The Classical Scattering of Neutral Mesons

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In a recent paper a theory of point particles interacting through neutral meson fields was developed from the point of view of action at a distance. Equations of motion differing from those of field theory were obtained. These equations are now used to calculate the scattering of scalar and vector mesons by nucleons. The resulting cross sections are somewhat larger than those previously obtained from field theory, but the difference is so small that an experimental distinction hardly appears possible.

I. INTRODUCTION

It is well established experimentally that the effect of an electromagnetic field on a slowly moving charge is given by the Lorentz force. The modifications of this force required to describe the radiation damping experienced by a particle undergoing a rapid acceleration are also well known. On the other hand, not enough is known experimentally about the behavior of a particle in a meson field to be certain of the correct fundamental equations describing the interaction of a nucleon and any of the different meson fields known from experiment.

While there is no doubt that such a description has ultimately to be quantum theoretical, the present quantum theory is largely based on classical concepts; thus, it may be that some of the difficulties of this theory can be resolved by a reinvestigation of the underlying classical theory.

Classical equations of motion of point particles interacting with a neutral vector meson field were first found by Bhabha[1] on the basis of field theory, following a method originally developed by Dirac[2] for the case of the electromagnetic field. Using the same method, the equations of motion for the case of a neutral scalar meson field were found by Harish-Chandra.[3]

Recently a theory of point particles interacting through neutral meson fields was developed from the point of view of action at a distance.[4,5] It was found that, in contrast to the case of electrodynamics, the equations of motion obtained from the field-theoretical and the action-at-a-distance point of view are slightly different.

If fields symmetric in time are used, the equations of motion developed from either point of view do not contain any terms describing radiation damping. Such terms do appear, however, if the method of Wheeler and Feynman[6] is applied to the equations. Its application to the field-theoretical equations using symmetric fields results in the same equations as originally obtained in field theory using retarded fields. Its application to the time-symmetric equations of action-at-a-distance theory, however, results in equations differing from those of the retarded case of both field theory and action at a distance.

Unlike electrodynamics, meson theory therefore appears to offer a possibility of decision between the two points of view by a comparison of their predictions with experiment. A clear-cut decision is not to be expected until these considerations have been applied to quantum theory; however, the classical results obtained should be a limiting case of the quantum-theoretical results and thus provide a preliminary orientation. With this in view, we have calculated the scattering of neutral mesons by nucleons according to the theory of action at a distance and compared the results with those of field theory.[1,7] The scattering of charged mesons, which is, of course, easier to investigate experimentally, is being calculated at present on the basis of an extension of the results of H to charged mesons.[8]

In a theory of action at a distance, only particles and the forces they exert on each other are considered as physically meaningful. Fields are introduced as auxiliary quantities only, and thus there are no "real" electromagnetic or meson fields to be scattered. When we are talking of the scattering of a plane wave by a particle, we mean that the total effect on the particle of all other particles takes the same form as if there were a plane wave present. This, in turn, will influence the motion of this particle in such a manner that its effect on other particles is the same as that of a scattered wave emanating from it. Once the equations of motion of the particles are established, we can, therefore, proceed using the more familiar language of field theory.

It was pointed out in H that, just as in electrodynamics,[9] the equations of motion obtained by an

[1] H. J. Bhabha, Proc. Roy. Soc. (London) **A172**, 384 (1939); hereafter referred to as B.
[2] P. A. M. Dirac, Proc. Roy. Soc. (London) **A167**, 148 (1938).
[3] Harish-Chandra, Proc. Roy. Soc. (London) **A185**, 269 (1946).
[4] P. Havas, Phys. Rev. **87**, 309 (1952); hereafter referred to as H.
[5] For the vector meson case some of the results of H have been obtained independently from different considerations by H. Kanazawa, Progr. Theoret. Phys. (Japan) **5**, 1050 (1950) and by H. Steinwedel, Sitzber. heidelberg. Akad. Wiss. Math.-naturw. Kl. 281 (1950).
[6] J. A. Wheeler and R. P. Feynman, Revs. Modern Phys. **17**, 157 (1945). For the relation with field theory and for the question of physical interpretation, see also references 9 and 10.
[7] Harish-Chandra, Proc. Indian Acad. Sci. **21**, 135 (1945); hereafter referred to as HC.
[8] P. Havas, Phys. Rev. **85**, 720, 701 (1952); a detailed account of this work is in process of publication.
[9] P. Havas, Phys. Rev. **74**, 456 (1948).

