The Transport of Added Current Carriers in a Homogeneous Semiconductor

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Taking into account the thermal equilibrium minority carrier concentration and employing the formulation which includes, as one of two fundamental differential equations, the continuity equation for added carrier concentration Δp, this equation is derived in a form which exhibits the ambipolar nature of the diffusion, drift, and recombination mechanisms under electrical neutrality. The general concentration-dependent diffusivity is given. The local drift velocity of Δp has the direction of total current density in an n-type semiconductor and the reverse in a p-type semiconductor, differing in general in both magnitude and direction from the minority-carrier drift velocity. Specifying a model for recombination fixes the dependence of a lifetime function for Δp on Δp and the electron and hole mean lifetimes. Negative Δp, or carrier depletion with electrical neutrality, may occur. For known total current density, the continuity equation alone suffices, as for the case of $|\Delta p|$ small, for which the equation is linear. A condition for this comparatively important case is derived, and theoretical relationships are given, with the aid of a parameter specifying the Fermi level, which determine for germanium the minority carrier-Δp drift velocity ratio as well as the ambipolar diffusivity and group mobility in terms of resistivity and temperature.

1. INTRODUCTION

CONCENTRATIONS of current carriers of one type injected into a homogeneous semiconductor are neutralized by substantially equal concentrations of additional carriers of the opposite type, the space charge associated with carrier injection being in general quite negligible. It is consistent with this electrical neutrality that the transport of the concentration of added carriers cannot be identified with the transport of injected carriers as such. The drift, diffusion, and recombination processes to which the added concentration is subject depend on the corresponding microscopic processes for both electrons and holes. The concentration transport is characterized by a diffusivity, an apparent mobility, and a decay time which equal the diffusion constant, mobility, and mean lifetime of the minority carriers only in the limiting case of relatively small added concentrations in suitably extrinsic semiconductor material. In this paper, the concentration transport problem is formulated in a manner which exhibits the general concentration-dependent diffusivity and local drift velocity; and it is indicated how a decay-time or lifetime function depends on the nature of the microscopic recombination process. These physical interpretations are based on a derivation of the continuity equation for the concentration of added carriers.

This continuity equation, at the same time, suffices as a formulation of the general transport problem if the total current density is known in terms of time and the space variables. Otherwise, two dependent variables are involved, and a second differential equation is required.[1,2] In particular, if the added carrier concentration is small so that the equation is linear, then the total current density is known as the solution of an ohmic flow problem. A simple example for arbitrary added concentrations is zero total current density, and cases of linear flow in one dimension and cylindrical or spherical symmetry may provide others.[3] For the small-signal case, expressions which are given for the diffusivity and the apparent or group mobility are evaluated for germanium, in their dependence on resistivity and temperature. With the decay time, these quantities provide the coefficients of the linear continuity equation. They differ appreciably from their values for strongly extrinsic material if the concentration excess of majority carriers is less than about 20 times the electron concentration in intrinsic material.[4] The relationships presented extend all formal small-signal solutions given elsewhere for the purely extrinsic case to semiconductor material of any resistivity.

The theoretical conclusion that the added carrier concentration may drift appreciably more slowly than the minority carriers in near-intrinsic material[5] has been applied to the design of transistors of large current amplification.[6] It provides an interpretation of various aspects of the behavior of transistors in which current multiplication is associated with increases in conductivity resulting from carrier injection.[7] Recent experi-

[1] W. van Roosbroeck, Bell System Tech. J. **29**, 560–607 (1950), Secs. 2.2, 2.3.
[2] R. C. Prim, III, Bell System Tech. J. **30**, 1174–1213 (1951), Sec. C.
[3] Employing an elegant alternative formulation, Prim has derived classes of formal solutions in three dimensions for arbitrary added concentrations. See reference 2, also reference 1, Eqs. (14).
[4] The practical importance of this range, which corresponds roughly to germanium resistivities at 300°K greater than 5 ohm-cm, has been enhanced by present purification techniques: W. G. Pfann and K. M. Olsen, Phys. Rev. **89**, 322–323 (1953).
[5] Reference 1, pp. 564, 577–578, 592–593 and Eq. (9).
[6] W. van Roosbroeck, J. Appl. Phys. **23** (12), 1411–1412 (1952).
[7] For example, theory and experiment were found consistent in the explanation of a cut-off frequency for current multiplication, in a filamentary germanium transistor, which was about 3 Mc/sec, rather than the 10 Mc/sec corresponding to the minority carrier drift velocity: The frequency data were fitted by use of the resistivity, 10 ohm-cm, in conjunction with an operating temperature of 72°C, which agrees fairly well with the operating temperature of 65°C determined, with Brattain, from the power required to melt on the filament a tiny flake of wax of known melting point.

ments on the transport of a pulse of injected carriers under an applied field,[8,9] in germanium and silicon filaments of various resistivities and at different temperatures, have supported the predicted relationship between the drift velocity of the pulse and that of the minority carriers.[10,11]

The assumptions of the theory and their justifications have previously been discussed.[1,11,12] It should be noted that the assumption that at any point electrons and holes always recombine at equal rates is consistent with a mechanism of recombination through trapping at imperfections[13] which can account for observed lifetimes. Trapping effects are ruled out which result from the immobilization of carriers in traps for times not small compared with their mean lifetime before their recombination or release.[14]

