Current Density Tables for Field Emission Theory*

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The Fowler-Nordheim field emission equation permits calculation of current density J as a function of surface electric field F and work function ϕ. Computed values of J for $1.00\times 10^7 < F < 1.00\times 10^8$ v/cm and for $2.00 < \phi < 6.30$ ev are tabulated; the table is of use in connection with several theoretical and experimental aspects of field emission.

INVESTIGATION of field emission phenomena requires frequent reference to the fundamental Fowler-Nordheim theory[1,2] in which the emission current density J in amperes/cm^2 is given as a function of the surface electric field F in v/cm and the work function ϕ in ev by the equation:[3]

$$J=(1.54\times 10^{-6}F^2/\phi)\exp[-6.83\times 10^7\phi^{\frac{3}{2}}f(y)/F], \quad (1)$$

where $f(y)$ is the Nordheim elliptic function[2,4] of the variable $y=3.79\times 10^{-4}F^{\frac{1}{2}}/\phi$. In these expressions, the numerical coefficients have been altered to conform with recent values of the physical constants. The comparison of experiment with theory necessitates repeated calculations of current densities from Eq. (1), or its logarithmic equivalent, for a wide range of F and for values of ϕ which depend upon the nature of the emitting surface.

Experimental results in these laboratories[5] have stimulated renewed interest in field emission theory for several reasons. First, pulse techniques have produced stable field current densities of the order of 10^8 amperes/cm^2 at fields of 8×10^7 v/cm for which the approximations involved in the derivation of Eq. (1) become questionable. Second, space charge effects[5] cause marked deviations from the current-voltage relationship expected in view of Eq. (1). Third, previous results here indicate that the transition from normal field emission to the vacuum arc occurs when the emitter is resistively heated, the problem thus involving electron emission in the presence of simultaneous high fields and high temperatures. This problem has had only partial theoretical treatment.[6–8]

In each of these studies, Eq. (1) is the basic reference for cold cathode emission in the absence of space charge. Table I has therefore been prepared, comprising

TABLE I. Common logarithms of field current density J in amperes/cm^2, from Eq. (1), for various values of electric field F in v/cm and work function ϕ in ev.

$F\times 10^{-7}$ (v/cm)	ϕ (ev) 2.00	2.50	3.00	3.50	4.00
1.0	3.04	−0.54	−4.45	−8.64	
1.2	4.51	+1.48	−1.80	−5.32	
1.4	5.56	2.94	+0.10	−2.94	
1.6	6.36	4.04	1.53	−1.14	
1.8	6.98	4.90	2.66	+0.26	
2.0	7.48	5.59	3.56	1.39	−0.90
2.2	7.90	6.16	4.30	2.32	+0.22
2.4	8.25	6.64	4.92	3.09	1.16
2.6	8.54	7.05	5.45	3.75	1.96
2.8		7.40	5.91	4.32	2.65
3.0		7.71	6.30	4.82	3.25
3.2		7.98	6.65	5.25	3.78
3.4		8.22	6.96	5.64	4.24
3.6		8.43	7.24	5.98	4.66
3.8		8.63	7.49	6.29	5.03
4.0		8.80	7.71	6.57	5.37
4.2		8.96	7.92	6.82	5.67
4.4			8.11	7.06	5.96
4.6			8.28	7.27	6.21
4.8			8.44	7.46	6.44
5.0			8.59	7.65	6.66
5.2			8.72	7.82	6.87
5.4			8.85	7.97	7.05
5.6			8.97	8.12	7.23
5.8			9.08	8.25	7.39
6.0			9.18	8.38	7.55
6.2			9.28	8.50	7.69
6.4				8.61	7.83
6.6				8.72	7.95
6.8				8.82	8.07
7.0				8.92	8.19
7.2				9.01	8.30
7.4				9.09	8.40
7.6				9.18	8.50
7.8				9.25	8.59
8.0				9.33	8.68
8.2				9.40	8.76
8.4				9.47	8.84
8.6					8.92
8.8					9.00
9.0					9.07
9.2					9.14
9.4					9.20
9.6					9.27
9.8					9.33
10.0					9.39

