each factor $(E-E_I)^{-1}$ in Eq. (4) by

$$(E-E_I)^{-1}-(E_1-E_I)^{-1}=-\Delta(E-E_I)^{-1}(E_1-E_I)^{-1}. \quad (8)$$

This subtraction changes the degree of divergence of each sum in Eq. (4) from linear to logarithmic, but does not make them converge. To make Eq. (4) converge absolutely, a second subtraction is required, replacing $(E-E_I)^{-1}$ by

$$(E-E_I)^{-1}-(E_1-E_I)^{-1}+\Delta(E_1-E_I)^{-2}=\Delta^2(E-E_I)^{-1}(E_1-E_I)^{-2}. \quad (9)$$

The second subtracted term in Eq. (9) appears in Eq. (3) as a term independent of Δ and so may be interpreted as an unobservable renormalization of the coupling constant G^2. We therefore take Eq. (9) as a renormalization prescription for use with the integral Eq. (3), both for the old and new TD methods. This prescription makes S finite and is justified by plausibility arguments; however it is noncovariant, and experience has shown that noncovariant subtraction prescriptions are often liable to error.[3]

Recently Cini[4] has made the important discovery of a covariant renormalization method for the TD equations. Cini's method is applied to the example under discussion, by writing S as a configuration-space integral

$$S=-\tfrac{3}{4}iG^2\int d_4z \exp[-ip\cdot z+i\Delta|z_0|](\bar{u}\gamma_5 S_F(z)\Delta_F(z)\gamma_5 u). \quad (10)$$

Here S_F and Δ_F are the Feynman Green's functions[5] for the nucleon and meson fields, and z_0 is the time-component of the four-vector z. Equation (10) holds in the old TD method; in the new TD method exactly the same formula for S holds, with z_0 replacing $|z_0|$. The divergence of S arises from the singularity of the product $S_F\Delta_F$ at $z=0$. Since $S_F\Delta_F$ is an invariant function of z, we can apply the usual covariant subtraction methods[6] and obtain

$$\gamma_5 S_F(z)\Delta_F(z)\gamma_5=A\delta_4(z)+B[\gamma_\mu(\partial/\partial z_\mu)+M]\delta_4(z)+R(z), \quad (11)$$

where A and B are divergent constants and $R(z)$ is a divergence-free function. In fact, A is proportional to the covariant nucleon self-mass, and B to the covariant coupling-constant renormalization. The Cini renormalization method gives the prescription that the A and B terms are to be subtracted, so that only the finite $R(z)$ is to be inserted in Eq. (10).

We have investigated the relation between the nonrelativistic and the Cini methods of renormalization as applied to this example. The results are the following. With the old TD method, the Cini renormalization is equivalent to the nonrelativistic subtraction (8), but not to (9). The Cini method does not lead to an absolutely convergent S. Thus with the old TD method the Cini renormalization is ambiguous; the nonrelativistic renormalization (9) gives a finite S and may be correct, but it is not trustworthy. With the new TD method, the Cini renormalization is precisely equivalent to the nonrelativistic subtraction (9). In this case the two types of renormalization give finite and consistent results, and the results may be used with some degree of confidence for practical calculations.

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Scattering of High-Energy Electrons and the Method of Nuclear Recoil*†

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In an effort to exhibit the finite dimensions and charge distribution within atomic nuclei, an electron scattering program has been initiated. The external electron beam of the Stanford linear electron accelerator is resolved in energy to about 1 percent, deflected, cleared of gamma-rays, and focused in vacuum at the center of a 20-inch diameter scattering chamber. Scattering targets are placed within the chamber and their position is remotely controlled. Scattered electrons emerge from the target foil, pass through the 0.006-inch aluminum window of the chamber and through a 0.003-inch aluminum foil into the vacuum chamber of a double-focusing magnetic spectrometer. The spectrometer is similar to that of Snyder *et al.*,[1] has a radius of 16 inches, and bends electrons of 135 Mev (maximum) through 180°. Scattered electrons are detected by a 4-inch long Čerenkov counter buried in a large lead shield. The main beam is monitored by a helium-filled ionization counter.