application of the Wheeler-Feynman method to the time-symmetrical equations have to be solved subject to the condition that the total retarded field of all the particles in the universe equals their total advanced field. In practice, however, we have to solve problems like the motion of a single particle in the only approximately known field of the other particles. The application of the above condition to such problems in electrodynamics leads to difficulties; as discussed elsewhere,[10] at present the most feasible method of proceeding appears to be to ignore the condition altogether. Similarly, we shall ignore it in the following.

II. THE SCATTERING OF SCALAR MESONS

Throughout this paper we shall use the same notation as in H except for dropping the subscript a, as we shall always deal with a single particle. Then we can write Harish-Chandra's[3] field-theoretical equation of motion of a particle in a scalar meson field in the form (H 21b)

$$M\dot{v}^\mu-\tfrac{1}{3}g^2(\ddot{v}^\mu+v^\mu\dot{v}^2)-\tfrac{1}{2}g^2\chi^2v^\mu+g^2\chi^2\int_{-\infty}^{\tau}\frac{s^\mu}{s^2}J_2(\chi s)d\tau'+g^2\chi\frac{d}{d\tau}\left[v^\mu\int_{-\infty}^{\tau}\frac{1}{s}J_1(\chi s)d\tau'\right]=gF^\mu+g\frac{d}{d\tau}(Uv^\mu),\quad(1)$$

where U and F are the external potential and field. The equation of motion obtained by application of the Wheeler-Feynman method to the time-symmetric equation of motion of the theory of action at a distance is (H 50b)

$$M\dot{v}^\mu-\tfrac{1}{3}g^2(\ddot{v}^\mu+v^\mu\dot{v}^2)-\tfrac{1}{2}g^2\chi^2v^\mu+\tfrac{1}{2}g^2\chi^2\left\{\int_{-\infty}^{\tau}\frac{s^\mu}{s^2}J_2(\chi s)d\tau'-\int_{\tau}^{\infty}\frac{s^\mu}{s^2}J_2(\chi s)d\tau'\right\}+\tfrac{1}{2}g^2\chi\frac{d}{d\tau}\left\{v^\mu\left[\int_{-\infty}^{\tau}\frac{1}{s}J_1(\chi s)d\tau'-\int_{\tau}^{\infty}\frac{1}{s}J_1(\chi s)d\tau'\right]\right\}=gF^\mu+g\frac{d}{d\tau}(Uv^\mu),\quad(2)$$

where U and F are the retarded potential and field of all other particles. To facilitate comparison of the results, we shall use the same assumptions and approximations as HC and B. Thus we take[11]

$$F_1=-\gamma\cos\omega_0 t,\quad F_2=F_3=0,\quad(3)$$

for the field of the incoming wave at the position of the nucleon (assuming that the amplitude of oscillation of the particle is small compared to the incident wavelength), and

$$z_1=(\beta/\omega_0)\sin\omega_0 t+\delta,\quad z_2=z_3=0,\quad(4)$$

for the coordinates of the nucleon. The amplitudes β and γ are assumed to be small so that quantities quadratic in them may be neglected. The calculation of β and δ is straightforward and quite analogous to that of HC. We shall therefore not give all the details. The values of the integrals appearing in Eq. (2) are given in the Appendix. Using them and equating the coefficients of $\sin(\omega_0 t+\delta)$ and $\cos(\omega_0 t+\delta)$ to zero in Eq. (2) we get

$$\beta=\frac{\gamma}{g[(\tfrac{1}{2}\chi^2-\tfrac{1}{3}\omega_0^2-\chi^3P_1/\omega_0)^2+(M/g^2)^2\omega_0^2]^{\frac{1}{2}}},\quad(5)$$

$$\cos\delta=\frac{\tfrac{1}{2}\chi^2-\tfrac{1}{3}\omega_0^2-\chi^3P_1/\omega_0}{[(\tfrac{1}{2}\chi^2-\tfrac{1}{3}\omega_0^2-\chi^3P_1/\omega_0)^2+(M/g^2)^2\omega_0^2]^{\frac{1}{2}}},\quad(6)$$

