which are substantially different from the actual masses of nuclides.

Unquestionably a better representation can be found for the shell stabilizing term than our present one. The local adjustment of the parameters incorporated in Eq. (21) probably would lead to a substantial improvement. It is also probable that the true shell correction may embody semimagic numbers and curved segments. At this time the experimental data is too sparse and too inaccurate in many regions of the mass surface to offer great encouragement to an effort to refine the shell correction on the basis of purely empirical considerations. Of necessity such a study will be a tedious one, since it will have to be made in conjunction with a more precise representation of the smooth trends of the nuclear mass surface and a more precise representation of the pairing effect than the simple expressions which we have used thus far. It appears therefore that we have reached a point at which we might best look to current nuclear theories to find a better representation of the shell stabilizing term.

We would like to express our appreciation to R. B. Minogue, N. J. Marucci, R. Oppenheim, J. S. Nader, N. Engler, R. Oswald, and Mrs. W. Steiger for their assistance in this study, and to Dr. C. D. Coryell for sending us a manuscript prior to publication.

PHYSICAL REVIEW VOLUME 91, NUMBER 1 JULY 1, 1953

The Parameters for the Slow Neutron Resonance in Rhodium*

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(Received March 30, 1953)

Measurements of the 1.26-ev resonance in rhodium have been made with a crystal spectrometer and the following values were obtained for the parameters: $E_0=1.260\pm0.004$ ev, $\sigma_0=(5000\pm200)\times10^{-24}$ cm^2, $\Gamma=0.156\pm0.005$ ev, $g\Gamma_n=(3.8\pm0.2)\times10^{-4}$ ev, and $\sigma_{fa}=(5.5\pm1.0)\times10^{-24}$ cm^2. The shape of the resonance agrees to very high accuracy with the one-level Breit-Wigner formula. The procedure is discussed for analyzing experimental data in cases where small corrections are required for instrument resolution and Doppler broadening.

I. INTRODUCTION

THE slow neutron resonance in rhodium at 1.26 ev offers a particularly favorable opportunity for studying the details of an absorption resonance. Several factors contribute to simplify greatly the analysis of this case: Rhodium is monoisotopic; the 1.26-ev resonance is well isolated from other resonances and presumably is not complicated by interference effects; the contribution to the cross section form resonant scattering is very small and may be neglected in the analysis; and the resonant energy lies within the range of very high resolution of modern neutron spectrometers.

Several previous measurements of the rhodium cross section have been made;[1-3] however, in these cases the instrument resolution was inadequate for a detailed analysis. The relatively high resolution which can be obtained with newer spectrometers has justified a remeasurement of the rhodium cross section. The purpose of these new measurements is to obtain accurate values of the Breit-Wigner parameters and to study the details of the shape of the resonance. The measurements reported below were made with the BNL crystal spectrometer, which has been described elsewhere.[4]

II. PROBLEMS IN ANALYZING DATA

The experimentally observed shape of a resonance differs from the "true" shape because of the distortion introduced by finite instrument resolution and by the Doppler broadening resulting from the thermal motion of the atoms in the specimen. The corrections required to account for these effects are appreciable even for cases of very high resolution and small Doppler broadening. The problem of fitting experimental data to a theoretical dispersion curve is greatly complicated by the need for the above corrections. As experimental technique advances and the data becomes more refined, the problem of finding an adequate and practical method of analysis becomes more pressing.

There are several possible approaches to the analysis problem. The "area" method of Havens and Rainwater,[5] which corrects for resolution, has recently been made more quantitative by Melkonian[6] and is now being extended to include the Doppler correction.[7,8] Another less elegant approach which could best be described as the "trial and error" method has been frequently used.[3,5,9] This consists of choosing trial values of the Breit-Wigner parameters and then computing the effect

* Research supported by the U. S. Atomic Energy Commission.
[1] Borst, Ulrich, Osborne, and Hasbrouck, Phys. Rev. **70**, 557 (1946).
[2] W. J. Sturm, Phys. Rev. **71**, 757 (1947).
[3] R. R. Meijer, Phys. Rev. **75**, 773 (1949).
[4] L. B. Borst and V. L. Sailor, Rev. Sci. Instr. **24**, 141 (1953).
[5] W. W. Havens, Jr., and J. Rainwater, Phys. Rev. **70**, 154 (1946).
[6] E. Melkonian, Phys. Rev. **90**, 362 (1953).
[7] G. v. Dardel and R. Persson, Nature **170**, 1117 (1952).
[8] E. Melkonian, Bull. Am. Phys. Soc. **28**, No. 3, 26 (1953).
[9] B. D. McDaniel, Phys. Rev. **70**, 832 (1946).

of resolution and Doppler broadening on the curve. Sets of such computed curves are compared with the observed curve and the best fit selected.

