the extended path of (3). Apparently (3) is a slow meson which gives rise to the decay electron (3′). The latter goes out of the illuminated region.

It is remarkable that in one relatively low energy interaction (classified as a star) there are produced both a slow V^0 and a slow π^+-particle.

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The Production of Mesons above 10 Bev

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12 000 local penetrating showers from carbon and paraffin have been compared using a 31-channel hodoscope in order to study nucleon-hydrogen collisions above 10 Bev. The average multiplicity of penetrating charged particles from the detected showers from carbon is found to be 4.42 ± 0.20 and the multiplicity from hydrogen considerably less. An upper limit of 2.4 is given. It is concluded that the multiple production of mesons is infrequent at this energy.

I. INTRODUCTION

It is well known that when a nucleus is struck by a high energy nucleon one or more mesons are often produced. Several theories have been proposed to account for their production.[1–5] Since their predictions differ considerably, it is of obvious importance to decide between them. However, these theories deal in the first place with the results of a collision of only two nucleons. But from the known collision mean free paths of the particles and the density of matter within the nucleus, it seems likely that there will be more than one interaction generally when a complex nucleus is struck. This view is supported by considerable experimental evidence.[6–8] Thus the results of a collision between a nucleon and a compound nucleus (which is easy to observe) is complicated by cascading and, as yet, no theory of the cascade within a nucleus has been demonstrated to be correct for the energies with which we are concerned. There is thus an obvious incentive for trying to observe the collisions of energetic nucleons with hydrogen. A number of experiments have been made with this object.[9–12] The experimental difficulties are considerable.

In the present experiment the method used was to observe, with a hodoscoped G-M counter system, the local penetrating showers from a thin layer of paraffin and later from a layer of carbon equivalent to that in the paraffin. Thus the method is similar to the well-known subtraction technique often used in the study of the low energy interactions of particles from accelerators. In our experiment the average primary energy was about 30 Bev.[13]

II. EXPERIMENTAL ARRANGEMENT

A sketch of the apparatus which was operated at sea level is given in Fig. 1. There were five trays of G-M counters, A, B, C, D, and E. Each of these trays contained 12 counters. All the counters were 50 cm long and 3.8 cm in diameter. The counters of trays A and B were in contact, and those of tray A had their axes at 90° to those of tray B. Tray C was separated from tray B by 60 cm of carbon and 7.5 cm of lead and there was a further 5 cm of lead between trays C and D. These last two trays were shielded at their sides and ends by 15 cm of Pb. Each of the counters in trays A and B could operate a neon lamp.

The condition for a master pulse was the discharge of at least two counters in each of trays B, C, and D.

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TABLE I. The rates of both extensive and nonextensive showers of various sizes in counts per hour corrected to a pressure of 1010 millibars.

Material in position Σ	Length of run, in hr	Average barometric pressure, in millibars	Type	Minimum possible number of particles 2	3	4	5	6	7	8	9	10	11	12	Total
0 cm	963	1010.5	Extensive	0.095 ±0.010	0.081 ±0.009	0.076 ±0.009	0.057 ±0.008	0.057 ±0.008	0.050 ±0.007	0.063 ±0.008	0.047 ±0.007	0.046 ±0.007	0.074 ±0.009	0.217 ±0.015	0.863 ±0.030
			Nonextensive	1.033 ±0.032	0.341 ±0.019	0.203 ±0.015	0.106 ±0.010	0.052 ±0.007	0.038 ±0.006	0.027 ±0.005	0.018 ±0.004	0.004 ±0.002	0.005 ±0.002	0.003 ±0.002	1.828 ±0.042
11.3 g/cm^2 carbon	1938.4	1012.0	Extensive	0.090 ±0.007	0.084 ±0.006	0.083 ±0.006	0.075 ±0.006	0.064 ±0.005	0.062 ±0.005	0.052 ±0.005	0.049 ±0.005	0.056 ±0.005	0.064 ±0.008	0.202 ±0.010	0.880 ±0.021
			Nonextensive	1.213 ±0.025	0.578 ±0.017	0.328 ±0.013	0.186 ±0.010	0.124 ±0.008	0.085 ±0.007	0.052 ±0.005	0.030 ±0.004	0.017 ±0.003	0.006 ±0.002	0.009 ±0.002	2.628 ±0.036
13.2 g/cm^2 paraffin	1912.9	1013.8	Extensive	0.082 ±0.007	0.075 ±0.006	0.070 ±0.006	0.077 ±0.006	0.060 ±0.005	0.065 ±0.006	0.052 ±0.005	0.055 ±0.005	0.049 ±0.005	0.068 ±0.006	0.187 ±0.010	0.844 ±0.021
			Nonextensive	1.297 ±0.025	0.559 ±0.017	0.339 ±0.013	0.191 ±0.010	0.129 ±0.008	0.081 ±0.007	0.046 ±0.005	0.030 ±0.004	0.017 ±0.003	0.009 ±0.002	0.009 ±0.002	2.704 ±0.036

