λ. The ratio of these two terms is

$$f=-5.7/\lambda a_0,$$

in which $a_0=0.529\times10^{-8}$ cm. The corresponding ratio for the ns contribution is the same.

The fractional decrease in the hfs of He^3 caused by the nuclear structure will be $(\mu_n/\mu_{He^3})f$. If the parameter λ is chosen so that the binding energy difference of 0.764 Mev between H^3 and He^3 is attributed to the electrostatic energy between the two protons in He^3, it is found that $\lambda=0.74\times10^{13}$ cm^{-1}. If the parameters of an exponential potential well are chosen to fit the two nucleon data and λ is chosen to give the highest binding energy for H^3, it is found that $\lambda=0.93\times10^{13}$ cm^{-1}. This value of λ yields a binding energy of 9.79 Mev for H^3 compared with the experimental value of 8.492 Mev.[4] Hence the uncertainty of the nuclear wave function limits the accuracy of this calculation.[7] For $\lambda=0.74\times10^{13}$ cm^{-1}, the fractional decrease in the He^3 hfs will be -1.3×10^{-4}, with an uncertainty of ±10 percent arising from the approximate integration.

This calculation has neglected nuclear motion effects,[3] admixture of D state in the nuclear wave function,[7] and exchange magnetic moments.[8]

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[1] G. Weinreich and V. Hughes, Phys. Rev. **90**, 377 (1953). See also the preceding letter [Phys. Rev. **91**, 195 (1953)].
[2] A. Bohr, Phys. Rev. **73**, 1109 (1948).
[3] F. Low, Phys. Rev. **77**, 361 (1950).
[4] J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).
[5] Fred, Tomkins, Brody, and Hamermesh, Phys. Rev. **82**, 406 (1951).
[6] W. Perl and V. Hughes, Phys. Rev. (to be published).
[7] R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952).
[8] E. N. Adams, II, Phys. Rev. **81**, 1 (1951).

Angular Correlation in Allowed β-Transitions

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THE purpose of the following is to examine the problem of deducing information concerning the β-interaction from β-recoil angular correlation experiments. We restrict our attention to allowed transitions. Then, in terms of well-established notation, the probability for emission of the β with energy W per dW and with angle ϑ between the β and neutrino, per $\sin\vartheta d\vartheta$, is

$$P(W)=\frac{g^2}{4\pi^3}pW(W_0-W)^2F(Z,W)|M|^2\left(1+\frac{ap}{W}\mu+\frac{b}{W}\right), \quad (1)$$

where $\mu=\cos\vartheta$, $|M|^2$ is the usual square matrix element weighted with the squares of the relative interaction coupling constants, and the constants b and a are defined elsewhere.[1]

Of course, the correlation in Θ, the angle between the β-particle and recoil nucleus, is the pertinent consideration. For β-particles of fixed energy this is

$$Q'(\Theta)=KP_0(W)J(\Theta,W)[1+af(\Theta,W)+b/W], \quad (2)$$

where $P_0=pW(W_0-W)^2F(Z,W)$ and K is a constant proportional to $g^2|M|^2$. In Eq. (2), $J(\Theta,W)$ is the Jacobian of the $\vartheta\to\Theta$ transformation and $f(\Theta,W)$ is $\mu p/W$ written in terms of the indicated variables.[2]

$$J(\Theta,W)=(1-\lambda^2\sin^2\Theta)^{-\frac{1}{2}}[-\epsilon\lambda\cos\Theta+(1-\lambda^2\sin^2\Theta)^{\frac{1}{2}}]^2, \quad (2a)$$

$$f(\Theta,W)=-(p/W)[\lambda\sin^2\Theta+\epsilon\cos\Theta(1-\lambda^2\sin^2\Theta)^{\frac{1}{2}}]. \quad (2b)$$

Here $\lambda=p/q$ and $\epsilon=1$ for $\lambda<1$ while for $\lambda>1$, $\epsilon=\pm1$ according as $\vartheta\lesssim\vartheta_0=\arccos(-1/\lambda)$; in the subsequent integration over W the contributions of both $\epsilon=\pm1$ must be added in the range $p>q$, $W>(W_0^2+1)/2W_0$.