A slight variation of the "trial and error" method has been chosen for analyzing the rhodium data because it is capable of any desired degree of accuracy. The steps in the analysis are thought to be of practical interest to many and therefore have been outlined in the next section. The method in its present form is too laborious for general application, but is of practical value for those cases in which the instrument resolution is very high.

III. METHOD FOR ANALYZING THE DATA

The total neutron cross section of an element containing an isolated resonance having negligible resonant scattering can be expressed in terms of the one-level Breit-Wigner formula,

$$\sigma(E)=\sigma_{fa}+(E_0/E)^{\frac{1}{2}}\sigma_0\Gamma^2[4(E-E_0)^2+\Gamma^2]^{-1}, \quad (1)$$

where σ is the total cross section at energy E, σ_{fa} is the free atom scattering cross section, σ_0 is the cross section at exact resonance E_0, and Γ is the total width of the resonance at half-maximum.

The parameter σ_0 is actually a composite quantity involving the more fundamental parameters as follows:

$$\sigma_0=4\pi\lambda\!\!\!^{-}{}_0^2 g\Gamma_n\Gamma_\gamma/\Gamma^2, \quad (2)$$

where $2\pi\lambda\!\!\!^{-}{}_0$ is the neutron wavelength at resonance; g is the statistical weight factor which for s neutrons is given by $g=\frac{1}{2}[1\pm(2I+1)^{-1}]$, I being the spin of the initial nucleus; Γ_n and Γ_γ are, respectively, the neutron and radiation widths.

The parameters which completely characterize the resonance are therefore E_0, σ_{fa}, g, Γ_n, and Γ_γ. From a total cross-section curve one might hope to obtain E_0, σ_{fa}, σ_0, and Γ. Additional measurements of the resonance scattering would be required to evaluate g, Γ_n, and Γ_γ separately.

(A) Analysis when Corrections are Required

Doppler Broadening

The theory of Doppler broadening has been presented by Bethe and Placzek[10] and elaborated by Lamb.[11] It was shown that the resonance term in Eq. (1) becomes

$$\sigma_c=\frac{\sigma_0}{\pi^{\frac{1}{2}}\Delta}\int_0^\infty\left(\frac{E_0}{E'}\right)^{\frac{1}{2}}\frac{\exp[-(E-E'/\Delta)^2]}{1+4(E'-E_0)^2/\Gamma^2}dE'. \quad (3)$$

For a free gas the Doppler width Δ is given by

$$\Delta=2(mE_0kT/M)^{\frac{1}{2}}, \quad (4)$$

where m is the mass of the neutron, M the mass of the nucleus, k the gas constant, and T the temperature of the sample. Lamb has shown that if the specimen is a solid, Eqs. (3) and (4) are valid only when the width of the resonance is much greater than the Debye temperature of the specimen; i.e., if $\Gamma\gtrless 4\theta$. If this condition is met it is possible to account for lattice binding merely by substituting in Eq. (4) for the real temperature T an effective temperature T_{eff} as defined by Lamb. For rhodium $T_{\mathrm{eff}}\approx 1.2T$.

In order to deal with Eq. (3), one may expand the $(E_0/E')^{\frac{1}{2}}$ term in a Taylor's series about $E'=E$. It is then possible to show that if $E_0>0.5$ ev, only the first term of the expansion need be considered. Equation (3) can now be written in the form

$$\sigma_c=\sigma_0(E_0/E)^{\frac{1}{2}}\psi(\xi, x), \quad (3a)$$

where

$$\psi(\xi, x)=\frac{\xi}{2\pi^{\frac{1}{2}}}\int_{-\infty}^{\infty}\frac{\exp[-\frac{1}{4}\xi^2(x-y)^2]}{1+y^2}dy, \quad (5)$$

and x, y, and ξ are defined as follows:

$$x\equiv(E-E_0)/\tfrac{1}{2}\Gamma, \quad y\equiv(E'-E_0)/\tfrac{1}{2}\Gamma, \quad \text{and} \quad \xi\equiv\Gamma/\Delta.$$

The function $\psi(\xi, x)$ has been evaluated only for a limited number of values of the arguments ξ and x.[8,12] For values applicable to the rhodium resonance, ψ was computed by numerical integration.

