deduced from an appropriate Lagrangian and are

$$\left\{-\frac{\partial^2}{\partial X_\mu \partial X_\mu}+F^{(r)}\right\}\varphi(X,r)=-g\Sigma_\alpha\bar{\psi}_\alpha(X+\tfrac{1}{2}r)\psi_\alpha(X-\tfrac{1}{2}r), \quad (6)$$

$$\gamma_\mu\frac{\partial\psi(x')}{\partial x_\mu'}+M\psi(x')=-g\int\psi(x'')(x''|\,\varphi|\,x')dx''. \quad (7)$$

We insert (5) in (6), multiply both sides by the complex conjugate $\chi_n^*(r)$, and integrate over the four-dimensional space of $r_1, r_2, r_3, r_0=-ir_4$. The result is

$$\left(\frac{\partial^2}{\partial x_\mu''\partial x_\mu''}-m_n^2\right)u_n(x'')=\int\Phi_n(x',x'',x''')\Sigma_\alpha\bar{\psi}_\alpha(x')\psi_\alpha(x''')dx'dx''', \quad (8)$$

where

$$\Phi_n(x',x'',x''')\equiv g\chi_n^*(x'-x''')\delta(\tfrac{1}{2}(x'+x''')-x''). \quad (9)$$

Similarly, we obtain from (7) the equation

$$\gamma_\mu\frac{\partial\psi(x')}{\partial x_\mu'}+M\psi(x')=-\Sigma_n\int\Phi_n(x',x'',x''')\times u_n(x'')\psi(x''')dx''dx'''. \quad (10)$$

If we compare these equations with the corresponding equations (19) of Møller and Kristensen[3] in the theory of nonlocal interaction between a local scalar (or pseudoscalar) field and a local spinor field, we notice that the internal eigenfunction $\chi_n(r)$ plays the role of a convergence factor. There is, however, an essential difference between their equations and ours. Namely, in our theory, we are obliged to take into account simultaneously all the particles with different masses m_n which were derived from an eigenvalue problem. Furthermore, the form function for each of these particles is uniquely determined by the same eigenvalue problem.

In the following letter, the above general considerations will be illustrated and further details will be examined.

* Now at Kyoto University, Kyoto, Japan, on leave of absence from Columbia University (July, 1953).

[1] H. Yukawa, Phys. Rev. **77**, 219 (1950); **80**, 1047 (1950).
[2] D. R. Yennie, Phys. Rev. **80**, 1053 (1950); J. Rayski, Acta. Phys. Polonica **10**, 103 (1950); Proc. Phys. Soc. (London) **A64**, 957 (1951); M. Fierz, Helv. Phys. Acta **23**, 412 (1950); Z. Tokuoka and Y. Katayama, Progr. Theoret. Phys. **6**, 132 (1951); C. Bloch, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **24**, No. 1 (1950); Progr. Theoret. Phys. **5**, 606 (1950); O. Hara and H. Shimazu, Progr. Theoret. Phys. **5**, 1055 (1950); **7**, 255 (1952); **9**, 137 (1953).
[3] P. Kristensen and C. Møller, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 7 (1952); C. Bloch, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 8 (1952); Y. Katayama, Progr. Theoret. Phys. **8**, 381 (1952).

Structure and Mass Spectrum of Elementary Particles. II. Oscillator Model

HIDEKI YUKAWA*
Columbia University, New York, New York
(Received May 25, 1953)

