n-Type Surface Conductivity on p-Type Germanium

W. L. BROWN
Bell Telephone Laboratories, Murray Hill, New Jersey
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A positive charge on the surface of a p-type germanium crystal induces a net negative space charge within the crystal adjacent to the surface. This space charge is composed of ionized acceptor atoms and also of electrons under certain conditions. When electrons occur they provide a layer of n-type conductivity immediately under the p-type germanium surface. Such a layer has been found on the p-type region of some n-p-n transistors. In the n-p-n structure the layer of electrons appears as an extra conducting path—"a channel"—across the p-type material between the two n-type ends. The conductance of a channel and the capacity between the channel and the p-type material have been measured and compared with the theoretical predictions based on a simple model.

1. INTRODUCTION

IN this paper the results of some experimental and theoretical investigations pertaining to surface conductivity will be described. Surface conductivity on germanium has previously been proposed as an explanation for certain aspects of transistor action.[1] These earlier examples now appear to have a different explanation. The phenomena to be discussed in this article, however, appear to be explicable only on the basis of a model in which we suppose that conduction by excess electrons occurs at the surface of a crystal of p-type material.

It is proposed that the previous chemical and electrical treatment of a specimen has resulted in a layer of positive charge on the surface of the p-type material. The presence of the positive surface charge requires a net deficit of positive charge within the crystal. Part of the deficit is provided by a loss of holes from the valence band near the surface of the crystal. Part of it is also provided by the presence of electrons in the conduction band. It is these electrons which form a layer of n-type conductivity at the germanium surface. The situation is represented diagrammatically in Fig. 1(a). If the specimen is in a field free region so that no electric field emerges from the surface, the net charge of the specimen must be zero. Deep in the interior of the crystal the charge density is, of course, zero. Thus, the positive surface charge must be just balanced by a negative space charge composed of negatively charged acceptors and excess electrons in a region adjacent to the surface.

It has been possible to study the layer of n-type conductivity shown in Fig. 1(a) by making use of single crystals of germanium having an n-p-n structure. In effect, at the surface of the crystal there is a continuous n-type region as a result of the positive surface charge on the p-type material. Figure 1(b) shows the potential energy of an electron in this structure. At the surface there is a channel of low potential energy across the p-layer. This channel provides a conducting path through which excess electrons may flow from one n-region to the other.

The n-p-n structures studied were actually experimental junction transistors. We will have occasion to refer to the two n-regions as emitter and collector and to the p-region as the base, consistent with transistor terminology.

In a sense the layer of n-type conductivity over the p-region forms a p-n junction. Between the excess electrons at the surface and the excess holes in the interior there is a region of substantially zero conductivity where there are neither electrons nor holes. This region acts like a dielectric, and changes in potential between the n-type region and the p-type region should result in charging currents like those for a condenser. According to the model, there should be a relationship between the conductance of the electrons in the channel and the charge of the condenser. A study of the relationship between conductance and capacity for the experimentally observed channels constitutes one of the strongest pieces of evidence for their existence. In order to explain some of these effects, however, it is necessary to suppose that the charge of the condenser does not reside entirely in mobile electrons but also in the charging and discharging of surface states. This is a familiar phenomenon in connection with the surface of germanium.[2,3]

The details of the calculation of the channel conductance and capacity that are a consequence of the model described above are given in Sec. 4.

2. EXPERIMENTAL EVIDENCE FOR CHANNELS

The presence of channels of n-type conductivity across the p-type base layers in experimental n-p-n transistors made itself conspicuous in the course of studying current-voltage characteristics of these units. One of the observations was made by leaving the emitter open-circuited while a reverse bias was applied between the base and the collector. This reverse potential tends to extract electrons from the base and

[1] J. Bardeen and W. H. Brattain, Phys. Rev. **75**, 1208 (1949).

[2] W. Shockley and G. L. Pearson, Phys. Rev. **74**, 232 (1948).

[3] J. Bardeen and W. H. Brattain, Bell System Tech. J. **82**, 1 (1953).

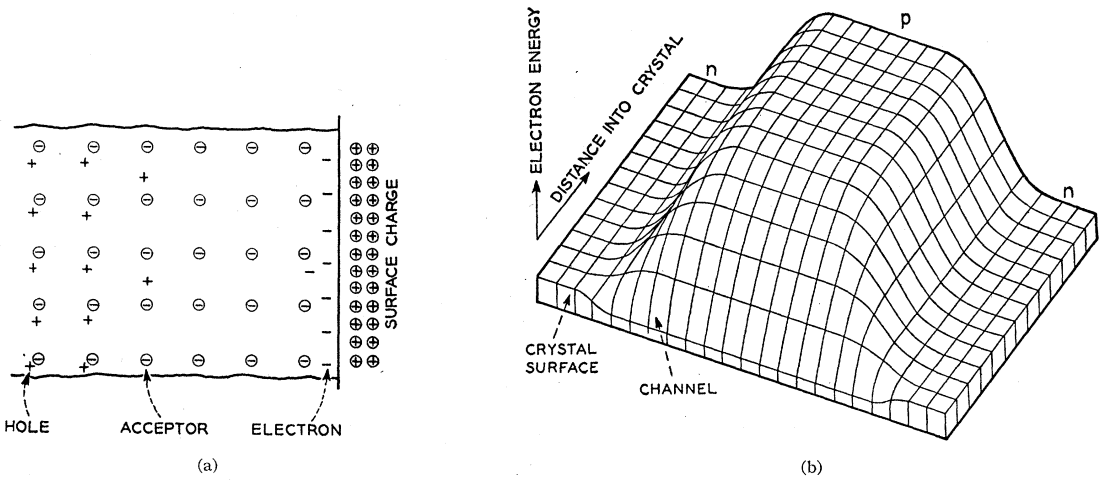


FIG. 1. (a) p-type germanium with a positive surface charge. (b) Potential energy of electrons in an n-p-n with a channel.

in fact to siphon some electrons from the emitter region over the potential maximum in the base layer and into the collector. This loss of electrons serves to charge the emitter positive so that it also assumes a reverse bias. The diffusion theory for p-n junction transistors[4,5] shows that the maximum potential which can be developed by this siphoning action is given by

$$V_f = (kT/q)\ln(1-\alpha),$$

where V_f is the emitter "floating potential," the open circuit potential of the emitter region with respect to the base, and where α is the efficiency of current transmission from emitter to collector through the base. Values of α as high as 0.998 have been observed. Such a value will correspond to a maximum floating potential of about 0.15 volt. It was found in experiments with these transistors that floating potentials as high as several volts could be observed. It was quite impossible to explain them by assuming that the connection between emitter and collector depended upon the diffusion of electrons through the base layer. The extra conducting path that is responsible for these floating potentials and that constitutes a defect in the n-p-n transistor is called a "channel."