Effect of Instrument Resolution

The total cross section at each energy is obtained by measuring the transmission of the specimen with monochromatic neutrons. The cross section can be computed from the relationship

$$T=e^{-N\sigma}, \quad (6)$$

where N is the thickness of the sample expressed as the number of nuclei per cm². If the instrument resolution were infinitely sharp the true transmission T would be obtained from the measurement. However, the instrument resolution will cause the measured transmission T_i to differ from true transmission as follows:

$$T_i(E_i)=\int_0^\infty T(E,\psi)R(E-E_i)dE\bigg/\int_0^\infty R(E-E_i)dE, \quad (7)$$

where $R(E-E_i)$ is the resolution function of the instrument. The experimentally determined resolution function of the BNL crystal spectrometer can be closely approximated by an error function modified by an additional $E^{-5/2}$ term which accounts for the reactor spectral distribution, the crystal reflectivity, and the $1/v$ variation of the detector efficiency:[4]

$$R(E-E_i)=AE^{-5/2}\exp[-a(E-E_i)^2], \quad (8)$$

where A is a numerical constant which will cancel in

[10] H. A. Bethe and G. Placzek, Phys. Rev. 51, 450 (1937); H. A. Bethe, Revs. Modern Phys. 9, 69 (1937), see p. 140.

[11] W. E. Lamb, Jr., Phys. Rev. 55, 190 (1939).

[12] M. Born, *Optik* (Verlag Julius Springer, Berlin, 1933), p. 486.

the normalization performed in Eq. (7). The nominal energy setting of the spectrometer is E_i, and $a \equiv 4\ln 2/\epsilon^2$, where ϵ is the full width at half-maximum of the error function.

Treatment of Data

The object of the analysis is to obtain the values of E_0, σ_0, and Γ which best satisfy the experimental points. First E_0 is obtained graphically and preliminary values σ_0' and Γ' are estimated from the experimental curve. Corresponding to each measured transmission T_{mi} a calculated value $T_{ci}(\sigma_0', \Gamma')$ is obtained from Eq. (7) using the preliminary parameters. The evaluation of Eq. (7) is carried out by numerical integration.

As the next step, corrections to σ_0' and Γ' will be found which give a better fit to the experimental points. T_{ci} is a function of σ_0 and Γ and can be expanded about σ_0' and Γ' in a Taylor's series:

$$T_{ci}(\sigma_0, \Gamma) = T_{ci}(\sigma_0', \Gamma') + \frac{\partial T_{ci}}{\partial \sigma_0}\Delta\sigma_0 + \frac{\partial T_{ci}}{\partial \Gamma}\Delta\Gamma + \cdots. \quad (9)$$

Higher terms in the expansion can be neglected provided the original choices were not too far in error. A set of residual equations can be written using the calculated values and the measured values as follows:

$$T_{ci}(\sigma_0', \Gamma') + \frac{\partial T_{ci}}{\partial \sigma_0}\Delta\sigma_0 + \frac{\partial T_{ci}}{\partial \Gamma}\Delta\Gamma - T_{mi} = r_i, \quad (10)$$

where

$$\frac{\partial T}{\partial \sigma_0} = \frac{-\int_0^\infty (N\sigma/\sigma_0) e^{-N\sigma} R(E-E_i)dE}{\int_0^\infty R(E-E_i)dE},$$

and

$$\frac{\partial T}{\partial \Gamma} \approx \frac{-\int_0^\infty \frac{N\sigma}{\Gamma}\left[\frac{8(E-E_0)^2}{4(E-E_0)^2+\Gamma^2}\right] e^{-N\sigma} R(E-E_i)dE}{\int_0^\infty R(E-E_i)dE}.$$

The latter partial derivative assumes $\psi = \frac{1}{2}\pi^{-\frac{1}{2}}\xi(1+x^2)^{-1}$, which gives only a small error in the case of rhodium. These terms must also be computed by numerical integration.

