf{K}-\mathbf{k})}\varphi^*(\mathbf{r}_N,\lambda_f)=-\frac{\hbar^2}{2M}[e^{i\mathbf{r}_N\cdot(\mathbf{K}-\mathbf{k})}\Delta\varphi^*-\varphi^*\Delta e^{i\mathbf{r}_N\cdot(\mathbf{K}-\mathbf{k})}]. \quad (36)$$

Hence, using Green's theorem in the integral over $\mathbf{r}_N$, expanding $\exp[i\mathbf{r}_N\cdot(\mathbf{K}-\mathbf{k})]$ in spherical harmonics with $\mathbf{K}-\mathbf{k}$ as the polar axis, and writing

$$\varphi=R_l(r_N)Y_l^m(\theta_N,\varphi_N),$$

Eqs. (33)–(36) imply

$$A(\mathbf{n})=-i^l\frac{(2l+1)^{\frac{1}{2}}}{(4\pi)^{\frac{1}{2}}}a^2\left[j_l(|\mathbf{K}-\mathbf{k}|r_N)\frac{\partial R_l}{\partial r_N}-R_l\frac{\partial}{\partial r_N}j_l(|\mathbf{K}-\mathbf{k}|r_N)\right]_{r_N=a}\int d\mathbf{r}w(r)e^{i\mathbf{r}\cdot(\mathbf{K}/2-\mathbf{k})}. \quad (37)$$

Equation (37) is valid for $m=0$, with $Y_l^m(\theta_N, \varphi_N)$ quantized along $\mathbf{K}-\mathbf{k}$. The integral vanishes for the other values of the magnetic quantum number. Equation (37) yields Butler's angular distribution, as has been previously[3] pointed out.

V. SUCCESS OF THE THEORY

It appears to be established that Butler's theory accounts for the observed angular distributions in (d,p) reactions. The success of the theory remains surprising, in view of the relatively low energy deuterons which have been used in the experiments. We have seen that Butler's assumption (d), Sec. III, is equivalent to neglecting as a source of proton waves the term $V_{NP}\Phi$ exterior to the nucleus, and that this neglect is identical with the Born approximation. It is well known that the Born approximation is often much better than expected; the theory of angular distributions in (d,p) reactions seems to be another such case, and we offer no explanation.

Assumption (b), Sec. III, led to Eq. (32) being integrated over $r_N>a$ rather than over all space, as in Eq. (19). In a sense neglect of V_{NP} for $r_N<a$ can be thought of as an an impulse approximation, i.e., the neutron-proton forces do not have time to act in the interval that the deuteron overlaps the nucleus. But this interpretation is hard to justify, since V_{NP} is not smaller than V_N, and since the deuterons are slow. In any event it is difficult to see why neglect of V_{NP} for $r_N<a$ should lead to a better result than including it. Integrating over all r_N in Eq. (32) permits V_{NP} to be replaced by V_N, as we have seen, and leads to a modified angular distribution[3] which, however, is generally not very different from Butler's original form. It is doubtful that the available data are accurate enough to choose between the two possibilities: integrating over $r_N>a$ and integrating over all r_N. With better data on selected nuclei in which the differences between the two forms are emphasized, together with comparisons of absolute cross sections with the theory, a decision between the two alternatives may be feasible.

An alternative means (to that in Sec. III) of converting the Schrödinger equation of the problem to an inhomogeneous differential equation is to replace the assumptions (a)–(d) of Sec. III by: (a') $V_P=0$; (b') assume $V_N=0$ for $r_N>a$; (c') for $r_N<a$ Ψ satisfies

$$r_N<a:\quad (T_N+T_P+V_{NP}-E)\Psi=-V_N\psi_D. \quad (38)$$

With these assumptions, V_{NP} is nowhere neglected. If ψ_D is replaced by Ψ on the right side of Eq. (38), we obtain the presumably correct Eq. (4), with of course $V_P=0$. Thus Eq. (38) amounts to neglecting $V_N\Phi$ as a source term, in the region $r_N<a$.