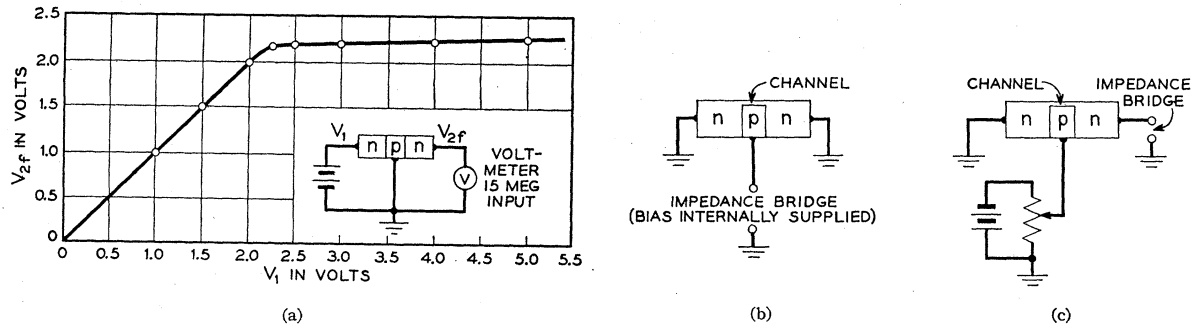
In addition to the appearance of anomalously high floating potentials there were other observations which indicated that the connection between emitter and collector was quite different in nature from the predictions of diffusion theory. In particular, there was the phenomenon of aging or "conditioning" the channel: The emitter floating potential would change slowly following a change in the reverse bias of the collector. Furthermore, the emitter floating potential depended upon the bias which had been applied to the emitter immediately before the floating potential measurement was made.

[4] W. Shockley, Bell System Tech. J. **28**, 435 (1949).
[5] Shockley, Sparks, and Teal, Phys. Rev. **83**, 151 (1951).

It is proposed that the conditioning effects described are a result of an increase in the positive charge on the surface of the p-layer when the p-n junctions are biased so as to extract electrons from the base. This can occur in either of two ways. Ions may actually move along the surface of the transistor so as to accumulate over the base layer. Alternatively, the removal of electrons from the base layer will result in a departure from equilibrium at the surface. Electrons emitted by molecules or ions on the surface will tend to drift away and not be replaced. The result will be to produce a positive charge on the surface. It would be possible to investigate the proposal further by shining light on an n-p-n structure while the conditioning voltages were applied, to see whether the presence of electrons produced by the light within the p-layer would prevent the formation of a channel.

As a consequence of their aging effects it is difficult to measure the properties of channels at room temperature. It has been found however that, if a channel is produced at room temperature and the n-p-n structure is then cooled to a temperature considerably below 0°C with the conditioning biases still applied, a much more stable situation is achieved. The low temperature effectively freezes the properties of the channel, and measurements can be made over a period of hours without appreciable drift in the channel characteristics.

If the connection between the emitter and the collector that leads to anomalous floating potentials is an n-type channel across the base, the floating potential of one n-region should exhibit a saturation effect as a function of the bias applied to the other n-region. When a reverse bias is applied between the two n-regions and the base, electrons will be removed from the channel. At a certain critical value of this bias, the pinch-off voltage, the number of electrons in the channel will be reduced substantially to zero. Under these conditions the channel conductance from one n-region to the other



(a) (b) (c)

FIG. 2. (a) Experimental floating potential curve. (b) Connections for measurement of channel capacity. (c) Circuit for channel conductance measurement.

should disappear and only the normal conductance by diffusion and conductance due to other leakages should remain.[6] If a potential equal to the pinch-off voltage is applied to the collector and the emitter is allow to float, electrons will flow from the emitter through the channel to the collector. This loss of electrons will make the emitter potential more positive until it approaches the pinch-off voltage. As it approaches this potential very closely, the emitter end of the channel will acquire a progressively lower conductance, and it will be continually more difficult for electrons to escape from the emitter region. If there is a leakage current of any sort between emitter and base, the replacement of electrons through this leakage path will prevent the floating emitter from quite reaching the potential at which electron flow through the channel would be impossible. Consequently, for any value of the collector bias at least as large as the pinch-off voltage, the emitter floating potential will approach the pinch-off voltage but will not quite reach it. For values of collector bias lower than pinch-off, the steady-state floating potential of the emitter must again be such that electrons escaping from the emitter through the channel will just equal the electrons arriving at the emitter from the base along other leakage paths. Since the channel conductance will not approach zero in this case, the emitter must float at a potential approximately equal to the collector potential. This argument has been given considering the emitter as floating, but it will apply equally well with the roles of collector and emitter interchanged.

These considerations account for the shape of the floating potential curves that have been observed. Figure 2(a) is an experimental curve for a channel which has been "frozen" at dry ice temperature to avoid conditioning effects. The potential of the floating n-region closely approximates the potential applied to the other n-region until a certain critical voltage is reached, after which its potential saturates.

The most important measurements made on channels were measurements at dry ice temperature of the capacity between the channel and the base layer and of the conductance of the channel between the emitter and collector regions. Both capacity and conductance were determined as a function of the reverse bias applied between the n-regions and the base. An impedance bridge was used to measure the capacity between the base and the emitter and collector connected together, Fig. 2(b). Since the capacities of the two internal p-n junctions are added to the channel capacity in such a measurement, a correction must be made for them. It was accomplished experimentally by letting the unit stand at room temperature without biases applied until the channel had disappeared. Then, cooling the structure to its previous low temperature, a capacity measurement showed the internal junctions alone. The channel capacity is the difference of these two measurements. The conductance of the channel between the n-regions was similarly determined with an impedance bridge, Fig. 2(c), as the difference of the conductances with and without a channel.