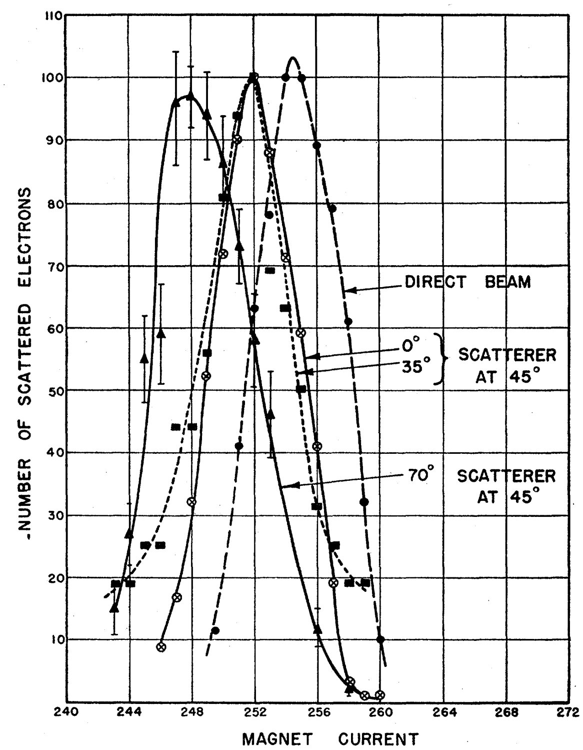

FIG. 1. Elastic scattering curves in beryllium (0.100 inch). The abscissa is proportional to the electron energy. The energy of the direct beam is 116 Mev and has about a 1 percent spread. The curve at 70° shows the effect of the nuclear recoil.

Elastic scattering profiles have been taken at various angles for incident electrons of 116 Mev and a 0.002-inch gold foil. The curves are quite similar at all angles. The energy loss in the gold foil may be observed as a shift towards lower energy between the direct beam and the scattered beams. The width at half-maximum of the elastic curves is about 2.8 percent and is largely due to the wide slit at the spectrometer exit.

Figure 1 shows elastic scattering profiles in beryllium. The elastic curves are shifted to lower energies due, as expected, to energy loss in the target. The elastic curve at 70°, however, is shifted considerably farther toward lower energies. This additional shift is a function of angle and is due to the recoil of the beryllium nucleus.

Figure 2 substantiates this interpretation and shows the two elastic curves observed with a polyethylene (CH_2) target. The magnitude of the energy shift of the hydrogen peak has been studied at various angles and agrees with the recoil hypothesis.

Thus a new method is available for studying "elastic" scattering of electrons in compounds or with unseparated isotopes. Each isotope should present its own elastic peak. The recoil shift is a

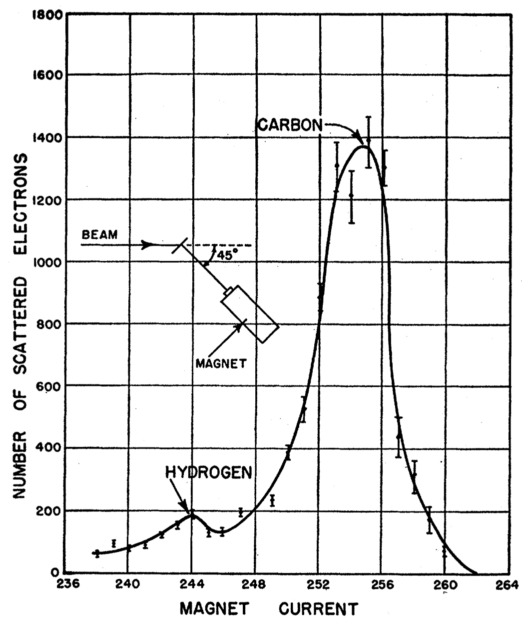

FIG. 2. Elastic scattering curves at 116 Mev for carbon and hydrogen in a polyethylene target.

quadratic function of the incident energy and therefore improved separation of isotope peaks will occur at higher incident energies. This new method clearly permits a study of the scattering from hydrogen and deuterium using solid targets.

