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Diffusion Currents in the Semiconductor Hall Effect

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The usual Hall effect equation for semiconductors applies if the concentration of carriers is undisturbed. In a magnetic field, however, holes and electrons are swept to the same side of the conductor. A gradient of carrier concentrations is thereby established, giving rise to diffusion currents across the sample. The magnitude of the diffusion currents depends on the recombination velocity at the surface and on the lifetime of excess carriers in the interior.

Partial differential equations governing the electric field and the variation of the carrier concentrations across the transverse cross section of the Hall sample are obtained. These are applied to rectangular and circular transverse cross sections. Equations for the Hall voltage as a function of both bulk lifetime and surface recombination velocity are derived and discussed. The analysis considers only disturbances which are linear in the applied magnetic field.

I. INTRODUCTION

IN semiconductors two current carriers are present in variable proportions: electrons having energies in the conduction band, acting as negative carriers; and vacant states or "holes" in the valence band, acting as positive carriers. When a current flows, electrons and holes move in opposite directions; however, a magnetic field tends to deflect the oppositely moving and oppositely charged carriers in the same direction. A counteracting electric field, such as is usually set up in the presence of a magnetic field by an initial movement of charge, cannot simultaneously balance the magnetic deflections of both carriers in view of the opposite forces exerted by the electric field on the two particles. Consequently, in the steady state, holes and electrons will flow in equal numbers in the direction of magnetic deflection. The equality of the two currents is required in order that the net transverse current be zero, in accordance with the principle of charge conservation. The Hall field is then that electric field which insures the necessary equality of transverse hole and electron flow.

Since the currents for the individual carriers perpendicular to the imposed direction of net steady-state current do not vanish, the particles themselves are not conserved. In fact, hole-electron pairs must be continuously generated in the region or on the surfaces from which the transverse current flows and must recombine in the region or on the surfaces to which it flows. If the processes of recombination and generation are slow, requiring large deviations of the carrier concentrations from their equilibrium values in order to be effective, holes and electrons will tend to accumulate on that side of the sample toward which they are deflected by the magnetic field. Since they are deflected in equal numbers, no space charge will result from this accumulation. However, once a gradient in carrier concentration is set up across the sample, further accumulation will be opposed by the resulting diffusion currents. The diffusion currents constitute a second means by which the effects of magnetic deflection may be balanced under steady-state conditions. In particular, when there is no generation or recombination of carriers whatever, the existence of diffusion currents makes possible the simultaneous vanishing of both the hole and the electron transverse currents.

The formula for the Hall constant usually quoted in the literature is appropriate for the case of zero lifetime, i.e., for the case in which any desired rate of generation or recombination can be maintained without an appreciable change in the carrier concentrations. This restriction upon the validity of the formula in question is usually not explicitly stated. The derivation of the formula is much simpler than that for the general case since no concentration gradients, and hence no diffusion currents, can exist when the lifetime is zero. Fowler[1] has given a discussion for the extreme case of infinite lifetime, and Welker[2] has, in addition, treated the intermediate case for intrinsic semiconductors. Welker's treatment neglects the surface recombination on two of the four long surfaces of the usual rectangular Hall specimen. It is the purpose of this paper to derive and discuss the more general equations needed for the interpretation of Hall experiments. It is instructive to compare the formulas for zero lifetime and infinite lifetime (no recombination). In Gaussian units they are

$$R=\frac{3\pi}{8ec}\frac{p-b^2n}{(p+bn)^2}\quad\text{(zero lifetime)},\tag{1}$$

and

$$R=\frac{3\pi}{8ec}\frac{p-bn}{(p+bn)(p+n)}\quad\text{(infinite lifetime)}.\tag{2}$$

Here e is the magnitude of the electronic charge, c the velocity of light; p and n are, respectively, the concentrations of holes and electrons, and b is the ratio of electron mobility to hole mobility. Equations (1) and (2) assume that the energy band surfaces are spherical and that holes and electrons are scattered by lattice

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[1] R. H. Fowler, *Statistical Mechanics* (Cambridge University Press, Cambridge, 1936), p. 428.