Figure 3 shows the results of the capacity and conductance measurements as a function of the bias applied between the n-regions and the base. These measurements were made on an n-p-n unit having a p-layer about 1.5 mils thick and 75 mils in circumference. The surface area of the p-type material was thus about 7×10^{-4} cm^2. The p-type resistivity was approximately 1 ohm cm at room temperature.

The left-hand ordinate of Fig. 3 refers to the experimental channel-to-base capacity and the capacity predicted from the constants of the p-layer (see Sec. 4). The experimental points are plotted with bars indicating the estimated limits of uncertainty in the measurements. The experimental curve at low voltages, although it is not well determined because of the uncertainties in the points, does not seem to have quite the theoretical slope. At biases above about 2 volts the theoretical curve should not apply since it is an approximation which neglects pinch-off. The magnitude of the capacity at low voltages agrees with the predictions about as well as the constants of the p-layer are known. In computing the theoretical curve the assumption has been made that the channel exists over the entire surface

[6] The channel is similar in its properties to a field-effect transistor. See W. Shockley, Proc. Inst. Radio Engrs. 40, 1365 (1952).

of the p-layer. This of course may not be the case,and assuming a smaller surface area would result in a proportionately smaller predicted channel capacity. The measurements seem to indicate that the whole p-layer surface is involved. These capacity measurements are typical of those made on several channels.

The right-hand ordinate of Fig. 3 refers to the measured channel conductance. In the preceding discussion of saturation floating potentials it was noted that the channel conductance was expected to decrease with increasing bias applied between n- and p-regions and to approach zero as the bias approached the critical pinch-off voltage. In this respect the conductance measurements verify the predictions. The conductance and capacity of Fig. 3 were measured for the same channel shown in Fig. 2(a). It is interesting to note that the saturation floating potential appears to be in good agreement with what one might select as the pinch-off voltage in Fig. 3. The residual conductance at the higher values of bias in Fig. 3 is apparently due to leakage through the coolant surrounding the unit.

We have noted previously that the conductance and the capacity of a channel are expected to be related to one another. In two respects the measurements do not bear out the expectations. In the first place when the channel is pinched off there should be no mobile electrons left to provide the charge on one plate of the germanium condenser. The capacity should thus approach zero as the conductance approaches zero at the pinch-off voltage. In the experimental curve there is no sharp decrease in capacity between 2 and $2\frac{1}{2}$ volts. The capacity which persists to higher voltages may be a result of nonuniformity in the positive surface charge which would tend to make the channel pinch off at the center of the p-layer before it would pinch off at the ends. With this type of nonuniformity, the channel conductance from emitter to collector could approach zero as the channel pinched off in the middle, but there would still be electrons near the ends to provide a capacity at higher values of the bias.

In the second place, the magnitude of the conductance is not at all what one would predict from the capacity data below pinch-off. With a change in bias ΔV_a there is a change in the condenser charge $\Delta Q = C\Delta V_a$. Corresponding to a decrease of V_a from $1\frac{1}{2}$ to 1 volts the experimental capacity is 14 $\mu\mu$f so that $\Delta Q = 7\times10^{-12}$ coulomb. On the channel side of the condenser this charge is provided by the addition of electrons to the channel. It should result in an increase in the channel conductance. If this charge is spread uniformly in a layer of length L (the thickness of the p-layer) across which there is a potential difference V, the drift velocity of the electrons in the layer will be $v_D = \mu V/L$. On the average each electron will drift through the channel in a time L/v_D, so that the increase in current corresponding to ΔQ is $\Delta I = \Delta Q v_D/L = \Delta Q \mu V/L^2$. This is equivalent to an increase in conductance

$$\Delta g = \mu \Delta Q/L^2.$$

With $L = 1.5$ mils, $\mu = 6700$ cm^2/volt sec (electrons at dry ice temperature) and $\Delta Q = 7\times10^{-12}$ coulomb, $\Delta g = 3\times10^{-3}$ mho. The observed increase in channel conductance for a bias change from $1\frac{1}{2}$ to 1 volt is 1×10^{-5} mho or about 300 times smaller.

There are two possible explanations for this discrepancy: (1) the effective mobility of the electrons in the channel may be lower than the mobility in bulk germanium or (2) the change in charge ΔQ may not all be provided by mobile channel electrons. The thickness of the channel within which the electrons must move is the order of a few hundred angstroms, considerably smaller than the mean free path of electrons in bulk germanium at dry ice temperature $l = 3800$A. The electrons will collide many times with the germanium surface for every time they are scattered by lattice vibrations. If the surface collisions are nonspecular, the first possibility indicated above arises. There is a reduction in the effective electron mobility, and hence in the conductance of the channel. The second possibility takes into account the role of surface states for electrons at the surface of the p-layer.[7] If

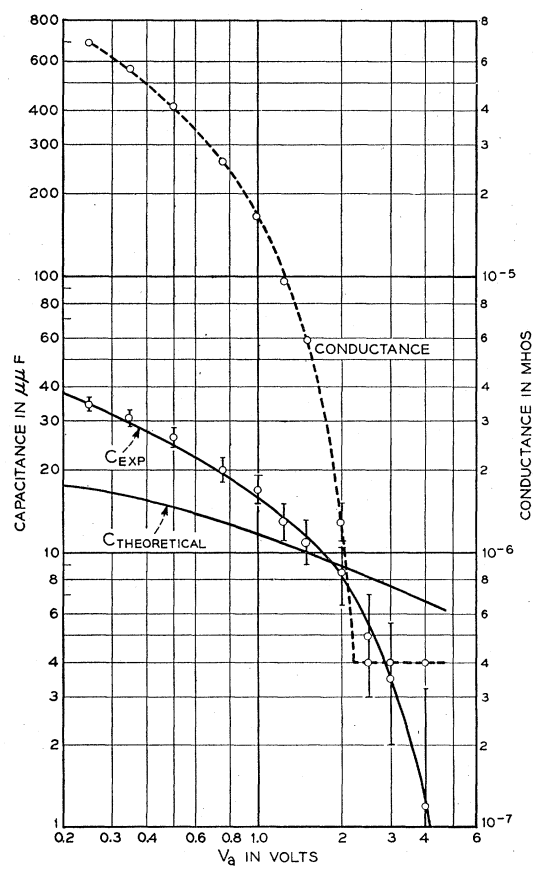


FIG. 3. Experimental channel capacity and conductance.