2. THE GENERAL CONTINUITY EQUATION

The continuity equations for holes and electrons are, in a familiar notation,[15]

$$\begin{aligned}\partial p/\partial t &= -e^{-1}\,\mathrm{div}\mathbf{I}_p-(p/\tau_p-g_0),\\ \partial n/\partial t &= e^{-1}\,\mathrm{div}\mathbf{I}_n-(n/\tau_n-g_0),\end{aligned}\tag{1}$$

where τ_n and τ_p are the electron and hole mean lifetimes, and g_0 is the thermal generation rate per unit volume. The current densities $\mathbf{I}_p$ and $\mathbf{I}_n$ are given by

$$\mathbf{I}_p=\sigma_p\mathbf{E}-eD_p\,\mathrm{grad}p,\quad \mathbf{I}_n=\sigma_n\mathbf{E}+eD_n\,\mathrm{grad}n;\tag{2}$$

the total current density is

$$\mathbf{I}=\mathbf{I}_p+\mathbf{I}_n,\tag{3}$$

and is solenoidal:[16]

$$\mathrm{div}\mathbf{I}=0.\tag{4}$$

Here σ_p and σ_n, given by

$$\sigma_p=e\mu_p p,\quad \sigma_n=e\mu_n n,\tag{5}$$

are the respective contributions of holes and electrons to the conductivity:

$$\sigma=\sigma_p+\sigma_n.\tag{6}$$

With the condition of electrical neutrality, namely

$$p-p_0\equiv\Delta p=\Delta n\equiv n-n_0,\tag{7}$$

these fundamental equations furnish equations for the ambipolar[17] transport of added carrier concentration Δp (or Δn). Equation (7) permits the elimination of a concentration and the field $\mathbf{E}$ from the current equations (2). It is found that

$$\sigma\mathbf{E}=\mathbf{I}-e(D_n-D_p)\,\mathrm{grad}\Delta p,\tag{8}$$

whence the current equations may be written as

$$\begin{aligned}\mathbf{I}_p&=(\sigma_p/\sigma)\mathbf{I}-eD\,\mathrm{grad}\Delta p,\\ \mathbf{I}_n&=(\sigma_n/\sigma)\mathbf{I}+eD\,\mathrm{grad}\Delta n,\end{aligned}\tag{9}$$

where D is the general concentration-dependent diffusivity:

$$D\equiv\frac{\sigma_nD_p+\sigma_pD_n}{\sigma}=\frac{n+p}{n/D_p+p/D_n}.\tag{10}$$

In this equation, the second form results because of the proportionality between the diffusion constants and the mobilities.[18] Substituting from (9) in (1), both continuity equations furnish the same continuity equation for Δp in a form which does not involve $\mathbf{E}$. It is instructive, however, to derive this continity equation also in a form which does involve $\mathbf{E}$ and which is readily obtained by substituting from the original current equations (2) in (1) to obtain

$$\begin{aligned}\partial p/\partial t=&D_p\,\mathrm{div}\,\mathrm{grad}p-e^{-1}\,\mathrm{grad}\sigma_p\cdot\mathbf{E}\\ &-e^{-1}\sigma_p\,\mathrm{div}\mathbf{E}-(p/\tau_p-g_0),\\ \partial n/\partial t=&D_n\,\mathrm{div}\,\mathrm{grad}n+e^{-1}\,\mathrm{grad}\sigma_n\cdot\mathbf{E}\\ &+e^{-1}\sigma_n\,\mathrm{div}\mathbf{E}-(n/\tau_n-g_0),\end{aligned}\tag{11}$$

then multiplying these equations by σ_n and σ_p and adding, so that $\mathrm{div}\mathbf{E}$ is eliminated. The continuity equation resulting from these derivations is

$$\partial\Delta p/\partial t=\mathrm{div}D\,\mathrm{grad}\Delta p-\mathbf{v}\cdot\mathrm{grad}\Delta p-\Delta p/\tau.\tag{12}$$

Here, writing r for $p/\tau_p=n/\tau_n$, a lifetime function τ for added carrier concentration has been introduced in accordance with

$$r-g_0\equiv\Delta p/\tau;\tag{13}$$

and $\mathbf{v}$ is given by

$$\begin{aligned}\mathbf{v}&\equiv(\mu_p/M)\mathbf{E}+\mathrm{grad}D=\mu^*\mathbf{E}+\mathrm{grad}D\\ &=\mu_p\mathbf{I}/M\sigma=e\mu_n\mu_pn_s\mathbf{I}/\sigma^2=\mu^*(\mathbf{I}/\sigma),\end{aligned}\tag{14}$$

with

$$\begin{aligned}&M\equiv\sigma/\sigma_s=1+(1+b^{-1})p/n_s,\\ &\sigma_s\equiv e\mu_nn_s,\quad n_s\equiv n_0-p_0,\quad b\equiv\mu_n/\mu_p,\end{aligned}\tag{15}$$

and

$$\mu^*\equiv\frac{\mu_p}{M}=\frac{e\mu_n\mu_pn_s}{\sigma}=\frac{n_s}{n/\mu_p+p/\mu_n}.\tag{16}$$

The velocity $\mathbf{v}$ may be identified as the local drift velocity for added carrier concentration[19] Δp. The

[8] Shockley, Pearson, and Haynes, Bell System Tech. J. **28**, 344–366 (1949).
[9] J. R. Haynes and W. Shockley, Phys. Rev. **81**, 835–843 (1951).
[10] M. B. Prince, Bull. Am. Phys. Soc. **28**, No. 2, 10 (1953); Phys. Rev. **91**, 271 (1953).
[11] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), Chap. 12.
[12] Conyers Herring, Bell System Tech. J. **28**, 401–427 (1949).
[13] W. Shockley and W. T. Read, Jr., Phys. Rev. **87**, 835–842 (1952).
[14] The theory can be extended, without essential change in its formal structure, to take such trapping effects into account.
[15] The notation employed is consistent with that of reference 1.
[16] Reference 1, Sec. 2.2. With negligible magnetic fields, it is also lamellar.
[17] This term has gained currency through the similarity with certain ionic transport phenomena in gases.
[18] The validity of this proportionality, as given by Einstein's relationship, is coextensive with the applicability of Boltzmann statistics, and these are consistent with the current equations (2). Also, see Phys. Rev. **88**, 1368–1369 (1952).
[19] This drift velocity $\mathbf{v}$ is, of course, to be distinguished from the differential transport velocity (reference 1, Sec. 2.2), which includes diffusive transport of Δp.