[2] H. Welker, Z. Naturforsch. **6a**, 184 (1951).

vibrations. Other cases are discussed by Shockley[3] and Klahr.[4]

When the concentration of one carrier very greatly exceeds that of the other, Eqs. (1) and (2) become identical. However, the distinction becomes quite important in the near-intrinsic region. In particular, the point at which the Hall constant is zero is quite different in the two cases. It is well known that the Hall coefficient for a P-type germanium or silicon sample reverses sign when the temperature is raised sufficiently, because the sample eventually becomes intrinsic; and when p is equal to n, the Hall constant must be negative, since b is greater than unity for silicon and germanium. The above two formulas show that the temperature at which this reversal occurs will be influenced by the lifetime of excess carriers in the particular sample measured. Considerations such as these indicate the possibility that the lifetimes of excess carriers might be measured on the basis of Hall effect experiments alone, without the necessity of observing the decay times of injected carriers. This possibility has in part motivated the following derivation of the Hall constant for the general case in which neither the bulk lifetime nor the surface recombination velocity is assumed to have an extreme value.

II. FUNDAMENTAL EQUATIONS

The total current density, i, is the sum of the current densities due to the magnetic deflection, to the concentration gradients, and to the electric fields present. Symbolically, we may write

$$\mathbf{i}=\mathbf{i}_H+\mathbf{i}_D+\mathbf{i}_C, \tag{3}$$

where $\mathbf{i}_H$ is the deflection current, $\mathbf{i}_D$ is the diffusion current, and $\mathbf{i}_C$ is the conduction current. The direction in which the net steady-state current flows will be called the "longitudinal direction," and will be designated by the subscript "l." Similarly, the subscript "t," standing for "transverse," will designate the directions contained in a plane perpendicular to the longitudinal direction. It is not necessary that all transverse currents lie along the direction of magnetic deflection; consequently, equations to which the subscript "t" are applied are two-dimensional vector equations.

Under steady-state conditions the transverse current must be zero. It follows that

$$\mathbf{i}_{Ht}+\mathbf{i}_{Dt}+\mathbf{i}_{Ct}=0, \tag{4}$$

$$\mathbf{i}_{Cl}=\mathbf{i}. \tag{5}$$

Before proceeding, it will be necessary to define a coordinate system. Let the magnetic field be oriented at right angles to the longitudinal direction, and let a right-handed Cartesian coordinate system be chosen such that the x axis lies along the longitudinal current direction, and the negative z axis along the direction of the magnetic field (i.e., $|H|=-H_z$). Then both holes and electrons will be deflected in the direction of the positive y axis. The deflection currents are given by

$$\mathbf{i}_{Hp}=(\mu_{Hp}i_{lp}H/c)\mathbf{y}_1, \tag{6}$$

$$\mathbf{i}_{Hn}=-(\mu_{Hn}i_{ln}H/c)\mathbf{y}_1, \tag{7}$$

where $\mathbf{y}_1$ is a unit vector along the y axis. In Eqs. (6) (6) and (7), as throughout the paper, the subscripts "p" and "n" refer to holes and electrons, respectively, so that $\mathbf{i}_{Hp}$ and $\mathbf{i}_{Hn}$ are the deflection currents for the two carriers, and i_{lp} and i_{ln} are the longitudinal currents. The constants of proportionality μ_{Hp} and μ_{Hn} are the Hall mobilities defined by Shockley.[5] If the energy surfaces at the top of the valence band and the bottom of the conduction band are spherical, if Boltzmann statistics apply, and if the scattering is primarily due to the lattice vibrations, then the Hall mobilities are related to the drift mobilities, μ_p and μ_n, in the following way:

$$\mu_{Hp}=3\pi\mu_p/8, \tag{8}$$

$$\mu_{Hn}=3\pi\mu_n/8. \tag{9}$$

However, these relations are not always valid, so that no use of Eqs. (8) and (9) will be made in the subsequent derivations.