[7] J. Bardeen, Phys. Rev. 71, 717 (1947).

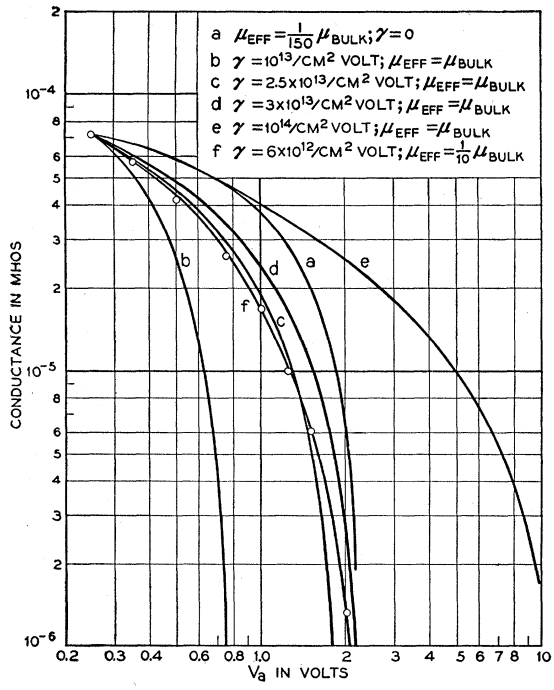


FIG. 4. Theoretical channel conductance.

such states exist, part of the charge associated with a change in bias on the channel will be provided by a change in the charge of the surface states. The corresponding change in conductance will then be smaller than if only mobile electrons had been added to or removed from the channel.

Figure 4 shows the comparison of calculations based on the two possible mechanisms mentioned above with the same experimental conductance points given in Fig. 3. All of the theoretical curves are fitted at the highest conductance point, corresponding to a bias of 0.25 volt. Curve (a) is fitted at this point by assuming the effective electron mobility is lower by a factor of 150 than in bulk germanium at dry ice temperature. It is assumed there are no surface states. There is, however, a second parameter at one's disposal in this calculation. That parameter has been fixed in curve (a) by choosing the pinch-off voltage as 2.2 volts. A factor of 150 reduction in the effective mobility is considerably larger than one would be inclined to select on the basis of the thickness of the channel compared with the normal electron mean free path. A factor an order of magnitude smaller would seem more reasonable. Furthermore, the shape of curve (a) does not agree very well with the experimental points.

Curves (b) through (e) have been computed assuming no reduction in mobility but different surface state densities as indicated. These curves illustrate how rapidly the pinch-off voltage varies with the surface state density. The density 3×10^{13} states/cm^2 volt gives pinch-off at the desired value of 2.2 volts. This curve (d) more nearly agrees with experimental points than curve (a), but it still is not a particularly good fit.

It is not unreasonable to suppose that both a reduced mobility and surface states play a part in determining the channel conductance. These effects have been introduced simultaneously in computing curve (f). It is fitted at $V_a=0.25$ volt, $V_a=1.0$ volt, and has a pinch-off voltage $V_0=2.2$ volts. The effective mobility for this curve is 10 times smaller than the bulk mobility, and the surface state density is 6×10^{12}/cm^2 volt. A second channel which has been analyzed in this same way gives an effective mobility about 5 times smaller than the bulk mobility and a surface state density about 2×10^{12}/cm^2 volt. These surface state densities are an order or magnitude smaller than the densities found by Shockley and Pearson.[2] The differences in the experiments do not make such a factor surprising.

Introducing surface states and a reduced mobility into the model has an insignificant effect on the predicted low voltage channel capacity. Thus by adding these concepts to the original model the relationship between channel capacity and conductance can be made consistent with the data at low voltages. The failure of the capacity to "pinch off" is not resolved by the model and apparently is to be attributed to nonuniformity in the surface charge.

3. ADDITIONAL CHANNEL EXPERIMENTS

The preceding experiments were done with *n-p-n* transistors completely encapsulated in plastic. With the expectation that changing the environment of the germanium surface would produce changes in the properties of the electron channel across the *p*-layer, a few experiments were done with bare *n-p-n* units.

The effect of water vapor on the saturation currents of *n-p-n* transistors was observed by Pietenpol and Ciccolella of the Bell Telephone Laboratories.[8] They found that large increases in current could be produced by increasing the relative humidity. Channels were found to show this same sensitivity to water vapor. An *n-p-n* with a high floating potential at moderate humidity would lose all evidence of a channel when put into a container with a desiccant.

It was also observed that the vapors of nitrobenzene and formamide, organic substances with large dipole moments, gave high floating potentials, but the possibility that water was a contributing factor in these experiments was not entirely eliminated. Benzene, carbon tetrachloride, and toluene, however, produced no channels.

It was found that the encapsulation process used on the *n-p-n* units of the previous experiments did not prevent the penetration of water vapor to the transistors. This strongly suggests that water vapor is responsible for the channels on these units.

[8] W. J. Pietenpol and D. F. Ciccolella (private communication).

In two bare n-p-n's no channels at all were developed by conditioning even in atmospheres of high humidity. These were units having a 25-mil and a 37-mil p-layer. This result is consistent with either of the proposed mechanisms by which the surface of the p-layer might obtain its positive charge.

In several respects the channels produced by conditioning n-p-n units are not altogether satisfactory. In the first place to examine the properties of the channel as a function of the potentials applied to it one must go to low temperatures to achieve stability. Furthermore, the actual surface area of the channel is unknown.

In the hope of producing a channel over which there was a more direct control a number of experiments were done with a drop of the electrolyte glycol borate covering the p-layer of a bare n-p-n. By means of a gold electrode a potential was applied to the electrolyte with respect to the p-type material. When this potential is positive, a layer of positive ions will be formed at the germanium surface, and a channel will be produced if the ions are numerous enough.