quantity μ^* is accordingly an ambipolar pseudomobility or group mobility for the drift of Δp under the local field $\mathbf{E}$. Both M and μ^* have the sign of n_s, being positive for an n-type and negative for a p-type semiconductor. Furthermore, as may readily be verified, M in an n-type, and $-bM$ in a p-type semiconductor are substantially unity for Δp small in strongly extrinsic material, and exceed unity in general. Thus, the drift velocity $\mathbf{v}$ has the direction of the total current density $\mathbf{I}$ in an n-type semiconductor and the reverse direction in a p-type semiconductor; it is in general smaller in magnitude than the drift velocity $\mu_p\mathbf{E}$ or $-\mu_n\mathbf{E}$ of the minority carriers, and differs also in direction.[20]

For the intrinsic semiconductor, for which the drift term is absent, $|M|$ being infinite and μ^* zero, and for which D equals[21] $D_i \equiv 2D_nD_p/(D_n+D_p)$, the continuity equation is linear in general, except, perhaps, for the dependence of τ on Δp; and the circumstance that Δp is not subject to drift follows most simply from the ambipolar form (9) of the current equations: With σ_p/σ and σ_n/σ, respectively, $1/(b+1)$ and $b/(b+1)$, the ohmic contributions to the hole and electron current densities do not depend on Δp.[22] Since, even in a strongly extrinsic semiconductor, $|M|$ may be large as a result of carrier injection, large total current densities do not in general insure the validity, as a good approximation, of the neglect of diffusion in the large-signal theory.

A current density associated with added carrier concentration Δp is properly defined by

$$\begin{aligned}\Delta\mathbf{I} &\equiv \mathbf{I}_p-(e\mu_p p_0/\sigma_0)\mathbf{I}=-\mathbf{I}_n+(e\mu_n n_0/\sigma_0)\mathbf{I}\\ &=(e\mu_n n_0/\sigma_0)\mathbf{I}_p-(e\mu_p p_0/\sigma_0)\mathbf{I}_n,\end{aligned} \tag{17}$$

in which σ_0 is the conductivity at thermal equilibrium. It is $\mathbf{I}_p$ minus the (drift) current density of holes in the semiconductor with no added carriers for the same total current density $\mathbf{I}$. The first two terms of the continuity equation (12) for Δp are given by $-e^{-1}\,\mathrm{div}\Delta\mathbf{I}$, since $\mathrm{div}\Delta\mathbf{I}=\mathrm{div}\mathbf{I}_p=-\mathrm{div}\mathbf{I}_n$. Substituting in (17) from (9) and introducing $\mathbf{v}$, $\Delta\mathbf{I}$ may be expressed in the form

$$\Delta\mathbf{I}=e(\sigma/\sigma_0)\Delta p\mathbf{v}-eD\,\mathrm{grad}\Delta p. \tag{18}$$

In this equation, the drift term, in which the factor σ/σ_0 is substantially unity for small added concentrations, gives a drift component of $\Delta\mathbf{I}$ which vanishes for the intrinsic semiconductor, and which for relatively large added concentrations in strongly extrinsic material equals $\mathbf{I}/(b+1)$ for n-type and $-b\mathbf{I}/(b+1)$ for p-type. These are, in magnitude, respectively the contributions of holes and electrons to the drift current density in intrinsic material.

To complete the continuity-equation formulation[23] for the general case, it is convenient to employ as the second dependent variable the potential

$$\psi \equiv V-[(b-1)/(b+1)](kT/e)\ln(\sigma/\sigma_0), \tag{19}$$

where V is the electrostatic potential. Then, from (8),

$$\mathbf{I}=-\sigma\,\mathrm{grad}\psi, \tag{20}$$

and the second differential equation (4) is

$$\mathrm{div}\sigma\,\mathrm{grad}\psi=0. \tag{21}$$

Making use of this equation, it is readily found that

$$\mathbf{v}\cdot\mathrm{grad}\Delta p=\bar{\mu}n_s\,\mathrm{div}\,\mathrm{grad}\psi,\quad \bar{\mu}\equiv\mu_n\mu_p/(\mu_n+\mu_p), \tag{22}$$

so that the continuity equation (12) may be written as

$$\partial\Delta p/\partial t=-e^{-1}\,\mathrm{div}\mathbf{I}'-\Delta p/\tau, \tag{23}$$

where

$$\begin{aligned}\mathbf{I}' &\equiv e\bar{\mu}n_s\,\mathrm{grad}\psi-eD\,\mathrm{grad}\Delta p\\ &=[b/(b+1)]\mathbf{I}_p-[1/(b+1)]\mathbf{I}_n\\ &=\Delta\mathbf{I}-[\mu_0^*/(\mu_n+\mu_p)]\mathbf{I},\end{aligned} \tag{24}$$

with μ_0^* the value of μ^* at thermal equilibrium.

Since the recombination rate r must equal the thermal generation rate g_0 for $\Delta p=0$, the introduction of the lifetime function τ in accordance with (13) entails no loss of generality.[24] For direct electron-hole recombination of the mass-action type,

$$r=(np/n_i^2)g_0=[1+(n_0+p_0+\Delta p)(\Delta p/n_i^2)]g_0, \tag{25}$$

where $n_i^2=n_0p_0$ is the square of the electron (or hole) concentration in the intrinsic semiconductor. Comparing (13) and (25),

$$\begin{aligned}\tau^{-1} &= (n_0+p_0+\Delta p)g_0/n_i^2=(n_0^{-1}+p_0^{-1}+\Delta p/n_i^2)g_0\\ &=(\tau_n^{-1}+\tau_p^{-1})\,|_{\Delta p=0}+(g_0/n_i^2)\Delta p.\end{aligned} \tag{26}$$

As this result indicates, τ equals τ_p or τ_n in strongly extrinsic n- or p-type material, if Δp may be neglected. An estimate has shown, however, that the direct electron-hole recombination process would give a lifetime τ much larger than those usually observed. A model which has been presented for recombination at imperfections and which gives τ in terms of Δp can account for observed lifetimes.[25] The τ_p and τ_n which must obtain with such a model may be found from r in Eq. (13).