The longitudinal currents are conduction currents, and hence

$$\mathbf{i}_{lp}=e\mu_p p\mathbf{E}_l, \tag{10}$$

$$\mathbf{i}_{ln}=e\mu_n n\mathbf{E}_l, \tag{11}$$

where $\mathbf{E}_l$ is the longitudinal electric field, which is constant both across and along the specimen, and p and n are the actual hole and electron concentrations as distinguished from the equilibrium concentrations, to be represented by p_0 and n_0. Thus (6) and (7) become

$$\mathbf{i}_{Hp}=e\mu_{Hp}\mu_p E_l H p\mathbf{y}_1/c, \tag{12}$$

$$\mathbf{i}_{Hn}=e\mu_{Hn}\mu_n E_l H n\mathbf{y}_1/c. \tag{13}$$

In materials that are not excessively P type or N type, both carriers obey Boltzmann statistics, so that the diffusion currents are

$$\mathbf{i}_{Dp}=-kT\mu_p\nabla p=-kT\mu_p\nabla_t p_1, \tag{14}$$

$$\mathbf{i}_{Dn}=kT\mu_n\nabla n=kT\mu_n\nabla_t p_1, \tag{15}$$

where k is Boltzmann's constant. In the right-hand terms of these equations, p_1 is the deviation from equilibrium of the two carriers:

$$p_1=p-p_0=n-n_0=n_1. \tag{16}$$

The deviations in n and p must be the same in order to

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[3] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), p. 277.

[4] C. N. Klahr, Phys. Rev. **82**, 109 (1951), Scattering due to sources other than lattice vibrations is considered by Klahr. Even more serious modifications are required by the nonspherical energy surfaces resulting from the calculations of Herman and Callaway, Phys. Rev. **89**, 518 (1953).

[5] Reference 3, p. 209, 270.

preserve space charge neutrality. The subscript "t" on the inverted delta signifies that only the transverse components of the gradient need be considered.

Formulas for the transverse conduction currents are written similarly to (10) and (11), with $\mathbf{E}_t$, the transverse electric field, replacing $\mathbf{E}_l$:

$$\mathbf{i}_{Cpt}=e\mu_p p\mathbf{E}_t, \qquad (17)$$

$$\mathbf{i}_{Cnt}=e\mu_n n\mathbf{E}_t. \qquad (18)$$

The consequences of Eqs. (12) to (18) are now summed up in three equations:

$$\mathbf{i}_{pt}=e\mu_p[\mu_{Hp}E_lHp_0\mathbf{y}_1/c-(kT/e)\nabla_t p_1+p_0\mathbf{E}_t], \qquad (19)$$

$$\mathbf{i}_{nt}=e\mu_n[-\mu_{Hn}E_lHn_0\mathbf{y}_1/c+(kT/e)\nabla_t p_1+n_0\mathbf{E}_t], \qquad (20)$$

$$\mathbf{i}_{pt}=-\mathbf{i}_{nt}. \qquad (21)$$

Equations (19) and (20) express the total transverse hole and electron currents in terms of the applied electric and magnetic fields, and the resultant transverse electric field and concentration gradient. Except in the diffusion current term, the hole and electron concentrations have been given their equilibrium values, since the neglected products p_1H and $p_1\mathbf{E}_t$ are of second order in the applied magnetic field. Throughout this paper we shall be concerned only with quantities which are of first order in the magnetic field. The discussion will, therefore, apply to the Hall effect in the limit of small fields, and will not reveal anything about the variation of the Hall constant with magnetic field.

III. PARTIAL DIFFERENTIAL EQUATIONS FOR TRANSVERSE VARIATION

The flow of excess carriers obeys the continuity equations

$$\operatorname{div}\mathbf{i}_p+ep_1/\tau=0, \qquad (22)$$

$$\operatorname{div}\mathbf{i}_n-en_1/\tau=0, \qquad (23)$$

where τ is the lifetime of excess carriers in the body of semiconductor. If Eqs. (19) and (20) are substituted in Eqs. (22) and (23), we find

$$\operatorname{div}\mathbf{E}_t-(kT/ep_0)\nabla_t^2 p_1+p_1/(\tau\mu_p p_0)=0, \qquad (24)$$

$$\operatorname{div}\mathbf{E}_t+(kT/en_0)\nabla_t^2 p_1-p_1/(\tau\mu_n n_0)=0. \qquad (25)$$

$\mathbf{E}_t$ can be eliminated by subtracting Eq. (25) from Eq. (24) leaving

$$\nabla_t^2 p_1-\beta^2 p_1=0, \qquad (26)$$

where β and another quantity,[6] D, which we shall need later, are defined by

$$\beta^2=\frac{1}{\tau D}=\frac{1}{\tau}\frac{e}{kT}\frac{\mu_n n_0+\mu_p p_0}{\mu_n\mu_p(n_0+p_0)}. \qquad (27)$$