This meant that generally at least two penetrating particles were necessary to discharge the apparatus. An observable rate of master pulses was produced by (a) local penetrating showers, (b) extensive penetrating showers, and (c) the random association of two μ-mesons. The treble knock-on rate of single μ-mesons was made negligible by wrapping each of the counters in trays C and D in lead sheet $\frac{1}{10}$ in. thick.

Extensive showers were detected by tray E which with all its counters in parallel operated a neon lamp. Showers in which the neon lamp was not discharged were classed as nonextensive.

III. RESULTS

The apparatus was first run with no material over tray A, then with 11.3 g/cm^2 of carbon in this position, next with 13.2 g/cm^2 paraffin, and finally with no absorber again as a check.

The results are given in Table I. The last column gives the total rate per hour for both extensive and nonextensive showers for all runs. The eleven preceding columns give the rates of showers containing different minimum possible numbers of ionizing particles. If, in the two top trays, m and n counters are discharged, $m \geqslant n$, then it is obvious that at least m particles must have fallen on these trays. If m is small, then the actual number of particles, as will be shown immediately, has a high probability of being m. If m is large, this is no longer so and the actual number will in general considerably exceed m.

The actual number of particles falling on these two trays may be estimated in the following manner. If N particles fall at random[14] on the trays, the probability $P(N, m, n)$ that m and n counters are discharged may be calculated following Schrödinger.[15] The probability that an m, n event will occur is then

$$\sum_N P(N, m, n)Q(N),$$

where $Q(N)$ is the probability of N particles occurring. By using this, one can compute the probability that an m, n event was caused by any particular number of particles, providing $Q(N)$ is known. In fact $Q(N)$ is not known, but the probability distributions are not very sensitive to its form. This can be seen from Fig. 2 where the probability of N particles giving (a) a 2, 2 event, (b) a 5, 3 event, and (c) an 8, 6 event are displayed. The dotted lines give the results for a $Q(N)$ of the form

$$Q(1)=Q(2)=\cdots=Q(70), \quad Q(71)=Q(72)=\text{etc.}=0.$$

This is a physically impossible form. The unbroken lines are for $Q(N) \propto 1/N$, which is at least an approximation to reality. However it will be seen that the results are not much different. It can also be seen from this figure that the resolution of the apparatus is excellent for small numbers of particles but deteriorates as the numbers increase. Fortunately most of the showers contain only a few particles.

Using this method then one can estimate the number of particles in a shower. Table II gives the rates of showers of different sizes corrected to 1010 millibars found in this way. Because of the decreasing resolution of the apparatus the larger showers are arranged in wider groups.

It will be seen that the rates for showers containing 2, 3, and 4 particles are almost identical with the rates for showers in which a maximum of 2, 3, or 4 counters were discharged in A or B.

The last two lines give the rates of showers of different

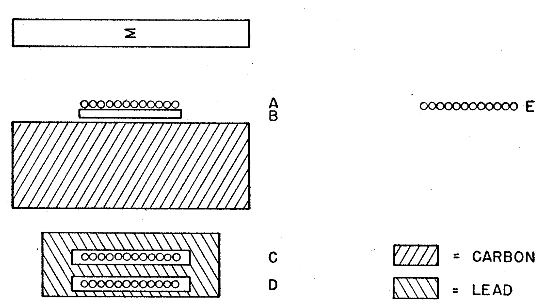

FIG. 1. Sketch of the apparatus.

[14] The departure from complete randomness in local penetrating showers has been shown to have little effect; see Froehlich, Harth, and Sitte, Phys. Rev. **87**, 504 (1952).