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[20] Introducing the minority carrier concentration p_0 into equations of Herring's solution by writing them for the case of no recombination with n_s as concentration unit rather than n_0, Shockley has shown, in effect, that $|M|$ is the ratio between $\mu_p|\mathbf{E}|$ and $|\mathbf{v}|$ if grad$D$ may be neglected, that is, for negligible diffusion or for $|\Delta p|$ small: reference 11, Chap. 12, Sec. 7; see reference 1, footnote 24.

[21] Note that $D/D_i-1=-(b-1)/2bM$.

[22] See reference 11, pp. 330–331; reference 1, pp. 564, 577–578.

[23] See reference 1, Eqs. (12), (13).

[24] The procedure is tantamount to the introduction of the combination function in Sec. 2.3 of reference 1.

[25] From Eqs. (5.5) and (6.1) of reference 13, a constant τ equal to $\tau_{p0}+\tau_{n0}$, for example, results if the ratio of the limiting lifetime τ_{p0} for holes in strongly extrinsic n-type material to the analogous lifetime τ_{n0} for p-type equals $(p_0-n_1)/(p_1-n_0)$, where n_1 and p_1 are the thermal equilibrium electron and hole concentrations if the Fermi level were at the energy level of the imperfections.

It is evident from these considerations that Δp need not be positive but can in principle be negative as well. Carrier depletion, with negative Δp, may be realized through a boundary condition which suppresses in part either the electron or hole component of total current, imposing a value smaller than that for ohmic transport. Such a condition, which holds at a reverse-biased junction with material of more strongly extrinsic conductivity of either the same or the opposite type, may give depletion without appreciable space charge over a region much wider than the adjoining space-charge region of the junction. For an extrinsic semiconductor, the width increases essentially in proportion to the magnitude of the total reverse bias current density, and substantially complete depletion of the minority carriers may be obtained, following which space-charge limited unipolar flow[26] of the majority carriers takes

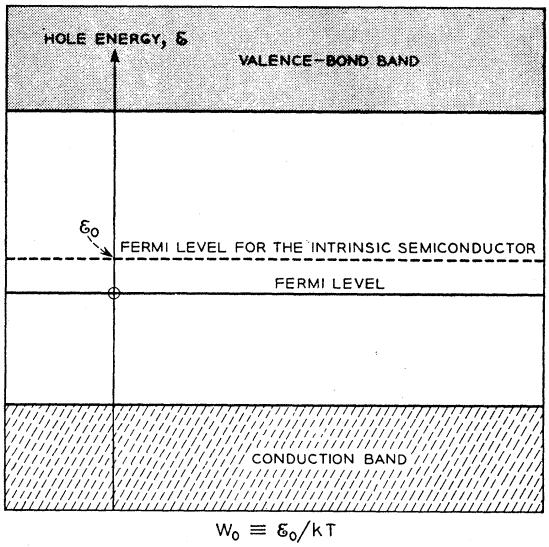

FIG. 1. Definition of W_0.

place. For an intrinsic semiconductor there is a saturation magnitude of the current density, and the width is of the order of a diffusion length, in which distance Δp approaches zero as a result of the thermal generation process.[27]

3. THE SMALL-SIGNAL CASE

(1) The Linear Continuity Equation

For sufficiently small added carrier concentrations, the general continuity equation (12) becomes an equation with constant coefficients for the diffusion and recombination terms.[28] If σ_i is the thermal equilibrium conductivity σ_0 for the intrinsic semiconductor, then for the condition[29]

$$|\Delta p| \ll (\sigma_0/\sigma_i) n_i = (\mu_n n_0 + \mu_p p_0)/(\mu_n + \mu_p) = (b n_0 + p_0)/(b+1), \tag{27}$$

the equation becomes[30]

$$\partial \Delta p/\partial t = D_0 \operatorname{div} \operatorname{grad} \Delta p - \mathbf{v}_0 \cdot \operatorname{grad} \Delta p - \Delta p/\tau_0, \tag{28}$$

in which τ_0 is the limiting value of the lifetime function, and D_0 and $\mathbf{v}_0$ are given by

$$D_0 \equiv kT \mu_n \mu_p (n_0 + p_0) = \frac{n_0 + p_0}{n_0/D_p + p_0/D_n}, \tag{29}$$

$$\mathbf{v}_0 \equiv (\mu_p/M_0)\mathbf{E}_a = \mu_0^* \mathbf{E}_a, \quad \mathbf{E}_a \equiv \mathbf{I}/\sigma_0, \tag{30}$$

with

$$M_0 \equiv \sigma_0/\sigma_s = 1 + (1 + b^{-1}) p_0/n_s \tag{31}$$

and

$$\mu_0^* \equiv \frac{e \mu_n \mu_p n}{\sigma_0} = \frac{n_s}{n_0/\mu_p + p_0/\mu_n}. \tag{32}$$

Equation (18) for the current density of added carriers becomes

$$\Delta \mathbf{I} = e \Delta p \mathbf{v}_0 - e D_0 \operatorname{grad} \Delta p. \tag{33}$$

The condition (27), which implies that the local conductivity σ is substantially σ_0, so that $\mathbf{v} \sim \mathbf{v}_0$, insures the linearity of (28) aside from a possible dependence of $\mathbf{v}_0$ on Δp. It shows that nonlinearities may appear at relative concentrations some b times smaller if the semiconductor is p-type than if it is n-type. The conditions,

$$|\Delta p| \ll [(b+1)/(b-1)][\sigma_0/\sigma_i][(n_0 + p_0)/|n_s|] n_i \tag{34}$$

for $D \sim D_0$, and

$$|\Delta p| \ll n_0 + p_0 \tag{35}$$

for $\tau \sim \tau_0$, are the analogous ones for the diffusion and recombination terms, the latter, obtained from (26), applying to the case of mass-action recombination. Either of these may provide linearity for the case of zero total current density. The stronger condition (27), not needed for linearity in the intrinsic case, is a necessary condition for the neglect of Δp in (4) or (21), which express the solenoidal property of $\mathbf{I}$. For (19) shows that with it condition ψ equals V to within a potential which is small compared with kT/e. If also, V varies at least by an amount of order kT/e, then grad $\psi \sim$ grad V and $\mathbf{v}_0$ is found simply from the applied field $\mathbf{E}_a$ of the solving an ohmic flow problem; and

[26] W. Shockley, Proc. Inst. Radio Engrs. **40**, 1289–1313; 1365–1376 (1952).