A preliminary angular distribution (Fig. 3) has been obtained for gold by using the peak intensity of the elastic curves as a measure of the scattering cross section. The width of the accepted

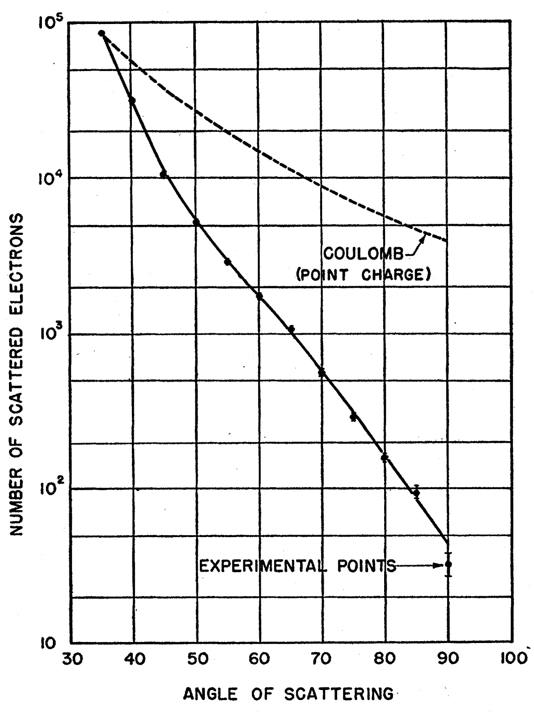

FIG. 3. Typical angular distribution obtained at 116 Mev with a 0.002-inch gold foil. The gold foil was oriented at 45° with respect to the incident beam for all angular settings of the spectrometer magnet.

energy range is approximately 1.3 percent and centers on the peak. While systematic errors have not been investigated, it appears probable from the results of many runs that the shape of the curve in Fig. 3 will not change appreciably. Thus the prominent diffraction peaks predicted by Parzen[2] and others (e.g., at 75°) for a uniform distribution of charge in heavy nuclei are not observed. Possible interpretations of the curve are: (a) the nuclear boundary in gold is not sharp and the charge distribution tapers off gradually, (b) inelastic effects may "fill up" the minima of the diffraction curves. Because of the very large fall-off in intensity as a function of angle of scattering (approximately 10^4 between 35° and 100°) the explanation (b) would not appear to be complete.

The angular distribution observed experimentally in beryllium at 116 Mev appears to fit (though not perfectly) a point charge distribution much more closely than in gold. This fact suggests that no large systematic error is present in the experiments.

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The Grain Density in the Tracks of 117- and 222-Mev π^- Mesons in Emulsions

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THE variation of the grain density in tracks of singly charged particles moving with velocities close to the speed of light has been the subject of a number of investigations, both experimental[1–8] and theoretical,[9–12] in the past few years. The studies have assumed particular interest, since the small increase that exists between the minimum at $W/mc^2\sim 4$ (W is total energy) and $W/mc^2\rightarrow\infty$, has provided, in certain favorable circumstances, a means of identifying heavy mesons produced in high-energy nuclear disintegrations.

The confirmation of this small increase has been facilitated by improvements in technique: one of the most important of these has been the introduction of the "blob" or "extended" counting convention.[4] A detailed analysis of the statistical fluctuations that occur when this counting convention is used has recently been published.[13]

While the existence of a minimum is now well established, certain finer details remain in doubt. These include the exact amount of the rise (it is about 10±5 percent), and the precise energy at which the plateau is reached. An accurate measurement in the region just below the minimum of ionization, where the polarization effects are very small, would also be of value. The production of fast π^--meson beams by the University of Chicago synchrocyclotron makes such an experiment possible.

Ilford G5 photographic emulsions, 400 microns thick, were exposed in succession to the 122-Mev and the 227-Mev π^--meson beams from the accelerator. The directions of the two beam into the plates were mutually perpendicular, in order to facilitate easy and rapid identification of their tracks. In addition some π^+ mesons were stopped in the plates. The electron tracks produced in the ensuing $\pi-\mu-e$ decay were used to give a measure of the "plateau" grain density existing in the plates. The blob counting convention was employed throughout this work.

Sufficient meson track length was found in 0.2 cm² area; thus, limiting the scanning to this region almost certainly eliminated any chance of nonuniform sensitivity or development. This area was 1 cm away from the edges of the plate that faced the beams. Therefore the energy of the mesons used was slightly less because of loss of energy by ionization. It is believed that they were well within the range of 117±5 and 222±5 Mev, respectively. A fur-