Equation (26) will determine the variation of p_1 in the interior of the transverse cross section, if we have boundary conditions available along the surface of the sample. These boundary conditions are determined by the rate of recombination at the surface and are given by[7]

$$\mathbf{i}_p\cdot\mathfrak{n}_1=sep_1, \qquad (28)$$

$$\mathbf{i}_n\cdot\mathfrak{n}_1=-sep_1, \qquad (29)$$

where $\mathfrak{n}_1$ is the unit outward normal along the surface, and s is a quantity characteristic of the surface called the surface recombination velocity. If we substitute Eqs. (19) and (20) in Eqs. (28) and (29), the resulting equations can be combined to give a boundary condition involving p_1 only

$$(\mu_{Hp}+\mu_{Hn})\frac{H}{c}E_l(\mathfrak{n}_1\cdot\mathbf{y}_1)-\left(\frac{kT}{ep_0}+\frac{kT}{en_0}\right)\frac{dp_1}{d\mathfrak{n}}$$
$$=sp_1\left(\frac{1}{\mu_p p_0}+\frac{1}{\mu_n n_0}\right). \qquad (30)$$

This boundary condition, in addition to Eq. (26), serves to determine p_1.

To find $\mathbf{E}_t$, once we have found p_1, either Eq. (24) or Eq. (25) can be used, which can be written with the use of Eq. (26) as

$$\operatorname{div}\mathbf{E}_t=-\frac{kT}{e}\left(\frac{\mu_n-\mu_p}{n_0\mu_n+p_0\mu_p}\right)\nabla_t^2 p_1. \qquad (31)$$

Now the potential V has the same variation in each transverse cross section, while its longitudinal variation is defined by E_l, so that we can write

$$V=-E_l x+V_t, \qquad (32)$$

where $\mathbf{E}_t=-\nabla_t V_t$, and hence

$$\operatorname{div}\mathbf{E}_t=-\nabla^2 V_t. \qquad (33)$$

Together with Eq. (31) this gives

$$\nabla_t^2 V_t=\frac{kT}{e}\left(\frac{\mu_n-\mu_p}{n_0\mu_n+p_0\mu_p}\right)\nabla_t^2 p_1, \qquad (34)$$

or

$$V_t=\frac{kT}{e}\left(\frac{\mu_n-\mu_p}{n_0\mu_n+p_0\mu_p}\right)p_1+V_L, \qquad (35)$$

where V_L is a solution of Laplace's equation (in the transverse coordinates). The boundary condition on V_t may be obtained by adding Eqs. (28) and (29) and substituting for the currents from (19) and (20) giving

$$\frac{\partial V_t}{\partial\mathfrak{n}}=\frac{kT}{e'}\left(\frac{\mu_n-\mu_p}{n_0\mu_n+p_0\mu_p}\right)\frac{\partial p_1}{\partial\mathfrak{n}}+\frac{\mu_{Hp}\mu_p p_0-\mu_{Hn}\mu_n n_0}{(n_0\mu_n+p_0\mu_p)}$$
$$\times\frac{H}{c}E_l\mathbf{y}_1\cdot\mathfrak{n}_1. \qquad (36)$$

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[6] This is the same diffusion coefficient defined by Harvey Brooks, Phys. Rev. **90**, 336 (1951).

[7] Reference 3, p. 321.

Substituting (35) in (36) leaves

$$\frac{\partial V_L}{\partial \mathfrak{n}}=\frac{\mu_{Hp}\mu_p p_0-\mu_{Hn}\mu_n n_0}{n_0\mu_n+p_0\mu_p}\frac{H}{c}E_l\mathbf{y}_1\cdot\mathfrak{n}_1. \tag{37}$$

A solution of Laplace's equation which satisfies (37) is given by

$$\mathbf{E}_{Lt}=-\nabla_t V_L=-\frac{\mu_{Hp}\mu_p p_0-\mu_{Hn}\mu_n n_0}{(n_0\mu_n+p_0\mu_p)^2}\frac{Hi}{ce}\mathbf{y}_1. \tag{38}$$

Here E_l has been replaced by

$$E_l=\frac{i}{e(n_0\mu_n+p_0\mu_p)}. \tag{39}$$

The field given by Eq. (38) is the one given by the usual Hall formula for zero lifetime. Hence V_t can be written as

$$V_t=V_D+V_L, \tag{40}$$

where

$$V_D=\frac{kT}{e}\left(\frac{\mu_n-\mu_p}{n_0\mu_n+p_0\mu_p}\right)p_1. \tag{41}$$

V_D is the part of V_t which depends on surface and volume recombination.