[27] These conclusions are readily obtainable from the relevant steady-state solutions. The width of the neutral region is determined on a self-consistency basis by computing for neutrality both the charge of the smallest concentration of remaining carriers which occurs and div$\mathbf{E}$, and comparing with the former the charge unbalance which the latter gives according to Poisson's equation.

[28] Note that the neglect of div$\mathbf{E}$ in the continuity equations (11) provides two (inconsistent) small-signal equations which apply respectively to the purely extrinsic n- and p-type semiconductors.

[29] The value of $|\Delta p|$ may, of course, be limited for Δp negative by the condition of complete minority carrier depletion.

[30] Harvey Brooks has independently derived results given here, in connection with the one-dimensional transport of a pulse of added carrier, and has also examined for this case the validity of the neglect of space charge: Phys. Rev. **90**, 336 (1953).

the transport of added carriers *per se* is then subject only to the linear continuity equation (28). The condition shows that linearity obtains for the complete depletion of minority carriers in an n-type semiconductor provided $p_0 \ll bn_0/(b+1)$, and in a p-type semiconductor provided $n_0 \ll p_0/(b+1)$.

(2) Application to Germanium

Writing (28) in the dimensionless form,

$$\partial\Delta P/\partial U = \text{div grad}\Delta P - (\mathbf{v}_0/v_D)\cdot\text{grad}\Delta P - \Delta P, \quad (36)$$

where the new variables[31] are given by

$$\begin{aligned} &\Delta P \equiv \Delta p/n_s; && U \equiv t/\tau_0; \\ &X \equiv x/L, \quad Y \equiv y/L, \quad Z \equiv z/L; && L \equiv (D_0\tau_0)^{\frac{1}{2}}, \end{aligned} \quad (37)$$

and where

$$v_D \equiv L/\tau_0 = (D_0/\tau_0)^{\frac{1}{2}}, \quad (38)$$

it appears that formal small-signal solutions differ essentially only according to the vector field, $\mathbf{v}_0/v_D = \mu_0^*\mathbf{E}_a/v_D$, which if the vector field $\mathbf{E}_a$ is specified, depends only on the scalar field intensity v_D/μ_0^*. For a particular semiconductor with τ_0 given this quantity will depend, in turn, through D_0 and μ_0^*, on resistivity and temperature.

The presentation of this dependence is facilitated by the use of a parameter W_0 whose definition is illustrated in Fig. 1: It is the energy for a hole measured from the actual Fermi level for the semiconductor to the location of the Fermi level for intrinsic material, and expressed in units of kT. From this definition and Boltzmann