The most satisfactory result of these experiments was obtained with an n-p-n slab having a 37-mil p-layer. The floating potential varied with the potential of the gold electrode in the way expected. With the p-layer at ground, grounding the gold electrode or applying a negative voltage to it produced a small floating potential, consistent with the predictions of diffusion theory. Positive voltages on the gold electrode gave floating potentials which increased with increasing applied voltage. The channel conductance and capacity measurements also showed the expected qualitative changes with bias and with the gold electrode potential. However, the channel was not stable enough in time to permit a satisfactory comparison with the detailed calculations made from the model. Furthermore, lowering the temperature did not improve the situation in this case. In fact it eliminated the channel altogether.

Other n-p-n units with glycol borate were considerably less satisfactory than this one, a common result being the appearance of an inductance when a capacity was expected. In all cases, including the best one, the currents drawn by the two junctions in reverse increased from 10- to 100-fold when the electrolyte was placed across them.

In addition to these experiments with double function devices, several attempts were made to use an electrolyte in conjunction with a single p-n junction. The idea was the following: If a channel could be developed extending out over the material on one side of the junction, hole electron pairs introduced into the channel by light might provide a measurable photo-current with the light farther from the junction than it would if the channel were absent. There was only a slight indication that such an effect occurred.

The use of glycol borate to develop channels was in general unsatisfactory. It did not provide a more direct and stable control over channels (the principal reason it was used). It further complicated the situation by always producing lower reverse junction impedances and often providing inductances when capacities were expected. These effects are not yet understood.

4. THEORETICAL CONSIDERATIONS

In this section the principal features of the n-type surface conductivity discussed in the preceding pages will be presented in a more mathematical form. The scheme of representation to be used is indicated in Fig. 5(a). Only the p-type base region is shown. The energy of an electron is plotted vertically with E_c the bottom of the conduction band and E_v the top of the valence band. The negatively charged acceptor centers are shown as energy levels in the energy gap just above E_v. The mobile holes are just below E_v in the valence band. The Fermi energy is $-q\varphi$, where φ is the Fermi potential and q the electronic charge. The electrostatic potential ψ in the crystal will be measured so that the energy $-q\psi$ is that appropriate to an electron in the middle of the energy gap.[9] With this definition of ψ,

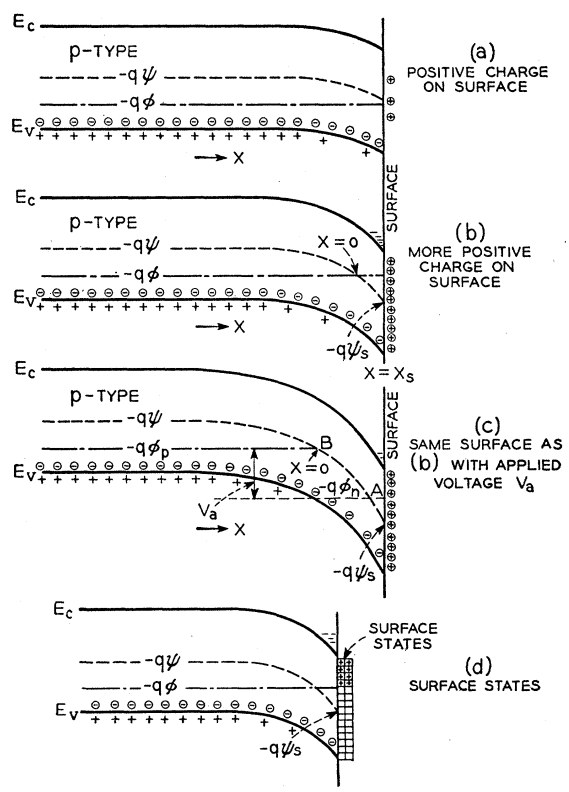


FIG. 5. Effective of positive charge at a p-type surface.

[9] This assumes equal effective masses for electrons and holes. If the masses are different, ψ is measured at a level slightly displaced from the center of the forbidden region. Details of this procedure together with a discussion of the quasi-Fermi potentials will be found in W. Shockley, *Electrons and Holes in Semiconductors* (D. van Nostrand Company, Inc., New York, 1950), Chap. 12.

when $\psi=\varphi$ the hole and electron concentrations are equal to the intrinsic concentration n_i. For the equilibrium case:

$$p=n_i e^{q(\varphi-\psi)/kT}, \quad n=n_i e^{q(\psi-\varphi)/kT}.$$

In a nonequilibrium case it will be convenient to define two quasi-Fermi potentials φ_n and φ_p such that

$$p=n_i e^{q(\varphi_p-\psi)/kT}, \quad n=n_i e^{q(\psi-\varphi_n)/kT}.$$

We will first examine the appearance of a channel from the standpoint of the energy bands in germanium. When positive charge is placed on the surface of p-type germanium it will produce an electric field which distorts the energy bands in the crystal, producing a negative space charge inside the surface to compensate for the positive surface charge. If the surface charge is small, the p-type material will simply have fewer holes near the surface, the space charge being made up entirely of ionized acceptor centers, Fig. 5(a). If the positive surface charge is large enough, the energy bands may be bent to such an extent that the electrostatic potential crosses the Fermi potential, and electrons become more numerous than holes, Fig. 5(b). This situation can apply only very close inside the germanium surface, but it results in a layer of n-type conductivity on the normally p-type material. It will constitute a channel if the p-type material appears as the base in an n-p-n transistor. The details of the space charge region for this case of larger surface charges provide the channel characteristics that have previously been described.