If one envisages an experiment in which only a very narrow band of β-energy is accepted, so that Eq. (2) applies, one cannot determine all the constants separately. Only Ka and $K(1+b/W)$ enter. If one considers pure Gamow-Teller transitions ($\Delta J=1$, He^6 for example) or pure Fermi transitions (say O^{14}), this implies that the ratio of coupling constants (C_A/C_T in the G-T case) is a double-valued function of a, and it would appear necessary to fix b within narrow limits to resolve the ambiguity.[3] However, this ambiguity is not at all present under the conditions of the experiment as it would presumably be carried out. Accepting a wide band of β-energies, one measures

$$Q(\Theta)=\int Q'(\Theta)dW. \quad (3)$$

This is of the form

$$Q(\Theta)=c_1A_1(\Theta)+c_2A_2(\Theta)+c_3A_3(\Theta),$$

where $c_1=K$, $c_2=Kb$, $c_3=Ka$, and the three functions A_i are *linearly independent* and well-defined. They would be obtained by numerical integration over the accepted band of W, except that in no case will energies greater than W_m contribute,

$$W_m=\sec^2\Theta[W_0-\sin\Theta(W_0^2-\cos^2\Theta)^{\frac{1}{2}}]. \quad (4)$$

The constants c_i are fixed from the data, by least squares for example, and then $a=c_3/c_1$. However, for pure G-T transitions a least-squares analysis should take into account the fact that the constants are not independent, but $c_1^2=c_2^2+9c_3^2$.

If one accepts the result of no Fierz interference[4] ($b=0$), the correlation experiment for a $\Delta J=0$ transition (other than $0\to0$) should give $-1<a<\frac{1}{3}$ for pure S and $\frac{1}{3}<a<1$ for pure V composition of the Fermi component. In addition, the value of a gives $C_F^2|\int1|^2/C_T^2|\int\sigma|^2$. If one does not assume $C_SC_V=0$, then the c_i are independent for $\Delta J=0$ transitions and a determination of $|C_F\int1/C_T\int\sigma|$ (for both $C_F=C_S$ and C_V) is possible in principle.

[1] S. R. de Groot and H. A. Tolhoek, Physica **16**, 456 (1950). Of course, the pseudoscalar term appearing in this reference makes no contribution here.
[2] These results appear in Reynolds, Biedenharn, and Beard, Oak Ridge National Laboratory Report ORNL-1444 (unpublished). Misprints occurring therein have here been corrected. As these authors point out, the singularity of the Jacobian is due to neglect of the finite mass of the recoil nucleus but, upon subsequent integration over W, this neglect introduces negligible error.
[3] O. Kofoed-Hansen and A. Winther, Phys. Rev. **89**, 526 (1953).
[4] J. Davidson and D. C. Peaslee (private communication). In this case $C_A=0$; see B. M. Rustad and S. L. Ruby, Phys. Rev. **89**, 880 (1953).

The Absolute Scintillation Efficiency of Anthracene

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MEASUREMENTS have been made of the absolute scintillation efficiency of single anthracene crystals at room temperature. The experimental method included the following observations:

(a) The transmission coefficient of a spectrophotelometer, of constant band width $d\lambda$, as a function of the wavelength λ.

(b) The relative spectral response of an EMI 6262 photomultiplier, using a standard lamp of known color temperature and the calibrated spectrophotelometer.

(c) Reduction of the intensity of the light from the standard lamp by a measured factor, using a diffuse reflector, the inverse square law, and calibrated filters.

(d) The absolute spectral sensitivity of the photomultiplier, using the reduced light source of known intensity and spectral distribution.

(e) The relative intensity distribution of the fluorescence transmission spectrum of single anthracene crystals, using the calibrated spectrophotelometer and photomultiplier.

(f) The flux and energy of a collimated beam of α-particles from a Po^{210} source, using standard scintillation counting techniques.

(g) The absolute light emission from an anthracene crystal excited by this α-particle flux, using the calibrated photomultiplier.

It was found that the fluorescence transmission spectrum and scintillation efficiency were independent of crystal thickness d, for $d>1.5$ mm, and the final results were obtained from the mean of measurements on 4 clear, polished anthracene crystals, exceeding this thickness. The absolute scintillation efficiency C_{IF} (total energy of fluorescence emission/total energy of incident radiation) was measured for several α-particle energies, from 0.042 percent at 1.18 Mev to 0.223 percent at 3.85 Mev, corresponding to 0.324 percent at 5.30 Mev. The mean wavelength of the fluorescence emission was 469 mμ, corresponding to a mean photon energy $E_F=2.65$ ev.

Hopkins[1] has observed that the scintillation efficiency of anthracene for excitation by 5.3-Mev electrons is 11.6 ($\pm$0.2) times that for excitation by α-particles of the same energy. Hence we obtain $C_{IF}=3.76$ ($\pm$0.07) percent for thick anthracene crystals at room temperature, excited by fast electrons. This corresponds to an energy expenditure $E_{IF}=70.5$ ($\pm$3.8) ev/fluorescence photon.

This value may be compared with that derived from the "photon cascade" theory.[2] On this theory, the scintillation process consists of m molecular fluorescence emissions, of decay time $(t_f)_0$, and quantum efficiency q_0. The mean value of m is given by

$$m=(t_I)_T/(t_f)_0, \tag{1}$$

where $(t_I)_T$ is the technical scintillation decay time of a thick crystal. The scintillation quantum efficiency is

$$q_I=(q_0)^m, \tag{2}$$

the scintillation energy efficiency is

$$C_{IF}=E_F q_I/E_Z, \tag{3}$$

and the energy expenditure/emitted photon is

$$E_{IF}=E_Z/q_I. \tag{4}$$

E_Z is the energy expenditure/primary photon, which on the theory is equated to the energy expenditure/ion-pair, i.e., ~30 ev/primary photon.