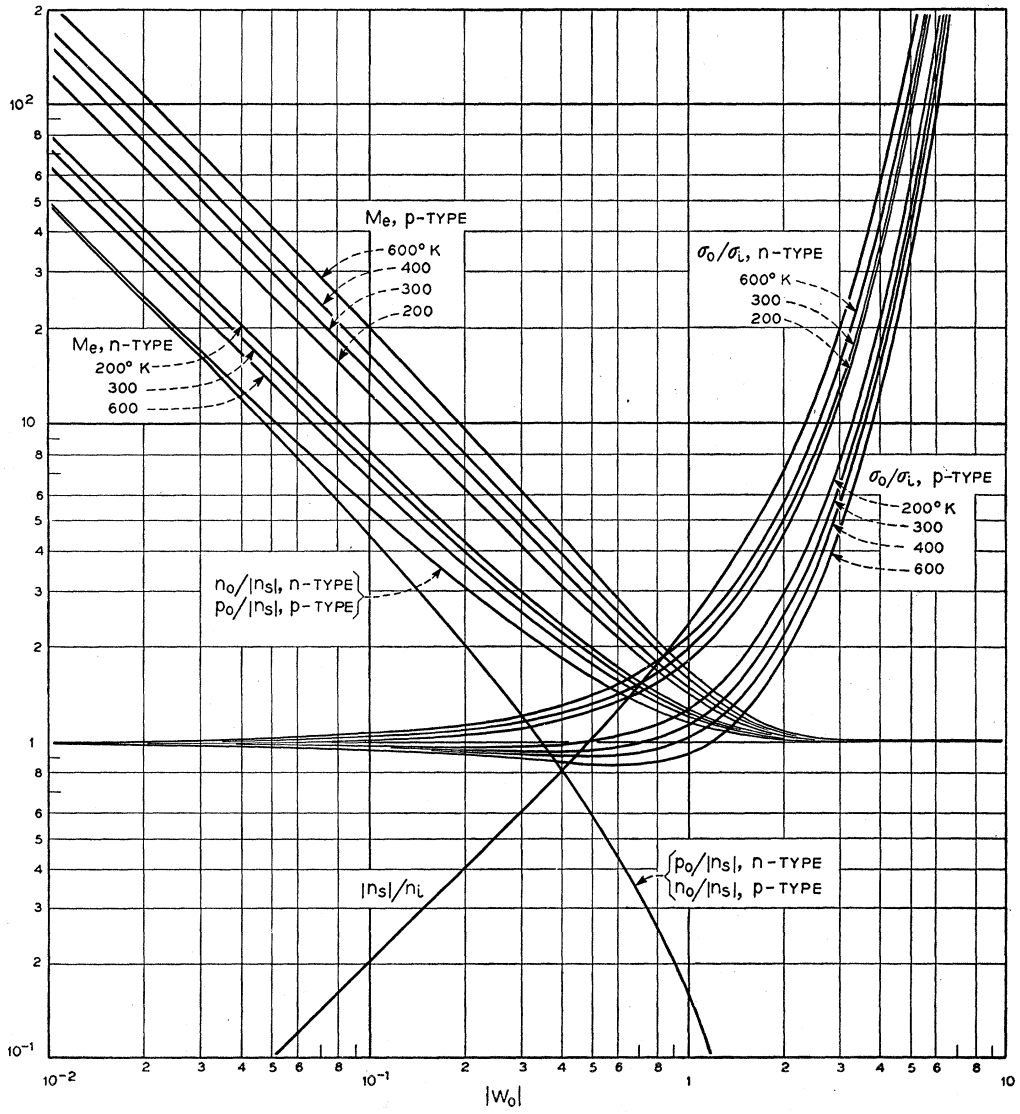

FIG. 2. The dependence for germanium of M_e, σ_0/σ_i, $|n_s|/n_i$, $p_0/|n_s|$, and $n_0/|n_s|$ on W_0.

[31] For the general case, the choice of L as length unit does not provide quite the formal concision of the choice of reference 1, Sec. 2.3.

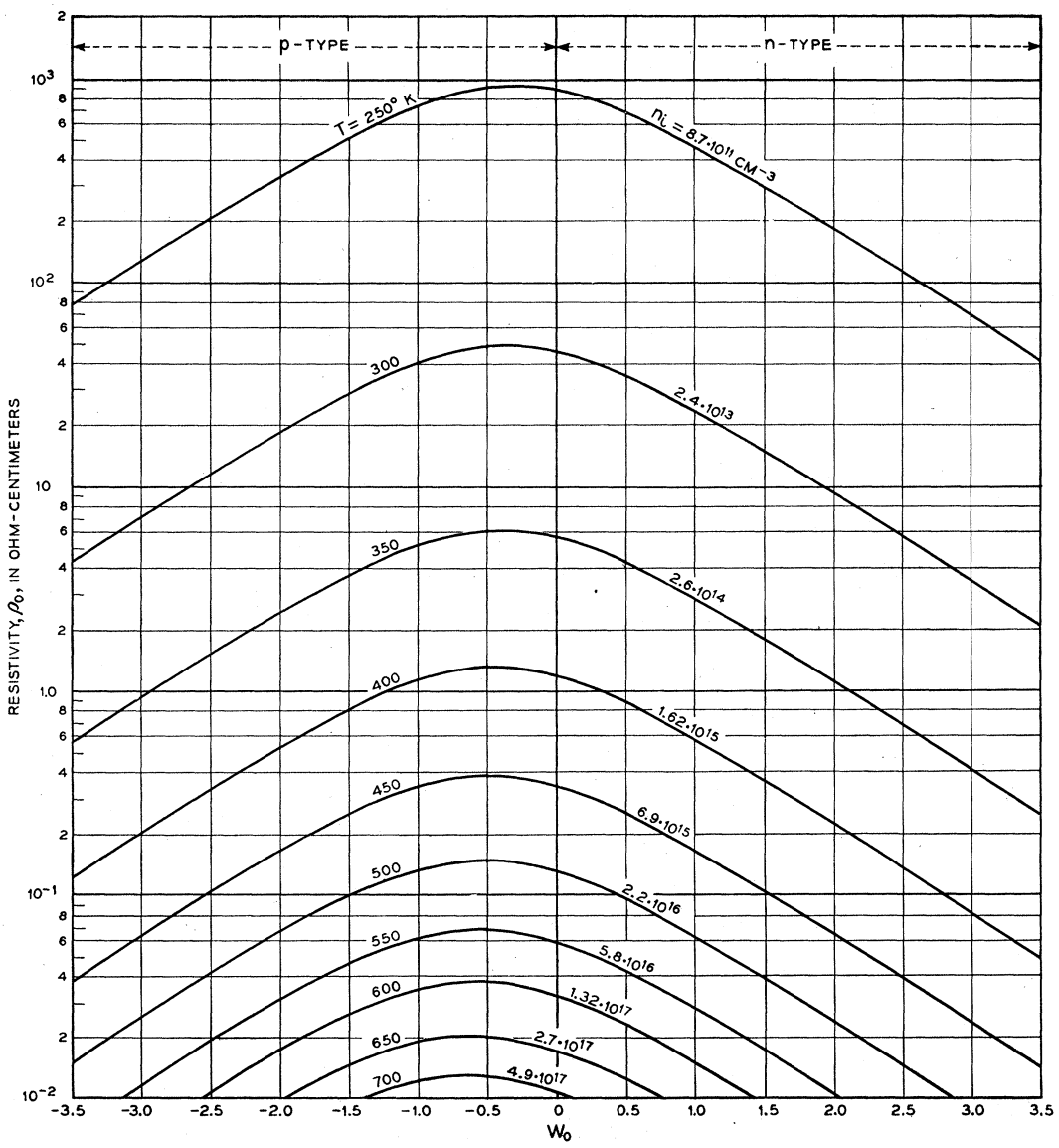

FIG. 3. The dependence of resistivity ρ_0 on W_0 for germanium at various temperatures.

statistics,

$$n_0 = n_i \exp(W_0), \quad p_0 = n_i \exp(-W_0);$$
$$W_0 = \tfrac{1}{2} \ln(n_0/p_0) = \sinh^{-1}(n_s/2n_i); \tag{39}$$

and the dimensionless quantity M_0 may be written as

$$M_0 = [(b+1)\coth W_0 + (b-1)]/2b. \tag{40}$$

While M_0 applies, of course, to either conductivity type, having the sign of W_0, it is usually more convenient to restrict its use to n-type and to define an analogous positive quantity for p-type. The alternative quantity, written for both types, is

$$M_e \equiv \sigma_0/\sigma_e = -\begin{cases} \mu_p/|\mu_0^*| = \mu_p/\mu_0^* = M_0, & n\text{-type}, \\ \mu_n/|\mu_0^*| = -\mu_n/\mu_0^* = -bM_0, & p\text{-type}, \end{cases} \tag{41}$$

where σ_e is the conductivity associated with the majority carrier excess:

$$\sigma_e \equiv -\begin{cases} e\mu_n(n_0 - p_0) = e\mu_n n_s = \sigma_s, & n\text{-type}, \\ e\mu_p(p_0 - n_0) = -e\mu_p n_s, & p\text{-type}. \end{cases} \tag{42}$$

Note that M_e is substantially unity for a strongly extrinsic semiconductor, and exceeds unity in general.