The general procedure that has been discussed has been applied to circular and rectangular cross sections, and the results will be described. In the limiting cases in which the excess carrier lifetime is zero (representing either zero bulk lifetime, or else an infinite surface recombination velocity) or infinite (representing infinite bulk lifetime accompanied by zero surface recombination velocity), a transverse electric field is set up which is independent of the shape of the transverse cross section. This transverse electric field has only a y component and is constant across the transverse cross section. The field established in the case of zero carrier lifetime is given by Eq. (38). The field established in the case of infinite carrier lifetime is

$$\mathbf{E}_t=\frac{(\mu_{Hp}p_0-\mu_{Hn}n_0)}{(n_0\mu_n+p_0\mu_p)(n_0+p_0)}\frac{iH}{ec}\mathbf{y}_1. \tag{42}$$

Equations (38) and (42) lead to Eqs. (1) and (2) under the assumptions of Eqs. (8) and (9).

IV. CIRCULAR CROSS SECTION

The voltage distribution around the circumference of the transverse cross section turns out to be of the form $V_t=V_0\cos\theta$, where θ is the angle that the radius makes with positive y-axis. A definition of the Hall constant R, which is consistent with Eqs. (1) and (2), is

$$V_0=RHia, \tag{43}$$

where a is the radius of the cylinder. R can best be written in the form

$$R=R_D+R_L, \tag{44}$$

where R_D is a measure of V_D and R_L is a measure of V_L. We then have

$$R_L=\frac{1}{ce}\frac{(\mu_{Hp}\mu_p p_0-\mu_{Hn}\mu_n n_0)}{(n_0\mu_n+p_0\mu_p)^2}, \tag{45}$$

and

$$R_D=\frac{I_1(\beta a)}{(s/D)aI_1(\beta a)+\beta aI_1'(\beta a)}\mathcal{R}, \tag{46}$$

where

$$\mathcal{R}=\frac{(\mu_n-\mu_p)(\mu_{Hp}+\mu_{Hn})n_0p_0}{(n_0\mu_n+p_0\mu_p)^2(n_0+p_0)}\frac{1}{ec}, \tag{47}$$

and $I_1(x)=-iJ_1(ix)$ is a Bessel function of an imaginary argument, whose values can be found in tables.[8] In the case of zero carrier lifetime ($s=\infty$ or $\tau=0$) we find $R_D=0$. In the case of infinite carrier lifetime ($s=0$ and $\tau=\infty$) we find $R_D=\mathcal{R}$. For intermediate cases:

$$0<R_D/\mathcal{R}<1. \tag{48}$$

It should be noted that the Hall measurement on a cylindrical conductor yields only one constant: the value of V_0. If all the mobilities and carrier densities are known and if one of the two quantities, s and τ, is known, the other one can be found directly from a Hall measurement. If both s and τ are unknown, their values cannot be found from Hall measurements on a single specimen. In actual practice it is likely that s is not uniform over the surface of the sample. This will be revealed by deviations from $V_t=V_0\cos\theta$.

V. RECTANGULAR CROSS SECTION

General Discussion

We shall assume that the Hall sample is oriented with respect to the magnetic field as shown in Fig. 1. Before discussing the detailed equations it will be in order to point out that the voltage distribution over the surface of the cross section depends on s and τ in different ways. If $s=0$, all the transverse currents are in the y direction, and the surfaces at $y=\pm w/2$ are equipotentials. If s is different from zero, diffusion currents flow toward all surfaces, and most of the quantities we are concerned with will show a variation

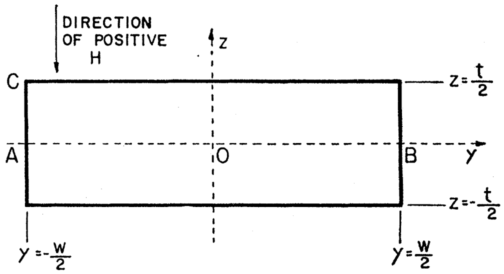

FIG. 1. Transverse cross section of rectangular Hall specimen. The longitudinal current is considered positive if directed out of paper.