AS an illustration of the general considerations on nonlocal fields in the preceding letter, let us assume that the operator F has a very simple form

$$F\equiv-\frac{\partial^2}{\partial X_\mu\partial X_\mu}+\frac{\lambda^2}{2}\left(-\frac{\partial^2}{\partial r_\mu\partial r_\mu}+\frac{1}{\lambda^4}r_\mu r_\mu\right)^2, \quad (1)$$

where λ is a small constant with the dimension of length. One may call this the four-dimensional oscillator model for the elementary particle, which was considered first by Born[1] in connection with his idea of self-reciprocity. However, our model differs from his model in that we have introduced internal degrees of freedom of the particles which are related to the nonlocalizability of the field itself. The internal eigenfunctions in our case are

$$\chi_{n_1n_2n_3n_0}(r)=H_{n_1}(r_1/\lambda)H_{n_2}(r_2/\lambda)H_{n_3}(r_3/\lambda)\times H_{n_0}(r_0/\lambda)\exp\{-(r_1^2+r_2^2+r_3^2+r_0^2)/2\lambda^2\}, \quad (2)$$

and the corresponding eigenvalues for the mass become

$$m_{n_1n_2n_3n_0}=(\sqrt{2}/\lambda)\,|\,n_1+n_2+n_3-n_0+1\,|, \quad (3)$$

where $r_0=-ir_4$ is a real variable and n_1, n_2, n_3, n_0 are quantum numbers which can take only zero or positive integer values. $H_n(x)$ denotes the Hermite polynomial of x of degree n. All these eigenfunctions (2) decrease rapidly in any direction whatsoever in the four-dimensional r space. Furthermore, the Fourier transform of each of these eigenfunctions has exactly the same form as the original function due to the self-reciprocity. Thus, the form function (9) in the preceding letter seems to be sufficient to cut off high energy-momentum intermediate states in such a way that each term corresponding to each Feynman diagram in the expansion of the nonlocal S-matrix according to the Bloch-Kristensen-Møller formulation is convergent. However, since we have to take into account all of infinitely many of different mass states of the nonlocal system, the number of terms in the S matrix increases very rapidly with the increasing power of the coupling constant, so that we can claim nothing for the moment concerning the convergence or divergence of the S matrix as a whole.

The totality of the internal eigenfunctions (2) constitutes a complete set of orthogonal and quadratically integrable functions in the four-dimensional r space and can be regarded as the eigenvectors for an infinite-dimensional unitary representation of the Lorentz group. The eigenvalues (3) for the mass are all infinitely degenerate. For instance, all those values of n's which satisfy $n_1+n_2+n_3-n_0=0$ give the same mass, $m_0=\sqrt{2}/\lambda$. This is not a peculiar feature of the oscillator model; it is common to all those models for which the operator F is separable, because there can be no unitary representation of finite dimensions for the Lorentz group. Presumably, such an undesired degeneracy could be removed either by introducing interaction with other fields or by first introducing the coupling between the external and internal degrees of freedom. The latter possibility can be illustrated by the addition of the coupling term,

$$-\beta^2\lambda^2\left\{-\left(\frac{\partial^2}{\partial X_\mu\partial X_\mu}\right)^2+\frac{1}{\lambda^4}\left(r_\mu\frac{\partial}{\partial X_\mu}\right)^2\right\}, \quad (4)$$

to the expression (1) for F, where β is a dimensionless real constant. The free field equation becomes

$$\left[k_\mu k_\mu+\frac{\lambda^2}{2}\left(-\frac{\partial^2}{\partial r_\mu\partial r_\mu}+\frac{1}{\lambda^4}r_\mu r_\mu\right)^2+\beta^2\lambda^2\left\{-\left(k_\mu\frac{\partial}{\partial r_\mu}\right)^2+\frac{1}{\lambda^4}(k_\mu r_\mu)^2\right\}\right]\chi(k_\mu,r_\mu)=0, \quad (5)$$

in the eight-dimensional space of k_μ and r_μ, where $\chi(k_\mu,r_\mu)$ is the Fourier transform of $\varphi(X_\mu,r_\mu)$ as defined by

$$\varphi(X_\mu,r_\mu)=\int\exp(ik_\mu X_\mu)\chi(k_\mu,r_\mu)(dk_\mu).^4 \quad (6)$$