In Fig. 2, M_e as well as the quantities,

$$\begin{aligned} &\sigma_0/\sigma_i = [b\exp(W_0) + \exp(-W_0)]/(b+1), \\ &|n_s|/n_i = 2\sinh|W_0|, \\ &p_0/|n_s| = \tfrac{1}{2}\exp(-W_0)/\sinh|W_0|, \\ &n_0/|n_s| = \tfrac{1}{2}\exp(W_0)/\sinh|W_0|, \end{aligned} \tag{43}$$

are plotted for germanium against $|W_0|$. Figure 3

relates W_0 to resistivity and temperature, resistivity $\rho_0=\sigma_0^{-1}$ being plotted against W_0 for values of absolute temperature T as parameter from the first of Eqs. (43). From these figures, M_e and other quantities may be found in terms of ρ_0 and T.

The mobilities employed for germanium and the corresponding temperature dependence of n_i are given by[32]

$$\mu_n=3800(300/T)^{1.66}\ \text{cm}^2\ \text{volt}^{-1}\ \text{sec}^{-1},$$
$$\mu_p=1820(300/T)^{2.33}\ \text{cm}^2\ \text{volt}^{-1}\ \text{sec}^{-1}, \quad (44)$$

and

$$n_i^2=n_0p_0=3.1\times10^{32}T^3\exp(-9100/T)\ \text{cm}^{-6}. \quad (45)$$

These empirical relationships have been obtained from conductivity measurements.[33] The temperature dependence of resistivity is associated principally with that of n_i, and to a lesser extent with the temperature dependence of the mobilities.

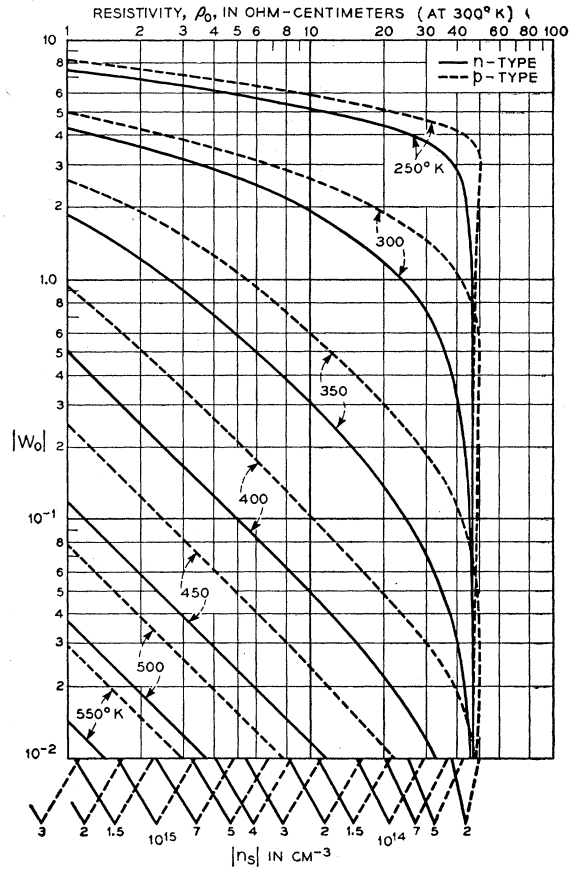

FIG. 4. The dependence of W_0 on the resistivity at 300°K or on $|n_s|$ for germanium at various temperatures.

[32] F. J. Morin, private communication.

[33] The coefficients in (44), for which the resistivity at 300°K is to exceed about 1 ohm-cm, are ones given by drift mobility measurements of Prince, reference 10; see reference 9. These measurements have essentially confirmed the differing exponents in (44). The intrinsic resistivity ρ_i at 300°K of 47 ohm-cm remains unchanged. See reference 11, Chap. 12, Sec. 8; E. M. Conwell, Proc. Inst. Radio Engrs. **40**, 1327 (1952).

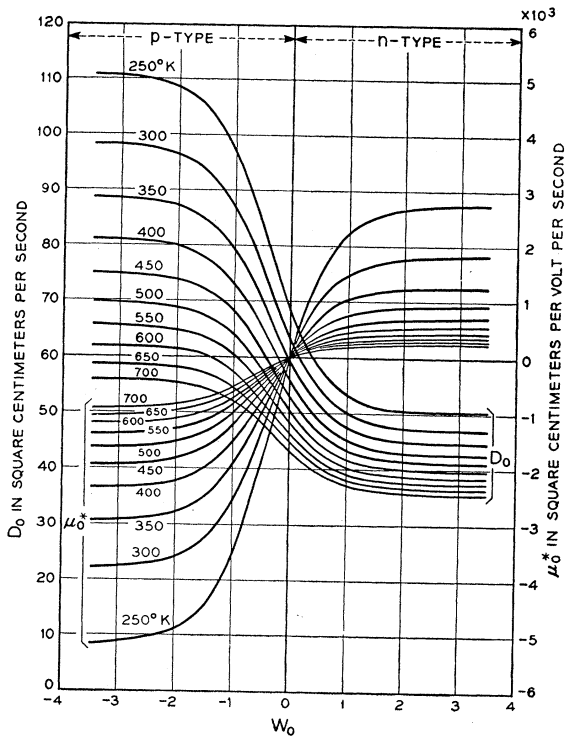

FIG. 5. The dependence of D_0 and μ_0^* on W_0 for germanium at various temperatures.