[15] E. Schrödinger, Proc. Phys. Soc. (London) **A64**, 1040 (1951).

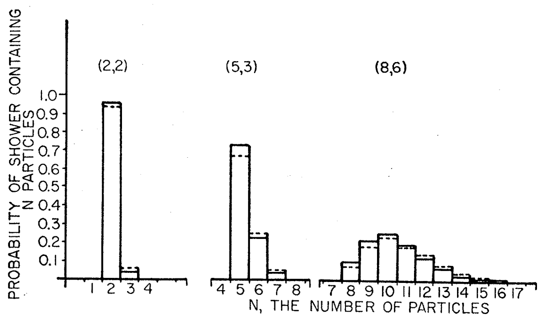

FIG. 2. Histograms showing the probability that a shower discharging m and n counters in the two top trays contains N particles. (1) 2 counters discharged in each tray (2, 2), (2) 5 counters discharged in one tray and 3 in the other (5, 3), and (3) eight counters discharged in one tray and six in the other (8, 6). The probability is calculated in each case for two different forms of $Q(N)$ (see text).

sizes from carbon and hydrogen obtained by subtracting the background rate from the carbon + background rate and the carbon + background rate from the paraffin + background rate. It will be seen that the only hydrogen rate which is significantly different from zero is that for twofold events.

The average number of ionizing particles per shower from carbon corrected for the electronic component resulting from π^0-mesons is 4.42 ± 0.20. The average number of ionizing penetrating particles from hydrogen was 2.4 but the small number of showers observed makes the statistical significance of this low.

IV. DISCUSSION OF RESULTS

It will be seen from Table I that the extensive showers are not significantly affected either in their absolute rate or in the distribution of multiplicities by the addition of the paraffin or carbon. The rates for different multiplicities have been calculated using the well-known integrals and excellent agreement with the experimental results was obtained.[13] It follows that the observed extensive showers were of the type normally observed at sea level and, since the predicted rate for such an extensive shower to trigger the master pulse without discharging the extensive tray was 1 in 1500 hours, that such events were a negligible contribution to the nonextensive rate.

Perhaps the most striking feature of the nonextensive results is the small number of recorded showers from hydrogen. This confirms the results of Walker, Duller, and Sorrels.[11] If we assume a geometric cross section and a similar multiplicity distribution from hydrogen as from carbon we would expect to get 619 events from hydrogen in 1912.9 hours whereas we got 145 ± 107. For showers other than twofolds the difference is even greater. We expect 467 showers from hydrogen with $N\geqslant 3$ and we get -15, that is to say the difference was negative due to statistical fluctuations, and not significantly different from 0. For carbon the number of showers with $N\geqslant 3$ was 1186. There is considerable evidence[16] in support of the view that the interaction cross section at these energies is geometric. It follows that collisions giving multiplicities of charged particles $N\geqslant 3$ must be considerably less frequent from hydrogen than from carbon. Even if we assumed a cross section of only half the geometric value, we would still expect 234 events from hydrogen with $N\geqslant 3$.

Since the results, confirming those of Walker *et al.*, seem to establish a considerable difference in the multiplicities from carbon and from hydrogen under identical conditions it is interesting to examine the actual multiplicities from carbon. Our average multiplicity of charged penetrating particles is 4.42 ± 0.20. As an estimate of the average multiplicities for all nucleon interactions with carbon at these energies this may be in error from two effects: (a) our counter method of measuring the numbers of particles in a shower; (b) the bias introduced by any apparatus which requires $\geqslant 2$ penetrating particles to trigger it.

The first of these effects would seem to be unimportant. Walker *et al.* working with a cloud chamber

TABLE II. The rates of nonextensive showers containing various numbers of ionizing particles in counts per hour corrected to a pressure of 1010 millibars. The last two lines give the rates for carbon and hydrogen obtained by the subtraction method.