The method of least squares is now applied to the Eqs. (10) to find the values of $\Delta\sigma_0$ and $\Delta\Gamma$ which reduce Σr_i^2 to a minimum, the statistical weights of the measured values T_{mi} all being approximately equal. The values of the parameters which give the best fit are, therefore, $\sigma_0 = \sigma_0' + \Delta\sigma_0$ and $\Gamma = \Gamma' + \Delta\Gamma$. At the conclusion a test can be carried out to prove that the higher terms in the Taylor's series, Eq. (9), were actually negligible. If not, it would be necessary to repeat the process.

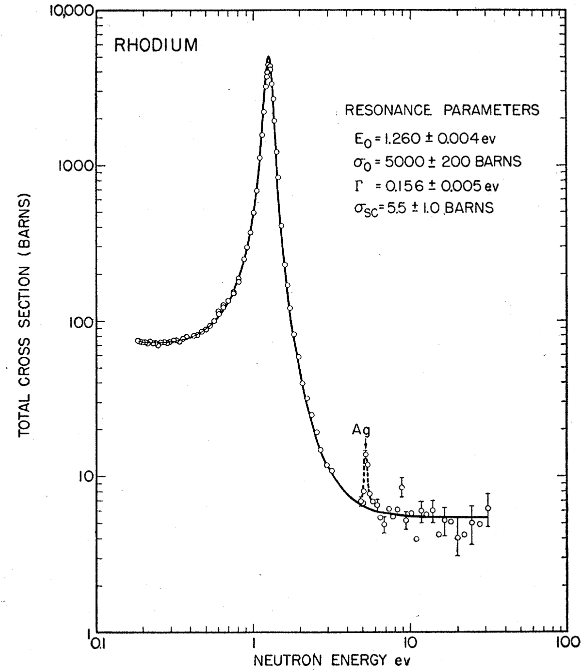

FIG. 1. The total cross section of rhodium as a function of energy. The curve is a plot of the one-level formula using $E_0 = 1.260$ ev, $\sigma_0 = 5000$ barns, $\Gamma = 0.156$ ev, and $\sigma_{fa} = 5.5$ barns. The experimental points do not deviate significantly from the theoretical curve except at the center of the resonance where corrections must be made.

The numerical integration mentioned above can be carried out using Weddle's rule with 12 intervals which for this case gives an accuracy of about 0.1 percent. The limits of integration 0 to ∞ can be replaced with the limits $(E_i - \frac{3}{2}\epsilon)$ to $(E_i + \frac{3}{2}\epsilon)$.

(B) Analysis in the Wings of the Resonance

In the wings of the resonance the variation of σ is negligible over the resolution width, and Eq. (3) reduces to Eq. (1) so that the Doppler correction can be ignored. The data may be fitted directly to Eq. (1) in these regions using the method of least squares to solve for σ_{fa} and $\sigma_0\Gamma^2$. The resonant energy E_0 can be obtained graphically and only an approximate value of Γ need be used in the denominator. Such an analysis is useful for comparison with the results obtained from the complicated curve fitting which is necessary near the center of the resonance.

The best value of σ_{fa} is obtained at higher energies where the effects of coherent scattering and resonant scattering are completely negligible.

IV. EXPERIMENTAL RESULTS

(A) Measurements

The total cross section of rhodium was measured over the energy range from 0.18 to 30 ev with samples of four different thicknesses (see Fig. 1). Only one

TABLE I. Parameters for the rhodium resonance derived from the analysis of several sets of experimental data. The resonant energy $E_0=1.260\pm0.004$ was obtained graphically. The uncorrected measured values are listed as the first entry for comparison.

Region analyzed	σ_0 $\times10^{-24}$ cm²	Γ ev	$\sigma_0\Gamma^2$ $\times10^{-24}$ cm² (ev)²	σ_{fa} $\times10^{-24}$ cm²
Uncorrected measured values	(4460)	(0.180)	(144.5)	
Near center of resonance	5000 ± 200	0.156 ± 0.005	121.7 ± 2	
Wings of resonance			118.1 ± 6	(8.6 ± 4)
Wings of resonance			122.2 ± 8	(13.3 ± 6)
Wings of resonance			131.0 ± 12	(5.3 ± 4)
6–30 ev				5.5 ± 1

resonance was found in this range except for a weak resonance at 5.2 ev attributed to a small impurity of silver. The variety of sample thicknesses is necessary to obtain reliable values over the wide range of magnitude of the cross section. Cross sections were computed only for those points having transmissions lying within the limits from 0.10 to 0.90. When the transmission lies outside these limits the uncertainty in the computed cross section becomes excessive unless an unreasonable amount of time is spent in improving statistics. Enough counts were taken on most points to give 1 percent statistics for the transmission.