For some purposes such as the estimation of the effects of joule heating, it is desirable to determine M_e in terms of T and the ρ_0 at some fixed temperature. In Fig. 4, $|W_0|$ is plotted against ρ_0 at 300°K or the corresponding $|n_s|$ for values of T as parameter, and a W_0 from this figure may be used to furnish M_e from Fig. 2 (or ρ_0 at temperature T from Fig. 3).[34] The curves of Fig. 4 are those of the third equation of (39), with n_s fixed in terms of ρ_0 at 300°K according to

$$n_s=(1/e\mu_nM_e\rho_0)\,|_{T=300°\text{K}}, \quad n\text{-type},$$
$$-n_s=(1/e\mu_pM_e\rho_0)\,|_{T=300°\text{K}}, \quad p\text{-type}. \quad (46)$$

The values of M_e in these equations may be found from Fig. 2 and the curve of Fig. 3 for 300°K. They may also be calculated from

$$M_e=-\begin{cases}[\{(b-1)^2+4b\lambda\}^{\frac{1}{2}}+(b-1)]/2b\lambda, & n\text{-type},\\ [\pm\{(b-1)^2+4b\lambda\}^{\frac{1}{2}}-(b-1)]/2\lambda, & p\text{-type},\end{cases} \quad (47)$$

where

$$\lambda\equiv1-(\rho_0/\rho_i)^2, \quad (48)$$

obtained by eliminating W_0 from (40). The double sign before the radical in the expression for M_e for p-type arises from the circumstance that, with $b>1$, ρ_0 has a maximum on the p-type side, as Fig. 3 shows. From

[34] The figures show, and it is easily verified analytically, that M_e is the same for both conductivity types if semiconductor material of a given initial resistivity for which $M_e\sim1$ is heated so that $|n_s|/n_i\ll1$.

(43), this maximum ρ_0 equals $(b+1)\rho_i/2b^{\frac{1}{2}}$, for which[35] the radicand in (47) vanishes; and the positive and negative signs, corresponding to the sign of λ, are to be taken in (47) for W_0 respectively to the extrinsic and near-intrinsic sides of this maximum in ρ_0.

The ambipolar diffusivity D_0 and group mobility $\mu_0{}^*$ may be written as

$$D_0=2\cosh W_0/[D_p{}^{-1}\exp(W_0)+D_n{}^{-1}\exp(-W_0)], \quad (49)$$

and

$$\mu_0{}^*=2\sinh W_0/[\mu_p{}^{-1}\exp(W_0)+\mu_n{}^{-1}\exp(-W_0)]. \quad (50)$$

[35] The corresponding W_0 is $-\ln b^{\frac{1}{2}}$, for which M_e is $2b/(b-1)$, or, for germanium, about 3.8, with $\rho_0=50$ ohm-cm at 300°K.

In Fig. 5, D_0 and $\mu_0{}^*$ are plotted against W_0 for germanium at various temperatures. The quantity $D_0{}^{-\frac{1}{2}}\mu_0{}^*$ multiplied by $\tau_0{}^{\frac{1}{2}}$ gives the reciprocal of the scalar field intensity $v_D/\mu_0{}^*$, the unitary field for the dimensionless equation (36).

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The Energy Loss of Hydrogen, Helium, Nitrogen, and Neon Ions in Gases*

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The rate of loss of energy of protons, deuterons, helium, nitrogen, and neon ions in the energy range of 150 to 450 kev has been measured in the gases hydrogen, helium, air, and argon. The ions were sent through a differentially pumped gas system and the energy loss in the forward direction due to the gas was determined with an electrostatic analyzer.

The results for protons agree with recent measurements at the California Institute of Technology. At the same energy, the stopping cross sections are roughly the same for neon and helium ions. The stopping power for nitrogen ions is greater than that for neon ions of the same energy by a factor ranging from 1.3 to 1.9, illustrating the importance of external electron configurations in determining stopping powers in our energy region. With the exception of hydrogen gas, the cross sections for the heavier ions follow a power law. The dependence ranges from $E^{0.33}$ to $E^{0.59}$, depending on the gas and ion, with several of the curves following an $E^{\frac{1}{2}}$ power law.

I. INTRODUCTION

FOR many experiments in nuclear physics it is important to know the rate of energy loss of charged particles in matter. Recently Taylor[1] has reviewed the field of energy loss and range energy relations. Since this review, Kahn[2] of this laboratory has measured the energy loss of protons from 500–1300 kev in various metals and mica.

The energy loss of protons in gases has recently been measured in several laboratories. At Los Alamos, Phillips[3] has measured the energy loss of protons from 10–80 kev in H_2, He, N_2, O_2, A, Kr, H_2O, and CCl_4. At the California Institute of Technology, Reynolds, Dunbar, Wenzel, and Whaling[4] have investigated the energy loss of protons from 25–550 kev in the gases H_2, He, N_2, O_2, air, A, Ne, CH_4, C_2H_2, C_2H_4, C_2H_6, H_2O, NH_3, NO, CO_2, and N_2O.

In the present investigation the energy loss of protons from 40–450 kev in air and argon was measured. The fact that a differentially pumped gas chamber was used made it possible to measure the energy loss of heavier ions. The stopping power of H_2, He, air, and A was investigated in the energy range from 150–450 kev for helium, nitrogen, and neon ions.

II. APPARATUS

A. The Source of Particles

The source of the particles was the "kevatron" (500-kev Cockcroft-Walton generator) at the Institute for Nuclear Studies, University of Chicago. The beam of particles was focused by allowing it to impinge on a quartz plate and observing the fluorescence and incandescence produced. (See Fig. 1.) Next, the beam underwent a 15-degree magnetic analysis in order to separate out the ionic component desired. The arc source was of the low voltage capillary type described elsewhere.[5] By changing the gas admitted to the arc, singly charged ions of hydrogen, deuterium, helium, nitrogen, and neon were obtained. The ion stream also contained

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