where the positive square root has to be taken. P_1 is given by

$$P_1=\begin{cases}\tfrac{1}{3}(-\nu^3+\tfrac{3}{2}\nu), & 0<\nu<1\\ \tfrac{1}{3}[(\nu^2-1)^{\frac{3}{2}}-\nu^3+\tfrac{3}{2}\nu], & \nu>1\end{cases}\quad(7)$$

where $\nu=\omega_0/\chi$. We note that for $\nu<1$ the square roots reduce to $\omega_0 M/g^2$. Therefore the effective mass of the particle equals M, which was not the case for the field-theoretical equations. There the field contributed to the effective mass an amount which for $\nu\ll1$ equalled $\tfrac{1}{2}g^2\chi$.

The calculation of the retarded field of the particle is the same as in HC, and we obtain therefore for the differential cross section of scattering of the incoming wave[12]

$$d\sigma=\begin{cases}0, & \nu<1\\ g^2\dfrac{\beta^2}{\gamma^2}\cos^2\theta\dfrac{(\omega_0^2-\chi^2)^2}{\omega_0^2}d\Omega, & \nu>1\end{cases}\quad(8)$$

where θ is the angle between the directions of propagation of the incident and the scattered wave. Inserting our value for β from Eq. (5), we obtain, for the case $\nu>1$,

$$d\sigma=\frac{\cos^2\theta(\omega_0^2-\chi^2)^2d\Omega}{\omega_0^2[(\tfrac{1}{2}\chi^2-\tfrac{1}{3}\omega_0^2-\chi^3P_1/\omega_0)^2+(M/g^2)^2\omega_0^2]},\quad(9)$$

and for the total cross section,

$$\sigma=\frac{\dfrac{4\pi}{3}\left(\dfrac{g^2}{M}\right)^2\dfrac{(\omega_0^2-\chi^2)^2}{\omega_0^4}}{1+\dfrac{1}{9}\left(\dfrac{g^2}{M}\right)^2\dfrac{(\omega_0^2-\chi^2)^3}{\omega_0^4}}.\quad(10)$$

[10] P. Havas, Phys. Rev. **86**, 974 (1952).

[11] The corresponding Eq. (3) and HC contains $\sin\omega_0 t$ instead. However, in the actual calculation HC used $\cos\omega_0 t$, as can be seen from his Eqs. (11) and (19).

[12] Owing to a misprint the factor $(\omega_0^2-\chi^2)^2$ in the corresponding Eq. (23) of HC appears without the square.

TABLE I. Total cross sections for the scattering of neutral mesons by a nucleon.

Incoming meson		Scattered meson	Total cross section according to theory of action at a distance		Total cross section according to field theory	
Scalar		Scalar	$\dfrac{\frac{4\pi}{3}\left(\frac{g^2}{M}\right)^2\frac{(\omega_0^2-\chi^2)^2}{\omega_0^4}}{1+\frac{1}{9}\left(\frac{g^2}{M}\right)^2\frac{(\omega_0^2-\chi^2)^3}{\omega_0^4}}$		$\dfrac{\frac{4\pi}{3}\left(\frac{g^2}{M}\right)^2\frac{(\omega_0^2-\chi^2)^2}{\omega_0^4}}{\left[1+\frac{g^2}{M}\chi-\frac{1}{3}\frac{g^2}{M}\frac{\chi^3}{\omega_0^2}\right]^2+\frac{1}{9}\left(\frac{g^2}{M}\right)^2\frac{(\omega_0^2-\chi^2)^3}{\omega_0^4}}$	
Vector	Transverse	Transverse	2	$\dfrac{\frac{4\pi}{3}\left(\frac{g^2}{M}\right)^2}{1+\frac{4}{9}\left(\frac{g^2}{M}\right)^2\left(\omega_0^2-\frac{3}{4}\frac{\chi^4}{\omega_0^2}-\frac{1}{4}\frac{\chi^6}{\omega_0^4}\right)}$	2	$\dfrac{\frac{4\pi}{3}\left(\frac{g^2}{M}\right)^2}{1+\frac{2}{3}\frac{\chi^3}{\omega_0^2}\frac{g^2}{M}+\frac{4}{9}\left(\frac{g^2}{M}\right)^2\left(\omega_0^2-\frac{3}{4}\frac{\chi^4}{\omega_0^2}\right)}$
		Longitudinal	χ^2/ω_0^2		χ^2/ω_0^2	
		Transverse and longitudinal	$2+\chi^2/\omega_0^2$		$2+\chi^2/\omega_0^2$	
	Longitudinal	Transverse	χ^2/ω_0^2		χ^2/ω_0^2	
		Longitudinal	$\frac{1}{2}\chi^4/\omega_0^4$		$\frac{1}{2}\chi^4/\omega_0^4$	
		Transverse and longitudinal	$\frac{\chi^2}{\omega_0^2}\left(1+\frac{1}{2}\frac{\chi^2}{\omega_0^2}\right)$		$\frac{\chi^2}{\omega_0^2}\left(1+\frac{1}{2}\frac{\chi^2}{\omega_0^2}\right)$	