The rhodium specimen was of high purity; a spectroscopic analysis[13] disclosed approximately 0.01 percent Ag, 0.01 percent Cu and traces of Pd and Pt. The specimens were in the form of metal foils which had been cut to carefully measured sizes. The thickness of each foil was determined by weighing.

The resolution of the spectrometer at the center of the resonance, 1.26 ev, was 0.026 percent; i.e., the resolution function had a full width at half-maximum of $\epsilon=0.033$ ev. The Doppler width for $T_{\mathrm{eff}}=1.2T=355°\mathrm{K}$ was $\Delta=0.0386$ ev.

(B) Results of the Analysis

The analysis described in Sec. III(A) was carried out using 13 experimental points. The values of the parameters which were obtained are listed in Table I. Included in the table are the uncorrected experimental values of maximum cross section and total width. The experimental values, when compared to the values obtained from the analysis, show the magnitude of the correction. Also listed in Table I are values of σ_{fa} and $\sigma_0\Gamma^2$ which were obtained from three sets of data by analyzing the wings of the resonance as outlined in Sec. III(B).

A curve was constructed by substituting the values $\sigma_0=5000\times10^{-24}$ cm², $\Gamma=0.156$ ev, and $\sigma_{fa}=5.5\times10^{-24}$ cm² into Eq. (1). This curve is shown with the experimental points in Fig. 1. A detailed linear plot of the central portion of the resonance is shown in Fig. 2, where curve A is the true shape of the resonance, curve B shows the effect of resolution and Doppler broadening, and curve C shows a plot of the function $R(E-E_i)$. It should be noted that the Breit-Wigner curve A and the corrected curve B become coincident in the wings of the resonance. The experimental points agree quite well with the calculated curve over the entire range of energy from 0.18 ev to 30 ev. It is apparent from this agreement that the one-level formula accurately fits the rhodium resonance over the entire range of energy for which observations were made. The error caused by neglecting resonant scattering is too small to be distinguished on these graphs.

[13] The author is indebted to Mr. M. Slavin of the Brookhaven National Laboratory for this analysis.

V. DISCUSSION OF RESULTS

The values in Table I may be used for computing several additional quantities of interest. Assuming $\Gamma\approx\Gamma_\gamma$ and substituting the numerical value of σ_0 in Eq. (2), one obtains $g\Gamma_n=(3.8\pm0.2)\times10^{-4}$ ev. The spin[14] of Rh^{103} is $\frac{1}{2}$ so the two possible choices of g are $\frac{1}{4}$ and $\frac{3}{4}$. If $g=\frac{1}{4}$, $\Gamma_n=(1.52\pm0.07)\times10^{-3}$ ev and $\Gamma_n/\Gamma=(9.7\pm0.8)\times10^{-3}$; while if $g=\frac{3}{4}$, $\Gamma_n=(5.08\pm0.20)\times10^{-4}$ ev and $\Gamma_n/\Gamma=(3.25\pm0.20)\times10^{-3}$. The absorption integral, $\Sigma_a\approx\frac{1}{2}\pi\sigma_0\Gamma/E_0$, is found to be $(980\pm70)\times10^{-24}$ cm². Finally the contribution of the resonance to the thermal absorption cross section σ_{th} may be computed by extra-

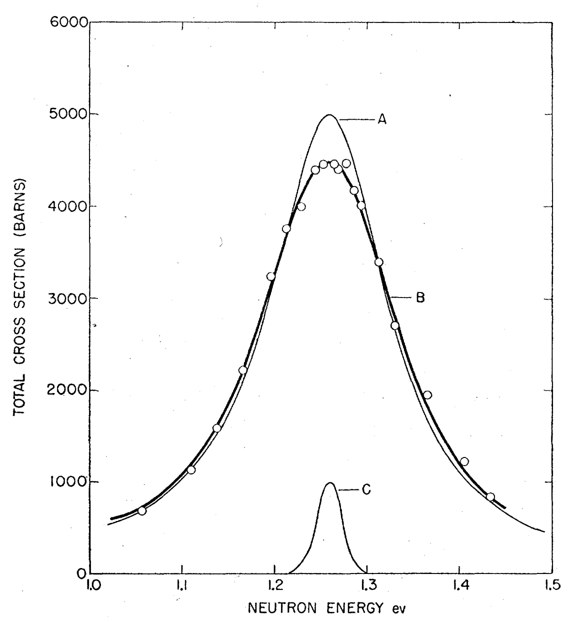

FIG. 2. A linear plot of the central region of the rhodium resonance. Curve A is the Breit-Wigner curve. Curve B is obtained by calculating the effect of instrument resolution and Doppler broadening on curve A. Curve C is a plot of the resolution function with an arbitrary scale as the ordinate.