For anthracene at room temperature, values of $(t_f)_0=3.5$ mμsec, $(t_I)_T=27$ mμ sec, and $q_0=0.9$ have been observed experimentally,[3] giving $m=7.73$ and $q_I=0.443$ from (1) and (2). Hence we obtain $E_{IF}=68$ ev/photon, in agreement with the direct experimental value.

A detailed account of this work will be published later. We wish to acknowledge the support received from the South African Council for Scientific and Industrial Research.

[1] J. I. Hopkins, Rev. Sci. Instr. **22**, 29 (1951).
[2] J. B. Birks, *Scintillation Counters* (Pergamon Press, London; McGraw-Hill Book Company, New York, 1953).
[3] J. B. Birks and W. A. Little, Proc. Phys. Soc. (London) (to be published).

Three-Body Scattering Problems

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IN a recent paper, Borowitz and Friedman[1] obtain an expression for the exchange scattered amplitude in the Born approximation which differs from that given by Mott and Massey.[2] They question the procedure of expanding the entire solution in terms of a complete set of eigenfunctions of a Hamiltonian apparently because the coefficients of such an expansion must contain δ-functions, and they give a detailed proof of this latter assertion. They state further that in the case of, for example, the ionization of atoms by electron impact, Mott and Massey's result diverges.

Firstly we point out that if we make the same assumptions as Borowitz and Friedman, it is obvious that such an expansion will contain δ-functions. Using their notation, it is consistent with their assumptions to choose $V(r)=0$, and since

$$\sin k_0 r=\int dk\delta(k-k_0)\sin kr$$

the statement is immediately proved. However, their assumptions are not comparable to those of Mott and Massey who use essentially

$$r\phi_k(r)\sim\sin(kr+k^{-1}\log 2kr+\eta),$$

and not

$$r\phi_k(r)\sim\sin(kr'+\eta).$$

Since it is not clear in what way the presence of singularities such as δ-functions in the expansion coefficients constitutes an objection to the treatment, the point will not be pursued further.

It is of more importance to note that the result of Mott and Massey, correctly interpreted, does *not* diverge and is in fact on the same basis as that of Borowitz and Friedman, the difference between the two being, of course, the familiar post-prior discrepancy. This discrepancy may be shown to equal

$$\int e^{-i\mathbf{k}_n\cdot\mathbf{r}_2}\phi_0(\mathbf{r}_2)d\mathbf{r}_2\Big[\lim_{r_1\to\infty}\int_{-1}^{+1}\int_0^{2\pi}\{e^{i\mathbf{k}_0\cdot\mathbf{r}_1}\operatorname{grad}(\phi_n^*(\mathbf{r}_1))-\phi_n^*(\mathbf{r}_1)\operatorname{grad}(e^{i\mathbf{k}_0\cdot\mathbf{r}_1})\}r_1^2 d\mu d\Phi\Big], \tag{1}$$

and if n is discrete it thus vanishes as has been pointed out by Bates, Fundaminsky, and Massey.[3] To prove that this result is, as would be expected, generally true it need only be noted that if n is in the continuum, (1) must be replaced by

$$\frac{1}{\Delta k_n}\int_{k_n}^{k_n+\Delta k_n}dk_n\{\text{expression (1)}\},$$

where Δk_n is arbitrarily small but not identically zero (see Gordon[4]); and clearly this expression vanishes as

$$\sin(r\Delta k_n)/r\Delta k_n.$$

Finally it may be worth while to point out that the formula obtained by Borowitz and Friedman is the same as would be found by following the Mott and Massey procedure for direct scattering and using a properly symmetrized wave function throughout.

I wish to thank Professor D. R. Bates for many helpful comments.

[1] S. Borowitz and B. Friedman, Phys. Rev. **89**, 441 (1953).
[2] N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, Chap. VIII.
[3] Bates, Fundaminsky, and Massey, Trans. Roy. Soc. (London) **A243**, 93 (1950).
[4] W. Gordon, Z. Physik **48**, 180 (1928).

Polarization of the Three-Photon Annihilation Radiation*

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WE have used a triple coincidence counter method to investigate the polarization of the gamma-rays resulting from the three-photon annihilation of positrons and electrons.[1] Of the many polarization effects that one might expect, we have chosen to investigate the simplest from the experimental point of view, i.e., the polarization of any one of the gamma-rays relative to the plane of emission of the three photons. The measurement was conducted for the symmetrical case (equal angles beangles between the photons).