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[8] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945).

with z as well as with y. Naturally, we still cannot find both s and τ from one Hall measurement. If, however, two potential differences are measured, on the same cross section, it will be possible to find both s and τ, providing all the mobilities and carrier concentrations are already known. This could be done, for instance, by measuring the potential between points A and C shown in Fig. 1, as well as the potential between A and B.

Detailed Equations

The field distribution due to V_L is specified by Eq. (38). This gives a potential difference between $y=w/2$ and $y=-w/2$:

$$V_L\left(\frac{w}{2}\right)-V_L\left(-\frac{w}{2}\right)=\frac{\mu_{Hp}\mu_p p_0-\mu_{Hn}\mu_n n_0}{(\mu_p p_0+\mu_n n_0)^2}\frac{wHi}{ec}. \tag{49}$$

In addition to this, there is a potential distribution of the form

$$V_D=Hig(z,y)\mathcal{R}, \tag{50}$$

where $\mathcal{R}$ is given by Eq. (47) and $g(z,y)$ is still to be defined. It is a quantity which satisfies

$$0<g(z,w/2)-g(z,-w/2)<w, \tag{51}$$

so that the Hall constant measured in the normal manner by probes at A and B will be between R_L and $R_L+\mathcal{R}$, as in the cylindrical case.

In order to exhibit $g(z,y)$, it is necessary to define a set of numbers γ_n which are positive roots of the equation

$$\gamma_n \tan(\gamma_n t/2)=s/D. \tag{52}$$

The roots are numbered in order of size; the lowest one is labelled $n=0$. The roots then satisfy

$$2n\pi<\gamma_n t<(2n+1)\pi. \tag{53}$$

Furthermore, for each γ_n we shall define an α_n:

$$\alpha_n=+(\beta^2+\gamma_n^2)^{\frac{1}{2}}. \tag{54}$$

Then, $g(z,y)$ is given by

$$g(z,y)=\sum_{n=0}^{\infty} g_n \sinh\alpha_n y \cos\gamma_n z, \tag{55}$$

with

$$g_n=\frac{4\sin(\gamma_n t/2)}{[(s/D)\sinh(\alpha_n w/2)+\alpha_n\cosh(\alpha_n w/2)]\times(\sin\gamma_n t+\gamma_n t)}. \tag{56}$$

Special Situations

The series given by (55) will be discussed in further detail for a number of special situations. If $s=0$, only the term for $n=0$ is present. Furthermore, $\gamma_0=0$, so that only variation along the y-axis exists. In that case the Hall coefficient, determined in the usual way from the potential difference between A and B in Fig. 1, will be given by[9]

$$R=R_L+\mathcal{R}\frac{2}{\beta w}\tanh\left(\frac{\beta w}{2}\right). \tag{57}$$

If we consider lifetime as a variable, Eq. (57) shows that R makes the transition from R_L to $R_L+\mathcal{R}$ when the diffusion length, $1/\beta$, is comparable to the width, w, of the specimen.

In order that the concentration of carriers across the thickness, t, of the sample (i.e., along the z direction) be constant, it is not necessary that s vanish. If s is small enough, diffusion can maintain equilibrium across the sample. The condition for this is $st\ll D$. Under this condition only the term for $n=0$ in (55) is important. In this case,

$$\alpha_0=(1/D\tau_e)^{\frac{1}{2}}, \tag{58}$$

where τ_e is an effective lifetime given by

$$1/\tau_e=1/\tau_s+1/\tau \tag{59}$$

and where τ_s is the lifetime which would be measured if the surfaces at $z=\pm t/2$ were the only place where recombination occurs. τ_s is in turn equal to $t/2s$.