Run	2	3	4	5 and 6	7, 8, and 9	$\geqslant 10$ ionizing particles
Background	1.033 ±0.033	0.340 ±0.014	0.202 ±0.014	0.140 ±0.012	0.073 ±0.009	0.041 ±0.006
Carbon +background	1.213 ±0.024	0.578 ±0.017	0.328 ±0.013	0.256 ±0.011	0.159 ±0.009	0.095 ±0.007
Paraffin +background	1.297 ±0.025	0.558 ±0.016	0.339 ±0.013	0.265 ±0.012	0.152 ±0.009	0.094 ±0.007
Carbon	0.180 ±0.041	0.238 ±0.026	0.126 ±0.019	0.116 ±0.017	0.086 ±0.013	0.054 ±0.010
Hydrogen	0.084 ±0.036	-0.020 ±0.024	0.010 ±0.018	0.009 ±0.016	-0.007 ±0.013	-0.001 ±0.010

found an average multiplicity of charged penetrating particles from carbon of 4.21 ± 0.30. The two results agree within their statistical errors and since in the Walker experiment the event was observed from carbon plates within a cloud chamber, it is difficult to see how their result could be in error apart from the statistical fluctuation.

The effect (b), however, may be expected to influence both results considerably. An interaction producing less than two ionizing penetrating particles will be missed, and even events with 2 or 3 penetrating particles will not be detected with complete efficiency. Thus the average multiplicity given by these experiments may be expected to be considerably greater than the average multiplicity of all interactions at these energies. Working with photographic plates which are not biased in this way, Salant and his co-workers[6] found an average multiplicity of charged penetrating particles from carbon, nitrogen, and oxygen of 1.4 ± 0.2 at an average energy of 3 Bev and 1.4 ± 0.3 at an average

[16] C. F. Powell, Repts. Progr. Phys. 13, 350 (1950).

energy of 14 Bev. It thus seems reasonable to conclude that at about 30 Bev the average multiplicity of charged penetrating particles from all interactions in carbon is about 2 and that from hydrogen is considerably less. The actual figure of 2.4 for the average multiplicity from hydrogen given by our experiment would seem to be a safe upper limit. It should be noted that the charged penetrating particles will include the high energy nucleons coming from the interactions but will not include the neutral mesons produced.

It is obvious that the primary energy for the events from hydrogen and carbon was the same and that the comparison of multiplicities does not depend on a knowledge of this energy. Nevertheless, it is of considerable interest to know the average energy of the particles producing these events. In a previous paper[13] the minimum triggering energy of this apparatus estimated by three different methods, was given as 20 Bev. These estimates depend, however, upon a number of factors (such as the form of the proton spectrum at sea level) which are not too well known. It is possible that the minimum is as low at 10 Bev, but it would be difficult to justify a lower limit than this.

V. COMPARISON WITH THEORY

It is obvious from a comparison of the multiplicities from hydrogen and carbon that a good deal of the production of charged particles in these encounters must be due to cascading within the nucleus. The average number of collisions in crossing a carbon nucleus (assuming a geometrical cross section) is 2.2. Our average multiplicity for carbon encounters (in agreement with Walker *et al.*) is 4.2 and, as we have seen, is probably an upper limit. Thus both the comparison of the carbon and hydrogen multiplicities and the absolute values of the multiplicities suggest that at these energies the multiple production of mesons must be of rare occurrence. This confirms the conclusion of Salant *et al.*[6]

Thus the experiment seems to agree best with the predictions of Heitler's theory. The theory of Fermi predicts an average number of charged particles in a nucleon-nucleon collision of 5.1 at 30 Bev and 3.9 at 15 Bev. To reach agreement even with the upper limit of our experimentally determined multiplicity for nucleon-nucleon encounters needs the assumption of an average primary energy of 3 Bev, and it then becomes difficult to explain both the average multiplicity from carbon and the absolute rate of events. The recent theory of Heisenberg[2] predicts an average multiplicity of 4.2 charged particles from a nucleon-nucleon encounter at 30 Bev and is thus also difficult to reconcile with our results. It should, however, be stressed that the experiment neither disproves the possibility of multiple production occasionally at these energies (Heitler theory predicts multiple production in a fraction of cases, 10 percent is given as an upper limit) nor generally at higher energies.

VI. CONCLUSION

The experiment confirms the result of Walker *et al.* that local penetrating showers from hydrogen are considerably less multiple than those from carbon and therefore adds to the evidence for cascading within the nucleus. The conclusion of Salant *et al.* that multiple production of mesons is rare at about 15 Bev is also confirmed and probably extended to about twice that energy.

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