Without going into as many details as previously, assumptions (a')–(c') imply the solution Ψ is

$$\Psi=\psi_D-G_1V_N\psi_D, \quad (39)$$

where G_1 is the outgoing Green's function satisfying

$$(T_N+T_P+V_{NP}-E)G_1=1. \quad (40)$$

To determine the scattering amplitude in closed form we must express G_1 in terms of G, Eq. (9), enabling us to employ the orthonormality of the set $\varphi(\lambda)$. From Eq. (40) the integral equation for G_1 is

$$G_1=G+G(V_N-V_{NP})G_1. \quad (41)$$

Substituting Eq. (41) in Eq. (39), the scattering amplitude $A(\mathbf{n})$ is seen to be

$$A(\mathbf{n})=-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_N d\mathbf{r}_P e^{-i\mathbf{k}\cdot\mathbf{r}_P}\varphi^*(\mathbf{r}_N,\lambda_f)V_N\psi_D$$
$$-\frac{1}{4\pi}\frac{2M}{\hbar^2}\int d\mathbf{r}_N d\mathbf{r}_P d\mathbf{r}_N' d\mathbf{r}_P' e^{-i\mathbf{k}\cdot\mathbf{r}_P}$$
$$\times\varphi^*(r_N,\lambda_f)[V_N(r_N)-V_{NP}(\mathbf{r}_P-\mathbf{r}_N)]$$
$$\times G_1(\mathbf{r}_N,\mathbf{r}_N';\mathbf{r}_P,\mathbf{r}_P')V_N(r_N')\psi_D(\mathbf{r}_N',\mathbf{r}_P'). \quad (42)$$

Equation (42) can be approximated by ignoring the term in $G_1V_N\psi_D=\psi_D-\Psi$, Eq. (39), which vanishes in the Born approximation $\Psi=\psi_D$. In first approximation therefore $A(\mathbf{n})$ of Eq. (42) is identical with the Born approximation to Eq. (19) with $V_P=0$. Other equally reasonable ways of estimating Eq. (42) lead to the same conclusion.

The above discussion demonstrates that a variety of different approaches can lead to angular distributions resembling Butler's. This helps to make understandable the success of this theory in accounting for observed angular distributions. As a corollary, the success of Butler's theory with presently available data does not strongly support his particular model.

We consider the physics of the (d,p) reaction still somewhat obscure, and until this is elucidated we see no good reason why Butler's original formula Eq. (32) should be superior to, say, the Born approximation in Eq. (19) or to Eq. (42) including the second correction term.

We add that it seems possible to carry through the calculations of this paper including spin without making the approximation that the nucleus is a center of force. By this means we would arrive at the selection rules,[1] but would not otherwise add enough to the simpler theory we have presented to warrant the extra formal complications. The chief desideratum of a more careful discussion would be to arrive at an improved estimate of the magnitude of the cross section,[1,2,4,6] but this we are not yet prepared to do.

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The Correction for Finite Angular Resolution in Directional Correlation Measurements*

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The correction of a measured directional correlation function for the finite angular resolution of the radiation detectors (scintillation counters) has been investigated experimentally in the case of the Ni^{60} $\gamma-\gamma$ cascade. The results show that the correction factors depend upon the pulse-height selection. The significance of this result for precision measurements is discussed.

INTRODUCTION

RECENT developments in the measurement of directional correlations, such as the investigation of mixed multipole transitions and the influence of external fields, have revealed the need for higher precision. Because the obtainable accuracy is in most cases limited by statistical errors, one tries to increase the number of measured events (coincidences) by using radiation detectors with high counting efficiency and large solid angles. The measured data then have to be corrected for all deviations from an ideal arrangement,[1] especially for the finite angular resolution of the radiation detectors (*solid angle correction*).

The present paper is confined to the discussion of this correction and its experimental investigation. The work originated from a precision measurement of the directional correlation of the Ni^{60} $\gamma-\gamma$ cascade.[2] We found there that the measured directional correlation function depended strongly on the settings of the pulse-height discriminators in the counting system. In order to explain this result, we assumed tentatively that the effective solid angle depends on the pulse-height selection and started measuring directly the angular resolution curve of the radiation detectors. Once the effective angular resolution curves were known, the calculation of the correction (for our very small source) was straightforward. It showed that the discrepancy actually was due to different solid angles. This result proves that a solid angle correction without experimental de-

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[1] H. Frauenfelder, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1953), Vol. 2, p. 145.

[2] Steffen, Lawson, Frauenfelder, and Jentschke (to be published).