The case in which one value of surface recombination velocity, s_w, applies for the surfaces at $y=\pm w/2$ and another value, s_t, applies at $z=\pm t/2$, is treated almost as easily as the case leading to (55) and (56). If $s_t\neq s_w$, (55) still holds, but instead of (56) we have

$$g_n=\frac{4\sin(\gamma_n t/2)}{[(s_w/D)\sinh(\alpha_n w/2)+\alpha_n\cosh(\alpha_n w/2)]\times(\sin\gamma_n t+\gamma_n t)}, \tag{60}$$

where

$$\alpha_n=+(\beta^2+\gamma_n^2)^{\frac{1}{2}}, \tag{61}$$

and instead of Eq. (52),

$$\gamma_n \tan(\gamma_n t/2)=s_t/D. \tag{62}$$

It is interesting to note that g can be made arbitrarily small by making s_w large enough. The value of g cannot be made arbitrarily small by increasing s_t without limit. Physically this comes from the fact that for very large values of s_t the rate at which recombination occurs at $y=\pm t/2$ is not determined by s_t but is limited by the time it takes the holes to diffuse to $y=\pm t/2$ from the interior of the specimen. Diffusion will determine the rate of recombination at $y=\pm t/2$, if $s_t t\gg D$. (For room temperature germanium mobilities, st is comparable to D if $s=750$ cm/sec and $t=1$ mm.) If $s_t t\gg D$, and if, furthermore, $s_w\geq s_t$, $w\geq t$, and $\tau=\infty$, then it is only the recombination at the surfaces at $y=\pm w$ that counts. In this case the series given by Eq. (55) can easily be summed, and the Hall coefficient,

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[9] A formally very similar expression has been derived by H. Welker, L'Onde Electrique **30**, 309 (1950). Welker's equation (33) applies to a semiconductor with no recombination, but with an alternating longitudinal current.

determined in the usual way from the potential difference between A and B in Fig. 1, will be

$$R=R_L+\frac{2D}{sw}\mathcal{R}. \tag{63}$$

Another case of interest occurs if s_t is zero. Then all terms of the series but the first vanish, and we find

$$R=R_L+\frac{1}{s_w w/2D+(\beta w/2)\coth(\beta w/2)}\mathcal{R}. \tag{64}$$

It should be pointed out that an experimental arrangement in which the voltage between two points on the same surface at $z=t/2$ is measured allows one to dispense with all terms of the series given by Eq. (56) except the first one, even if s_t is not zero. In such arrangements we rely on the fact that the hyperbolic functions with larger values of α_n decay more rapidly with y as we move away from $y=\pm w/2$.

VI. CONCLUDING REMARKS

In conclusion we wish to point out that a complete interpretation of the experimental Hall effect in near-intrinsic semiconductors requires a knowledge of the precise nature of the probes used to detect the Hall voltage. The voltage of a metal probe placed on the surface of a semiconductor is not a direct indication of the electrostatic potential of the underlying semiconductor unless the carrier concentrations have their equilibrium values. This is because a metal contact acts as a recombination center for excess carriers; and if there is a barrier to holes or electrons associated with the contact (as is usually the case), a potential change across the barrier is required to equalize the flow of holes and electrons toward the contact.[10] In the first approximation this extra contribution to the probe voltage will be proportional to the excess carrier concentration. Since V_b [see Eq. (41)] is also proportional to the excess carrier concentration, one may take the extra probe voltage into account by replacing the constant $\mathcal{R}$, defined by Eq. (47), by an effective constant $\mathcal{R}'$ in the subsequent equations. If one writes

$$\mathcal{R}'=\mathcal{R}(1+\alpha), \tag{65}$$

then α will be a constant depending on the nature of the probe. The maximum value of α is $(n_0\mu_n+p_0\mu_p)/p_0(\mu_n-\mu_p)$, corresponding to an infinitely high barrier to electrons. The minimum value of α is $-(n_0\mu_n+p_0\mu_p)/n_0(\mu_n-\mu_p)$, corresponding to an infinitely high contact barrier to holes. The special case $\alpha=-1$ corresponds to a contact having no barrier. Over the surface of such a contact the recombination velocity is equal to thermal velocity (10^7 cm/sec), and hence no appreciable deviation of the carrier concentrations in the immediate vicinity is possible.

The authors wish to thank L. P. Hunter for pointing out that the usual interpretation of Hall effect data leads to inconsistencies and also for many discussions.

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[10] J. Bardeen, Bell System Tech. J. 29, 469 (1950).