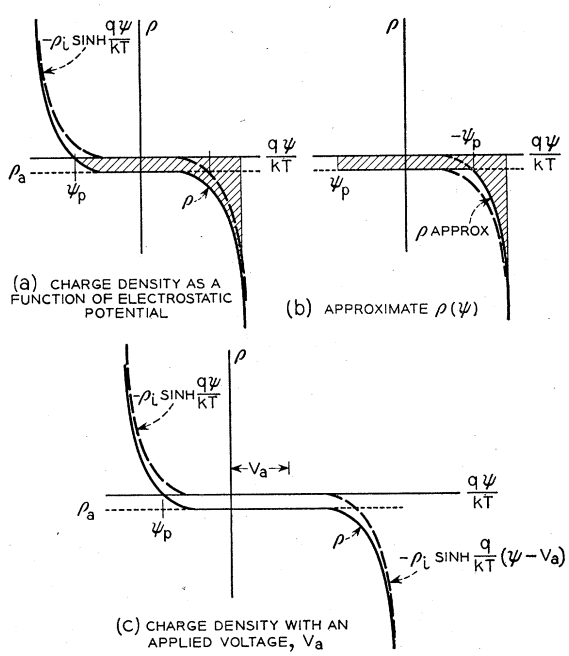


FIG. 6. Theoretical charge densities.

We will assume a one-dimensional problem: p-type germanium with uniform properties extending indefinitely to the left of a plane on which there is a uniform surface charge with a density Q_s. We will also assume that there is no electric field outside of the germanium so that the system as a whole is electrically neutral. We will measure potentials with respect to the Fermi potential in the equilibrium case and will select as origin of the x-coordinate the plane in the p-type material at which $\varphi=\psi$.

Within the semiconductor of dielectric constant κ and having a charge density ρ, Poisson's equation,

$$d^2\psi/dx^2=-4\pi\rho/\kappa, \tag{1}$$

is to be satisfied with the following boundary conditions: (a) $x=-\infty$, $\psi=\psi_p$, $d\psi/dx=0$. Deep in the interior of the p-type material the potential approaches ψ_p, the value which it would have in the absence of a surface charge. The electric field vanishes at this point.

(b) $(d\psi/dx)_{x_s}=4\pi Q_s/\kappa$. Gauss' theorem applied to the charged surface leads to this boundary condition for the electric field in the germanium at $x=x_s$.

Multiplying (1) through by $d\psi/dx$ and integrating,

$$\left(\frac{d\psi}{dx}\right)^2=\frac{-8\pi}{\kappa}\int_{\psi_p}^{\psi}\rho d\psi. \tag{2}$$

In the equilibrium case, $\rho(\psi)$ has the form

$$\rho=\rho_i \sinh(q\psi/kT)+\rho_a \tag{3}$$

where $\rho_i=2qn_i$ and $\rho_a=-qN_a$. N_a is the density of acceptors. Figure 6(a) shows $\rho(\psi)$. The dashed line is the charge density due to mobile carriers, predominately holes for $\psi<0$, predominately electrons for $\psi>0$. The dotted line is the charge density of ionized acceptors. The solid line is the sum of these, the total charge density. Here ψ_p is the value of ψ for which $\rho=0$ since electrical neutrality exists in the undisturbed p-material. The square of the electric field corresponding to some value of ψ is then proportional to the area under the curve and between ψ_p (a negative quantity) and ψ. One such area is shown shaded in Fig. 6(a). It is clear that with known values of Q_s and ρ_a the total electrostatic potential difference between the interior and the surface can be found by simply extending the integral to a value of $\psi=\psi_s$ satisfying the boundary condition (b) above. This is equivalent to saying that in order for the system to be electrically neutral the charge per unit area in the space charge region must be equal and opposite to Q_s.

For values of ψ close to zero the charge density of holes and electrons is small compared with the fixed charge density of acceptors. As ψ approaches $-\psi_p$, n-p increases rapidly and greatly exceeds N_a for values of ψ larger than $-\psi_p$ by a few times kT/q. To simplify the integral (2) we will approximate the actual $\rho(\psi)$ as

given in (3) by: $\rho=\rho_a$ for $|\psi|<\psi_p$, and

$$\rho=-\rho_i\sinh(q\psi/kT)\cong-qn_ie^{q\psi/kT} \quad \text{for} \quad \psi>-\psi_p. \quad (3)$$

This approximates the area shown shaded in Fig. 6(a) by that shaded in Fig. 6(b). For values of $\psi<-\psi_p$, the square of the electric field will be larger in this approximation than it actually is. For $\psi>-\psi_p$, it will be smaller. For the experimental conditions which have been encountered, the field given by this approximation is in error by less than 10 percent for any $\psi>-\psi_p$, and for most values of ψ in this range the error is considerably smaller.

In considering the properties of channels, we will only be concerned with values of Q_s large enough so that electrons will be present at the germanium surface with a density $n>N_a$. This is equivalent to restricting our attention to surface charges so large that boundary condition (b) will require extension of the field integral to values of $\psi>-\psi_p$.

In the nonequilibrium case when a voltage is applied between the p- and n-regions, the carrier densities are to be expressed in terms of the quasi-Fermi potentials φ_n and φ_p. The variations in the quasi-Fermi potentials in the present situation are quite similar to those discussed by Shockley[3] for normal p-n junctions. The applied potential establishes a difference between φ_n in the n-type material and φ_p in the p-type material. Furthermore the quasi-Fermi potentials will have only a small slope in the material for which they specify the density of the majority carrier. In the n-p-n structure with a channel, the high electron density at the surface of the p-region is in effect an n-type layer connected to the normal n-regions at either end. The potential applied between the p-region and the two n-regions connected together thus appears as a difference $V_a=\varphi_n-\varphi_p$ between φ_n which describes the concentration of electrons in the channel and φ_p which describes the hole concentration in the base. The situation is shown in Fig. 5(c). φ_n and φ_p have been drawn horizontal over the range for which they represent the majority carrier. We will not be concerned with the shape of φ_n to the left of A or with the shape of φ_p to the right of B. They will certainly not recross the electrostatic potential line so that between A and B the hole and electron densities will both be less than n_i.