[14] H. Kuhn and G. K. Woodgate, Nature **166**, 906 (1950).

polating Eq. (1) back to an energy of 0.025 ev. This extrapolation gives $\sigma_{th}=(142\pm10)\times10^{-24}$ cm^2.

Table II summarizes the various parameters and compares these with values previously reported. In general the agreement is good but not always within the experimental errors. It should be noted that there is a disagreement with Meijer[3] on the values of σ_0 and Γ, however the product $\sigma_0\Gamma^2$ is in good agreement. The individual values obtained for σ_0 and Γ depend strongly on the resolution correction, but the product $\sigma_0\Gamma^2$ is relatively insensitive to resolution. It appears possible that Meijer underestimated the correction required for instrument resolution.

Scattering measurements by Brockhouse[15] indicate that $\frac{3}{4}$ is probably the proper choice for g; however, the uncertainty is large. Regardless of the choice of g, there is a large discrepancy with the value of the ratio Γ_n/Γ reported by Harris *et al.*[16]

Comparison between the values of σ_{th} listed in Table II indicates that most of the thermal absorption cross section must be attributed to the 1.260-ev resonance. Experimental accuracy is not sufficient to permit a reliable estimate of the residual σ_{th} not due to this resonance.

TABLE II. Various parameters of the rhodium resonance compared to results previously reported. Cross sections are in units of 10^{-24} cm^2 and widths are given in electron volts.

Parameter	Reported in this paper	Previously reported
E_0	1.260 ± 0.004	1.30,[a] (1.21 ± 0.02),[b] 1.28[c]
σ_0	5000 ± 200	2750 ± 200[b]
Γ	0.156 ± 0.005	0.14,[d] (0.21 ± 0.01)[b]
$\sigma_0\Gamma^2$	121.7 ± 2.0	117 ± 8[b]
$g\Gamma_n$	$(3.8\pm0.2)\times10^{-4}$	4×10^{-4} [d]
g		$\frac{3}{4}$[e]
Γ_n	$(5.1\pm0.2)\times10^{-4}$	
Γ_n/Γ	$(3.25\pm0.2)\times10^{-3}$	$(4\pm3)\times10^{-3}$,[e] 4.3×10^{-2} [f]
Σ_a	980 ± 70	529,[f] 790[d]
σ_{fa}	5.5 ± 1.0	5.3,[e] 6 ± 2[b]
σ_{th}	142 ± 10	150 ± 7.5[g]

[a] Reference 1.
[b] Reference 3.
[c] Reference 2.
[d] L. L. Lowry and M. Goldhaber, Phys. Rev. **76**, 189 (1949).
[e] Reference 15.
[f] Reference 16.
[g] H. Pomerance, Phys. Rev. **83**, 641 (1951).

VI. CONCLUSION

Only one resonance has been observed in the rhodium total cross section between 0.18 and 30 ev. It is unlikely that any other resonances of normal width (i.e., $\Gamma\approx0.10$ ev) exist in this interval unless they are unusually weak. The observed resonance may be fitted to a Breit-Wigner one-level formula with excellent agreement over the entire range of observation, provided that small corrections are made near the center of the resonance to account for instrument resolution and Doppler broadening. The curve fitting yielded accurate values of the characteristic parameters with the exception of g which must be determined by other means.

The author is indebted to Mr. J. Chernick, Dr. H. H. Landon, and Mr. H. L. Foote, Jr., for many helpful discussions relating to the analysis problem; and to Dr. E. Melkonian for pointing out the proper method of making the Doppler correction.

[15] B. N. Brockhouse, Can. J. Phys. **31**, 432 (1953).
[16] Harris, Muehlhause, and Thomas, Phys. Rev. **79**, 11 (1950).