This is compared with the field-theoretical value in Table I.

III. THE SCATTERING OF VECTOR MESONS

Bhabha's field-theoretical equation of motion of a particle in a vector meson field is (H 21a)

$$M\dot{v}^\mu-\tfrac{2}{3}g^2(\ddot{v}^\mu+v^\mu\dot{v}^2)$$
$$-g^2\chi^2 v_\rho\int_{-\infty}^{\tau}\frac{s^\mu v^\rho(\tau')-s^\rho v^\mu(\tau')}{s^2}J_2(\chi s)d\tau'=gG^\mu{}_\rho v^\rho,\qquad(11)$$

where $G^\mu{}_\rho$ is the external field. The equation of motion obtained by application of the Wheeler-Feynman method to the time-symmetric equation of motion of the theory of action at a distance[4,5] is (H 50a)

$$M\dot{v}^\mu-\tfrac{2}{3}g^2(\ddot{v}^\mu+v^\mu\dot{v}^2)$$
$$-\tfrac{1}{2}g^2\chi^2 v_\rho\left\{\int_{-\infty}^{\tau}\frac{s^\mu v^\rho(\tau')-s^\rho v^\mu(\tau')}{s^2}J_2(\chi s)d\tau'\right.$$
$$\left.-\int_{\tau}^{\infty}\frac{s^\mu v^\rho(\tau')-s^\rho v^\mu(\tau')}{s^2}J_2(\chi s)d\tau'\right\}=gG^\mu{}_\rho v^\rho,\qquad(12)$$

where $G^\mu{}_\rho$ is the retarded field of all other particles. Again assuming that the amplitude of oscillation of the particle is small compared to the incident wavelength, we take for the field of the incoming wave at the position of the nucleon

$$G_{10}=\gamma\cos\omega_0 t,\quad G_{20}=G_{30}=0,\qquad(13)$$

and for the coordinates of the particle

$$z_1=(\beta/\omega_0)\sin(\omega_0 t+\delta),\quad z_2=z_3=0.\qquad(14)$$

The calculation of β and δ proceeds as for scalar mesons; the necessary integrals are again given in the Appendix. We obtain[13]

$$\beta=\frac{3\gamma}{2g\omega_0[(3M/2g^2)^2+\omega_0^2(1-P_2)^2]^{\frac{1}{2}}},\qquad(15)$$

with
$$\cos\delta=\frac{\omega_0(1-P_2)}{[(3M/2g^2)^2+\omega_0^2(1-P_2)^2]^{\frac{1}{2}}},\qquad(16)$$

$$P_2=\begin{cases}1, & 0<\nu<1\\ 1-\dfrac{1}{\nu^2}\left(\nu+\dfrac{1}{2\nu}\right)(\nu^2-1)^{\frac{1}{2}}, & \nu>1.\end{cases}\qquad(17)$$

We note that for $\nu<1$ the square roots reduce to $\frac{3}{2}M/g^2$. Thus the effective mass is again M, while for the field-theoretical equations the field contributes an amount $-\frac{1}{2}g^2\chi$, if $\nu\ll1$.