Figure 6(c) indicates the nonequilibrium dependence of ρ on ψ. We now measure potentials with respect to φ_p in the body of the p-material. As in the equilibrium case we get the square of the electric field at any point as the area under ρ between ψ_p and the value of ψ appropriate to the point. Since the surface field is fixed by (b), as V_a is increased the number of electrons in the channel will decrease, a greater fraction of the total space charge that is required by Q_s being provided by acceptor ions. Eventually as V_a is increased to a critical voltage V_0, the surface field will be provided by the acceptor ions alone, no electrons will be required, and the channel will be "pinched off." As in the equilibrium case we will approximate the sum of fixed and mobile charges by the larger of the two.

$$\begin{aligned} \rho&=\rho_a, && \psi_p<\psi<V_a-\psi_p,\\ \rho&=-qn_ie^{q(\psi-V_a)/kT}, && V_a-\psi_p<\psi<\psi_s. \end{aligned} \quad (4)$$

With this approximation the criterion for pinch-off will be that value of $V_a=V_0$ for which $\psi_s=V_0-\psi_p$ or $\psi_s-\varphi_n=-\psi_p$; the difference between the electrostatic potential and the quasi-Fermi level for electrons is just just such as to make $n=N_a$ at the surface.

The actual charge Q_s on the surface has not been measured, but a knowledge of V_0 will suffice since it allows an approximate calculation of the surface field to be made if ρ_a is known. It was indicated in the preceding section that a measurement of the floating potential of one n-region as a function of the bias applied to the other n-region in an n-p-n structure should show a saturation floating potential approximately the pinch-off voltage. If the leakage paths for electrons to the floating n-region have a very low conductance, the saturation floating potential will be a good approximation to V_0. Of course, by measuring the channel conductance directly as a function of bias, V_0 may be determined.

When the channel is pinched off, the charge density making up the whole space charge layer will be approximately ρ_a. The electric field integral (2) is then

$$E_s^2=\left(\frac{d\psi}{dx}\right)_{x_s}^2=\frac{-8\pi}{\kappa}\int_{\Psi_p}^{\psi_s=V_0-\psi_p}\rho_a d\psi=\lambda(V_0-2\psi_p), \quad (5)$$

where $\lambda=(8\pi/\kappa)(-\rho_a)$. If the applied voltage is insufficient to pinch off the channel the charge density will be approximately given by (4). The electric field integral gives a value at the surface:

$$E_s^2=\lambda\left\{V_a-2\psi_p+\frac{kT}{q}\left[e^{q[\psi_s-(V_a-\psi_p)]/kT}-1\right]\right\}. \quad (6)$$

Since the surface charge Q_s is assumed to be constant with variations in V_a, the electric field at the surface must be the same for $V_a=V_0$ or for any smaller value: (5) and (6) must be equal. This provides a condition determining ψ_s as a function of V_a;

$$\psi_s=V_a-\psi_p+\frac{kT}{q}\ln\left[\frac{V_0-V_a}{kT/q}+1\right]. \quad (7)$$

This value of the electrostatic potential at the surface, or more directly the difference ψ_s-V_a, gives the electron density at the surface from (4). We could make use of this density to compute the total number of electrons in the channel per unit of surface area, but we will see presently that there is a simplier method of achieving the same result.

We shall have occasion later to use an expression for the electric field at points throughout the space charge region. The field corresponding to any value of ψ is obtained using (2) and (4).

$$E^2=\left(\frac{d\psi}{dx}\right)^2=\lambda(\psi-\psi_p) \quad \text{if} \quad \psi_p<\psi<V_a-\psi_p,$$

$$E^2=\lambda\left\{V_a-2\psi_p+\frac{kT}{q}\left[e^{q[\psi_s-(V_a-\psi_p)]/kT}-1\right]\right\} \quad \text{if} \quad V_a-\psi_p<\psi<V_0-\psi_p. \tag{8}$$

In Eq. (6) which gives the square of the surface electric field, the first two terms, $\lambda(V_a-2\psi_p)$, represent the contribution that is made by the acceptor ions alone. The difference between the surface field, E_s, and the field produced by the acceptors alone $E_1=[\lambda(V_a-2\psi_p)]^{\frac{1}{2}}$ is proportional to N, the total number of channel electrons per unit of surface area. By Gauss' theorem,

$$4\pi qN/\kappa=E_s-E_1, \tag{9}$$

and using (5):

$$4\pi qN/\kappa=[\lambda(V_0-2\psi_p)]^{\frac{1}{2}}-[\lambda(V_a-2\psi_p)]^{\frac{1}{2}}. \tag{10}$$

The conductance of the channel is directly related to the quantity qN. If Q is the total electron charge in the channel, L is the channel length, and μ the electron mobility, the channel conductance is

$$g=Q\mu/L^2.$$

using the same reasoning applied for a similar case in Sec. 2. However,

$$Q=(qN)wL,$$

where w is the width of the channel, so that

$$g=qN\mu w/L.$$

From (9)

$$g=\mu(w/L)[\kappa(-\rho_a)/2\pi]^{\frac{1}{2}}[(V_0-2\psi_p)^{\frac{1}{2}}-(V_a-2\psi_p)^{\frac{1}{2}}]. \tag{11}$$

This expression indicates a monatonic decrease in conductance as V_a is increased from zero toward V_0. At $V_a=V_0$ the conductance apparently becomes zero. The detailed dependence of g on V_a for very small values of g is not properly represented by (11) because we have neglected electron densities $n<N_a$. The value of the conductance below which this detail becomes important is, however, much smaller than the smallest channel conductance that has been measured.

It was mentioned earlier that the channel electrons were confined to move in a very thin layer adjacent to the germanium surface. We are now in a position to make an approximate evaluation of this thickness. The density of electrons decreases by $1/e$ for a decrease of kT/q in ψ as we go away from the surface. ψ changes by kT/q in a distance which falls within the limits

$$kT/qE_1>\tau>kT/qE_s.$$

The upper limit is the distance over which the potential would change by kT/q is there were only the electric field of the acceptor ions. The lower limit is the distance corresponding to the electric field at the surface. Most of the channel electrons are constrained to move in a layer of thickness τ. E_1 of course increases with V_a, but for material which is 1 ohm cm p-type at room temperature and a floating potential of 2 volts, the upper limit for τ will never exceed a few hundred angstroms. The lower limit for this case will be about 80A.

We have already indicated in Fig. 5(b), (c) that the space charge layer increases in thickness as the applied voltage is increased. As a result, some holes are expelled from the p-material and an equal number of electrons are removed from the channel. This effect is describable as a capacity when V_a has a small ac component. On a unit area basis,

$$C=(-\rho_a)dx/dV_a, \tag{12}$$

since $(-\rho_a)dx$ is just the charge of the holes expelled by an increase dV_a which results in a width increase dx.