One can solve Eq. (5) in the coordinate system in which only one component of the wave vector is different from zero.[2] Thus, one obtains the mass spectrum

$$m_{n_1n_2n_3n_0}=\frac{\sqrt{2}}{\lambda}\frac{|\,n_1+n_2+n_3-n_0+1\,|}{[1-2\beta^2(n_0+\frac{1}{2})]^{\frac{1}{2}}}, \quad (7)$$

where n_0 is restricted by the condition

$$n_0<\tfrac{1}{2}(1/\beta^2-1). \quad (8)$$

If we take, for instance, $\beta=1/\sqrt{2}$, only $n_0=0$ is allowed and the mass spectrum reduces to

$$m_{n_1n_2n_3n_0}=(2/\lambda)(n_1+n_2+n_3+1), \quad (9)$$

and the degree of degeneracy of the mass eigenvalues is now finite. In particular, the lowest mass, $m_0=2/\lambda$, is free from degeneracy and the corresponding solution of (5) is given by

$$\chi_{0000}(k_\mu,r_\mu)=\exp\left\{-\frac{1}{2\lambda^2}\left(r_\mu r_\mu+\frac{2(k_\mu r_\mu)^2}{m_0^2}\right)\right\} \quad (10)$$

in an arbitrary coordinate system, where $k_\mu k_\mu=-m_0^2$.

The above advantage of introducing the coupling between external and internal degrees of freedom is offset, however, by a

complication which is almost prohibitive if we further take into account the interaction with other fields, because the general method of reducing the theory of nonlocal fields to that of the nonlocal interaction between local fields as discussed in the preceding letter can no longer be applied straightforwardly to our case. On the other hand, it may well be that one could arrive at the desired removal of the infinite degeneracy as a consequence of the interaction between nonlocal fields without assuming the coupling between external and internal degrees of freedom for each of the nonlocal fields. This is plausible, because the submatrix of the S matrix corresponding to one-particle states can always be represented by an equivalent coupling between the external and internal variables for the particle in question, so that one can hope that a reasonable mass spectrum which is free from the infinite degeneracy may come out even without assuming the coupling between external and internal degrees of freedom at the outset.

A detailed account of all these points, including the quantization of nonlocal fields, will be given in a forthcoming paper.

* Now at Kyoto University, Kyoto, Japan, on leave of absence from Columbia University (July, 1953).

[1] M. Born and H. S. Green, Proc. Roy. Soc. Edinburgh **62**, 470 (1949); M. Born, Revs. Modern Phys. **21**, 463 (1949).

[2] Equation (5) has no solution which is quadratically integral unless k is time-like, i.e., $k_\mu k_\mu < 0$.

Inverse Oppenheimer-Phillips Process in the $Cu^{65}(He^3,\alpha)$ Reaction

D. N. KUNDU AND M. L. POOL
The Ohio State University, Columbus, Ohio
(Received May 25, 1953)

AN inverse Oppenheimer-Phillips process may be conceived where the bombarding particle picks up a nucleon from the target nucleus without actual penetration when the two are in close proximity of each other. A high binding energy of the nucleus composed of the projectile and the pickup nucleon would be a desirable condition.

A situation favorable to this inverse O−P process arises in connection with (He^3, α) reactions. Experimental evidence is now available to test this phenomenon.

Copper was bombarded with 13-Mev He^3 in the cyclotron and the 12.9-hour Cu^{64} was produced according to the reaction $Cu^{65}(He^3, \alpha)$. The excitation curve obtained for this reaction is shown in Fig. 1 which also shows, for comparison, a corresponding curve for the 9.4-hour Ga^{66} produced from the reaction $Cu^{65}(He^3, 2n)$.

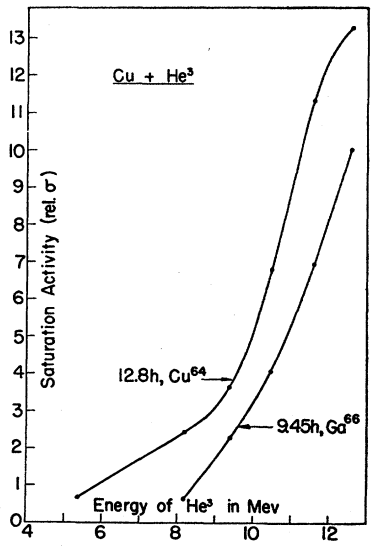

FIG. 1. The excitation curve for the 12.9-hour Cu^{64} and 9.4-hour Ga^{66} produced by He^3 bombardment of copper.