The calculation of the retarded field is the same as in B. We again get no scattering if $\nu<1$. If $\nu>1$, the flow of energy into the solid angle $d\Omega$ is given by

$$\omega_0 g^2\beta^2(\omega_0^2-\chi^2)^{\frac{1}{2}}\sin^2\theta d\Omega/8\pi\qquad(18)$$

for transverse scattered waves, and

$$g^2\beta^2\chi^2(\omega_0^2-\chi^2)^{\frac{1}{2}}\cos^2\theta d\Omega/8\pi\omega_0\qquad(19)$$

for longitudinal scattered waves.[14] Here θ is the angle between the direction of oscillation of the particle and the direction of propagation of the scattered wave. The energy flow associated with the incoming plane wave is

$$(\omega_0^2-\chi^2)^{\frac{1}{2}}\gamma^2/8\pi\omega_0,\qquad(20)$$

[13] Equations (15) and (16) have also been obtained by Kanazawa (reference 5), who, however, did not give the value of P_2 and did not calculate the scattering.

[14] In the equations corresponding to Eqs. (18)–(21) given in B the factor $\omega_0(\omega_0^2-\chi^2)^{\frac{1}{2}}$, which follows from (B17), has been omitted erroneously. The cross sections, however, which only involve the ratios of these expressions, are given correctly.

if the wave is transverse, and

$$(\omega_0^2-\chi^2)^{\frac{1}{2}}\omega_0\gamma^2/8\pi\chi^2, \qquad (21)$$

if it is longitudinal. The differential cross sections for the different cases can be obtained by dividing the appropriate energy flows by each other and using Eq. (15). Integrating those, we obtain the total cross sections listed in Table I together with their field-theoretical counterparts.[15]

IV. DISCUSSION

In this paper we have calculated the scattering of mesons according to the theory of action at a distance as developed in H. The resulting total cross sections are summarized in Table I, which also lists the corresponding field-theoretical results.

It can be seen that corresponding results are very similar. Furthermore, they approach the same limit as $\chi\rightarrow 0$, as required, since in this limit (ordinary or scalar electrodynamics) there is no difference between the equations of motion of the two theories, as can be seen directly from Eqs. (1), (2), (11), and (12).

The only difference in the cross sections for scalar mesons is the appearance of an added term in the denominator of the field-theoretical cross section. As these formulas hold only for $\omega_0>\chi$, this added quantity is always positive, and therefore the cross section is always smaller than that predicted by the theory of action at a distance. In the vector meson case, there is a term $2\chi^3g^2/3\omega_0^2M$ in the denominator of the field-theoretical expression, which has no counterpart in the action-at-a-distance result, which, on the other hand, contains a term $-(\chi^3g^2/3\omega_0^2M)^2$ in its denominator, which does not appear in field theory. Both of these terms have the effect of increasing the cross section predicted by the theory of action at a distance as compared with that of field theory.

At very high frequencies the corresponding expressions of the two theories become equal. At the other limit of $\omega_0=\chi$, both theories predict zero cross section for scalar mesons. For vector mesons, the ratio of any two corresponding cross sections of the theory of action at a distance and of field theory becomes $1+(g^2\chi/M)(\frac{2}{3}+\frac{1}{9}\chi g^2/M)$. The value of $g^2\chi/M$ is different for the various kinds of mesons known experimentally and has not been determined with great accuracy. Even in the most favorable case it is only of the order of 1/20. Using this value we obtain about 1.03 for the above ratio. At intermediate values of ω_0/χ, the ratio of the cross sections is even closer to 1 for vector mesons; for scalar mesons it is approximately equal to 1.1 over a fairly wide range.

[15] Equation (B43) for the scattering of longitudinal mesons contains several misprints, which have been corrected in the table. The misprints were repeated in a paper by Vachaspati, Phys. Rev. **80**, 973 (1950), who also erroneously quoted the cross sections for the *total* scattering of transverse and of longitudinal mesons obtained in B as being those for the transverse-transverse and for the longitudinal-longitudinal scattering, respectively.

Furthermore, the general dependence on frequency is very similar in both theories, and the angular dependence of the differential cross sections [Eqs. (9), (18) and (19)] is exactly the same. Thus, considering the experimental difficulties involved, an investigation of the scattering of neutral mesons cannot be expected to furnish an experimental decision between the two theories.