We can obtain the width of the space charge layer by a second integration of the field integral (2). If $(d\psi/dx)^2=f(\psi)$, the distance from $x=0$ (the point at which $\psi=0$) to the surface will be

$$x_s=\int_{\psi=0}^{\psi_s}\frac{d\psi}{\sqrt{f(\psi)}}. \tag{13}$$

This is not the total extent of the distortion of the energy band because we have neglected the part corresponding to $\psi<0$. As ψ decreases until it only exceeds ψ_p by the order of kT/q, it subsequently approaches ψ_p exponentially with a characteristic length, the Debye length. Thus the disturbance produced by the surface charge actually extends indefinitely far into the interior of the p-material. However as we change V_a we do not change the electric field for any values of $\psi<0$, so that, whatever the shape of the energy bands to the left of $x=0$, it will remain the same for any V_a and will simply move as a whole with respect to the surface as x_s changes. Thus what we require in (11) is the variation of x_s with V_a.

We can think of x_s as made up of two parts: x_1 the distance to the right of $x=0$ over which the acceptors contribute the predominate charge, and x_2 the width of the channel, i.e., the distance adjacent to the surface over which electrons out-number the acceptors. As V_a increases, x_1 increases and x_2 decreases. Except near pinch-off, x_2 varies much more slowly with V_a than x_1 does. We will make the approximation that dx_s/dV_a

$\simeq dx_1/dV_a$. From (8) and (12),

$$x_1=\int_{\psi=0}^{V_a-\psi_p}\frac{d\psi}{\sqrt{f(\psi)}}=\int_0^{V_a-\psi_p}\frac{d\psi}{[\lambda(\psi-\psi_p)]^{\frac{1}{2}}},$$

so that

$$dx_1/dV_a=1/[\lambda(V_a-2\psi_p)]^{\frac{1}{2}},$$

and

$$C=\frac{(-\rho_a\kappa/8\pi)^{\frac{1}{2}}}{(V_a-2\psi_p)^{\frac{1}{2}}}. \tag{14}$$

For 1 ohm-cm p-type material this differential capacity is

$$C=2\times10^{-8}/(V_a-2\psi_p)^{\frac{1}{2}}\quad \text{farad/cm}^2,$$

where the potentials are in volts.

Expression (14) has been plotted in Fig. 3. As a result of neglecting the variation of x_2 with V_a (14) does not include any upper limit for V_a. Such a limit does exist, however, and for $V_a=V_0$ the capacity should be zero. The details of the way in which C approaches zero as V_a approaches V_0 require a more complete treatment than has been given above.

To a rough approximation C varies as $V_a^{-\frac{1}{2}}$ (the dependence to be expected in a p-n junction formed with a sudden change in impurity concentration). However, no values of V_0 have been observed which are large enough to make this approximation valid over any reasonable range of V_a. Simply adding more ions to the surface of the p-region to increase the floating potential by a large amount does not in general produce a simpler situation. In the first place, if V_0 becomes very high, at $V_a=0$ the number of electrons required to satisfy the surface field will be so large that degeneracy will exist, the Fermi level will rise above the bottom of the conduction band, and the simple charge density expression (6) will no longer hold.

In the second place, as the number of ions on the surface increases, the electric field at the surface will also increase and ultimately will exceed the Zener field in germanium. At this point the normal high impedance junction between the channel and the p material will be shunted by the low impedance Zener region, and the simple channel conductance from one n-region to the other will be drastically changed. As V_0 is increased by putting more and more ions on a particular p-layer, at room temperature, the Zener condition will be reached in germanium before degeneracy occurs. In fact, except for values of $N_a>10^{18}/\text{cm}^3$, for a Zener field of 10^5 volt/cm degeneracy will not become the primary consideration until the temperature is lowered below about 80°K.

It is interesting to note how high the electric fields actually are at the surface to give floating potentials of the order measured. From (6)

$$(d\psi/dx)_{x_s}=[8\pi(-\rho_a)/\kappa]^{\frac{1}{2}}(V_0-2\psi_p)^{\frac{1}{2}},$$

which for the same special case we have previously considered ($V_0=2.0v$, 1Ω cm p-type) gives

$$(d\psi/dx)_{x_s}=4\times10^4\quad \text{volt/cm}.$$

This is a field not a great deal smaller than the Zener field.

The effect of introducing surface states into the problem will be considered briefly. We will assume that there are electron surface states distributed uniformly in the energy gap, Fig. 3(d). These states will be assumed neutral when filled with an electron and positively charged when empty. They are characterized by two parameters: γ the density of states per unit area per unit potential, and Q_0 the net charge per unit area in the surface states if they are filled up to the middle of the energy gap. The Fermi potential indicates the actual level of filling, and since ψ by definition lies at the middle of the energy gap, the charge in the surface states in general is

$$Q_0-q\gamma(\psi_s-\varphi_n).$$

This changes the boundary condition (b) to

$$(d\psi/dx)_{x_s}=(4\pi/\kappa)[Q_0-q\gamma(\psi_s-\varphi_n)].$$

Or, measuring potentials with respect to φ_p as we have done before,

$$(d\psi/dx)_{x_s}=(4\pi/\kappa)[Q_0-q\gamma(\psi_s-V_a)]. \tag{b'}$$

If the same approximations are made for $\rho(\psi)$ as were made in the previous case, we again obtain Eqs. (5) and (6) by simply integrating the charge density from deep in the interior of the p-type material to the surface. However, the boundary condition at the surface no longer requires that the surface field be a constant, independent of V_a, because of the presence of the surface states. Expressions (5) and (6) cannot simply be equated. For each V_a, ψ_s must be determined from (6) and (b′). From the value of ψ_s and using (b′) and (9) the density of the channel electrons, N, can be found.

In calculating ψ_s the two surface state parameters are at our disposal. As in the previous case Q_0 is directly related to the pinch-off voltage V_0 and may be determined from it. γ can then be chosen so that the channel conductance fits the experimental value at some V_a assuming a value for the electron mobility.

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