A study of the shapes of these curves indicates that the $Cu^{65}(He^3, 2n)$ curve fits smoothly a theoretical curve involving compound nucleus formation, as expected. The high-energy part of the experimental curve for the $Cu^{65}(He^3, \alpha)$ reaction can be similarly fitted to a theoretical curve down to about 9.0 Mev. For lower energies the shape of the curve changes in a marked way. The cross section is less energy-sensitive and the reaction can be traced to as low as 5.4 Mev where essentially no penetration of the potential barrier would be expected.

The experimental data appear to indicate that the inverse O−P phenomenon comes into evidence from very low energies up to an energy where compound nucleus formation mechanism takes over and, from there on, the latter plays the major role in the $Cu^{65}(He^3, \alpha)$ reaction.

It may be pointed out that the 5.10-minute Cu^{66} activity was not produced in measurable amount by the reaction $Cu^{65}(He^3, 2p)$. The similar reaction $Cu^{63}(He^3, 2p)$ has a Q value of only $+0.18$ Mev as compared with the Q value of $+10.7$ Mev for the reaction $Cu^{65}(He^3, \alpha)Cu^{64}$. Analogous to the latter is the reaction $Cu^{63}(He^3, \alpha)$ which gave a substantial yield of the 9.9-minute Cu^{62} activity.

Bremsstrahlung at High Energies

H. A. BETHE, *Laboratory of Nuclear Studies, Cornell University, Ithaca, New York;*
L. MAXIMON, *Graduate Division of Applied Mathematics, Brown University, Providence, Rhode Island*
AND
F. LOW, *Physics Department, University of Illinois, Urbana, Illinois*
(Received June 1, 1953)

TWO of us[1] have published a formula for the cross section for bremsstrahlung which was derived without the use of the Born approximation. The matrix element was taken to be

$$M = \int \psi_f^* \alpha_\lambda \psi_i e^{-i\mathbf{k}\cdot\mathbf{r}} d\tau, \tag{1}$$

with α_λ the Dirac matrix in the direction of polarization of the emitted quantum, and ψ_i and ψ_f wave functions of the electron in the Coulomb field of a nucleus.

Like most authors before us,[2] we took *both* electron wave functions to be plane waves plus outgoing spherical waves. This is of course correct for the initial state ψ_i, but it is wrong for the final state ψ_f; the latter must be taken as a plane wave plus *ingoing* spherical waves.

To show this, we note that in the process of determining a differential cross section, we *observe* (e.g., by means of a counter) an electron moving in direction $\mathbf{p}_2$ some time *after* the radiation process has taken place. This observation is described by a plane wave packet concentrated around the point $\mathbf{v}_2 t$; this wave packet is moving away from the nucleus and is not accompanied by any other waves at this late time. We now follow the development of this wave packet *backwards in time* by use of the Dirac equation. As long as the packet is outside the field of the nucleus, it remains a plane wave packet. When the time t approaches zero (from above), the Coulomb field will scatter the wave packet, and for $t<0$ the plane wave packet will be accompanied by scattered spherical waves. As t becomes more and more negative, both the plane wave and the spherical waves will move farther away from the nucleus. But in the usual language in which one describes the motion with increasing time, both waves will, for $t<0$, move *towards* the nucleus. This shows that we have indeed ingoing spherical waves associated with the plane wave, and this result can be carried over into the time-independent formalism.

In fact, the use of converging spherical waves in ψ_f is the natural counterpart of the use of outgoing spherical waves in ψ_i. Here the observation of an electron going in the direction $\mathbf{p}_1$ is made in the beginning; therefore the electron will be described by a pure plane wave until it hits the nucleus, and by a plane wave plus outgoing spherical waves thereafter.