However, in contrast to electrodynamics, meson theory does at least in principle provide the opportunity for an experimental decision between the points of view of field theory and the theory of action at a distance. It is hoped that an investigation of other phenomena involving neutral or charged mesons may lead to results sufficiently different in the two theories to allow such a decision.

APPENDIX

Because of the form chosen for the incoming fields [Eqs. (3) and (13)] we only have to consider the $\mu=1$ component of the equations of motion (2) and (12) in the approximation used. We shall first evaluate the expression

$$I_1=\int_{-\infty}^{\tau}\frac{s^1}{(s^2)}J_2(\chi s)d\tau'-\int_{\tau}^{\infty}\frac{s^1}{(s^2)}J_2(\chi s)d\tau'$$

occurring in Eq. (2). In our approximation $\tau=t$ and $s=\tau-\tau'$. Thus

$$I_1=\int_0^{\infty}\frac{s^1}{(s^2)}J_2(\chi s)ds+\int_0^{-\infty}\frac{s^1}{(s^2)}J_2(\chi s)ds.$$

But

$$s^1=-s_1=-\frac{\beta}{\omega_0}[\sin(\omega_0 t+\delta)-\sin(\omega_0 t'+\delta)],$$

and therefore

$$I_1=-\frac{2\beta\cos(\omega_0 t+\delta)}{\omega_0}\int_0^{\infty}\sin\omega_0 s\frac{J_2(\chi s)}{(s^2)}ds.$$

Now let $u=\chi s$ and $\nu=\omega_0/\chi$. Then we get

$$I_1=-\frac{2\beta\chi\cos(\omega_0 t+\delta)}{2i\omega_0}\times\left[\int_0^{\infty}e^{i\nu u}\frac{J_2(u)}{u^2}du-\int_0^{\infty}e^{-i\nu u}\frac{J_2(u)}{u^2}du\right].$$

Using the values of these integrals as given by HC, Eq. (9), we obtain finally

$$I_1=-\frac{2\beta\chi P_1}{\omega_0}\cos(\omega_0 t+\delta),$$

where P_1 is given by Eq. (7).

The other term needed is

$$\int_{-\infty}^{\tau}\frac{1}{s}J_1(\chi s)d\tau' - \int_{\tau}^{\infty}\frac{1}{s}J_1(\chi s)d\tau',$$

which equals

$$\int_{0}^{\infty}\frac{1}{s}J_1(\chi s)ds - \int_{-\infty}^{0}\frac{1}{s}J_1(\chi s)ds,$$

which is zero.

The expression to be calculated in Eq. (12) is

$$I_2 = v_\rho\left[\int_{-\infty}^{\tau}\frac{s^1v^\rho(\tau') - s^\rho v^1(\tau')}{(s^2)}J_2(\chi s)d\tau' - \int_{\tau}^{\infty}\frac{s^1v^\rho(\tau') - s^\rho v^1(\tau')}{(s^2)}J_2(\chi s)d\tau'\right].$$

The only nonvanishing term is that for which $\rho=0$. Substituting for the components of s and v, we obtain

$$I_2 = \int_0^\infty ds\frac{J_2(\chi s)}{(s^2)}\left[2\beta s\cos(\omega_0 t+\delta)\cos(\omega_0 s) - \frac{2\beta}{\omega_0}\cos(\omega_0 t+\delta)\sin(\omega_0 s)\right].$$

By making the substitutions $u=\chi s$ and $\nu=\omega_0/\chi$, and by changing the trigonometric functions to exponential form, we get

$$I_2 = \beta\cos(\omega_0 t+\delta)\left[\int_0^\infty du\frac{J_2(u)}{u}e^{i\nu u} + \int_0^\infty du\frac{J_2(u)}{u}e^{-i\nu u} - \frac{\chi}{i\omega_0}\int_0^\infty du\frac{J_2(u)}{u^2}e^{i\nu u} + \frac{\chi}{i\omega_0}\int_0^\infty du\frac{J_2(u)}{u^2}e^{-i\nu u}\right].$$

Using the values of the integrals as given on p. 395 of B, we obtain

$$I_2 = -\tfrac{4}{3}\beta\nu^2\cos(\omega_0 t+\delta)P_2,$$

where P_2 is given by Eq. (17).