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TABLE I.—*Continued.*

$F\times10^{-7}$ (v/cm)	ϕ(ev) 4.35	4.50	4.65	5.00	5.50	6.30
2.0	−2.58	−3.32	−4.06	−5.85	−8.50	−12.97
2.2	−1.31	−1.99	−2.66	−4.30	−6.71	−10.78
2.4	−0.25	−0.87	−1.50	−2.99	−5.22	−8.96
2.6	+0.65	+0.08	−0.50	−1.89	−3.95	−7.41
2.8	1.43	0.90	+0.36	−0.94	−2.86	−6.08
3.0	2.11	1.61	1.10	−0.11	−1.91	−4.92
3.2	2.70	2.23	1.75	+0.61	−1.08	−3.91
3.4	3.22	2.78	2.33	1.25	−0.34	−3.02
3.6	3.69	3.27	2.84	1.82	+0.32	−2.22
3.8	4.11	3.71	3.30	2.34	0.90	−1.50
4.0	4.49	4.11	3.72	2.80	1.44	−0.85
4.2	4.84	4.47	4.10	3.22	1.92	−0.27
4.4	5.15	4.80	4.45	3.60	2.35	+0.26
4.6	5.44	5.11	4.76	3.95	2.76	0.75
4.8	5.71	5.38	5.06	4.28	3.13	1.20
5.0	5.95	5.64	5.33	4.58	3.47	1.62
5.2	6.18	5.88	5.58	4.85	3.78	2.00
5.4	6.39	6.10	5.80	5.11	4.08	2.35
5.6	6.59	6.31	6.02	5.35	4.35	2.68
5.8	6.77	6.50	6.22	5.57	4.60	2.99
6.0	6.94	6.68	6.41	5.78	4.83	3.28
6.2	7.10	6.85	6.59	5.97	5.06	3.55
6.4	7.26	7.01	6.76	6.16	5.28	3.81
6.6	7.40	7.16	6.91	6.33	5.48	4.04
6.8	7.53	7.30	7.06	6.50	5.66	4.27
7.0	7.66	7.43	7.20	6.65	5.84	4.48
7.2	7.78	7.56	7.33	6.80	6.00	4.68
7.4	7.90	7.68	7.46	6.93	6.16	4.88
7.6	8.01	7.80	7.58	7.07	6.31	5.06
7.8	8.11	7.90	7.69	7.19	6.46	5.23
8.0	8.21	8.01	7.80	7.31	6.60	5.40
8.2	8.31	8.11	7.91	7.43	6.72	5.55
8.4	8.40	8.20	8.00	7.54	6.85	5.70
8.6	8.48	8.29	8.10	7.64	6.97	5.85
8.8	8.57	8.38	8.19	7.74	7.08	5.98
9.0	8.65	8.46	8.28	7.84	7.19	6.12
9.2	8.72	8.54	8.36	7.93	7.30	6.24
9.4	8.80	8.62	8.44	8.02	7.40	6.36
9.6	8.67	8.69	8.52	8.10	7.49	6.48
9.8	8.94	8.76	8.59	8.18	7.58	6.59
10.0	9.00	8.83	8.66	8.26	7.67	6.69
12.0	9.54	9.40	9.25	8.91	8.41	7.58
14.0		9.83	9.69	9.39	8.95	8.23
16.0				9.76	9.37	8.72
18.0					9.70	9.12
20.0					9.97	9.44

common logarithms of J predicted by Eq. (1) for a range of fields and work functions found useful in the present series of investigations here. Recent corrections by Houston[4] of Nordheim's elliptic function have been employed. The values of ϕ included in the table were chosen for various experimental reasons. $\phi=4.5$ ev is the accepted average value for tungsten, the cathode material most commonly used here. The values 4.35 and 4.65 ev are those found by Nichols[9] for tungsten crystal faces (116) and (110), respectively, which become individually significant for single crystal cathodes. $\phi=5.0$ ev is an estimate[10,11] of a possible corrected value for the (110) face. $\phi=6.3$ ev is a published[12] average value for platinum, one of the materials of higher work function. The values below 4.5 ev are useful in the interpretation of experiments with the barium-coated tungsten cathode. In the latter cases, the table is terminated at values of F such that $f(y)=0$, a limit imposed on Eq. (1) by the lowering of the surface potential barrier to the top Fermi level. For higher work functions, this limit falls outside the range of the table.

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