## Two-Particle Potential from the Bethe-Salpeter Equation

WILHELM MACKE
*Instituto de Fisica Teórica, Sao Paulo, Brazil*
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A NONRELATIVISTIC derivation of the two-particle potential between two fermions, due to interaction with a boson field, was outlined by Tamm[1] and Dancoff[2] and generalized by Lévy,[3] including higher-order terms in the interaction potential. Following Lévy's treatment, the interaction terms of higher order can be represented by nonrelativistic graphs, distinguishing in addition the sequence of transitions and intermediate states. In this treatment, it does not seem to be understandable why the strongly diverging vacuum graphs are not considered, although according to the derivation they should appear, because the influence of them on the energy denominators is such that they are not separable. Further it is impossible to recognize the other divergent parts of the graphs as terms of mass and charge renormalization. This is impossible because the relativistic invariance has been destroyed in the very beginning of the derivation of the one-time formalism. In these cases Lévy goes back to the Bethe-Salpeter formalism,[4] showing an approximate correspondence of the two formalisms in two special examples and presuming it for the whole.

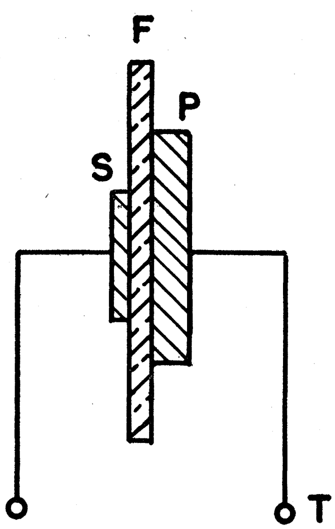


FIG. 1. Example of a Lévy graph with $n=2$ and $m=0$. The energy denominators are

$$\Delta E_1=E^{(1)}(\mathbf{p}+\mathbf{k}_1)+E^{(1)}(\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)+\omega(\mathbf{k}_2),$$
$$\Delta E_2=E^{(1)}(\mathbf{p})+E^{(1)}(\mathbf{p}+\mathbf{k}_1)+E^{(1)}(\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)+E^{(2)}(-\mathbf{p}-\mathbf{k}_2)-W,$$
$$\Delta E_3=E^{(1)}(\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)+E^{(2)}(-\mathbf{p}-\mathbf{k}_2)+\omega(\mathbf{k}_2)-W.$$

To clear up this situation, a rigorous treatment of the two-particle potential was carried through, starting with the relativistic Bethe-Salpeter equation[4] and passing on from this to the one-time formalism. We give the results briefly here, whereas explicit calculations and results will be outlined in detail in an another place.[5] The results correspond to those of Lévy in those special cases he took into consideration. For the difference coordinates in momentum space the wave equation

$$[W-(m_1^2+\mathbf{p}^2)^{\frac{1}{2}}-(m_2^2+\mathbf{p}^2)^{\frac{1}{2}}]a(\mathbf{p})=-Va(\mathbf{p}') \tag{1}$$

holds, where $W$ is the total energy of the system, $-V$ is the operator of the interaction potential, and $a(\mathbf{p})$ is a four-component wave function, containing the spins of both particles. $V$ is represented by the totality of all Feynman graphs with two fermion lines running from the left to the right (see Fig. 1), but without vacuum graphs. The right ends should not contain self-energy parts but run into the wave function $a(\mathbf{p}')$ of (1). The different graphs should be distinguished by the number of meson lines ($n$), the number of closed loops ($m$), and the different topologies $\tau$ which are possible with given $n$ and $m$. Further, we have to distinguish the order of the $2n$ points where the meson lines end. In this way, every Feynman graph ($nm\tau$) consists of $(2n)!$ ordered graphs, built up by all permutations ($\pi$) of the points without alteration of the topology. Our interaction operator consists of the sum over all these graphs,

$$V=\Sigma V_{nm\tau\pi}, \tag{2}$$

where for every term $V_{nm\tau\pi}$ there is an analytic representation of the general form

$$V_{nm\tau\pi}=(-1)^l\lambda^n\int\frac{d\mathbf{k}_1\cdots d\mathbf{k}_{n-m}d\mathbf{q}_1\cdots d\mathbf{q}_m}{2\omega_1\cdot 2\omega_2\cdots 2\omega_n}\cdot\frac{\Gamma_1\cdots\Gamma_{2n}}{\Delta E_1\cdots\Delta E_{2n-1}}. \tag{3}$$

The meaning of the terms of this formula is

$$\lambda=g^2(2\pi)^{-3},\quad \omega_\nu=(\mu^2+\mathbf{k}_\nu^2)^{\frac{1}{2}},\quad \Delta E=E_{\text{ex}}-W, \tag{4}$$

where $g$ is the coupling constant of the meson field, and $\mu$ is the mesonic mass. Every point of the graph means a transition to another virtual state; $E_{\text{ex}}$ is the energy of the virtual state and depends on the lines lying between the two points. If two fermion lines of such a virtual state are both in the initial state or in the final state, the energy of both is $W$ (see Fig. 1). All other fermion lines give $E(\mathbf{p})=(m_{1,2}^2+\mathbf{p}^2)^{\frac{1}{2}}$, and the meson lines $\omega(k)=(\mu^2+\mathbf{k}^2)^{\frac{1}{2}}$. Graphs that contain a virtual state with only two fermion lines are to be cut out. The connection between the variables $\mathbf{p}_\nu$ and $\mathbf{k}_\nu$ is given by the graph in the usual way. The $\Gamma$'s have the same meaning as in Lévy's paper.[3] The lines running backwards are related to negative energy states, the lines with normal direction are related to positive energy states. $l$ in (3) is the number of lines running backwards.

If a graph contains no closed loop ($m=0$), the integrations go over all meson variables $d\mathbf{k}_\nu$. If it contains one closed loop ($m=1$), one variable $\mathbf{k}_\nu$ of the adjacent meson lines can be eliminated, because $\Sigma\mathbf{k}_\nu=0$ holds for those lines. But here the integration goes over the momenta $d\mathbf{q}$ of the particle in the closed loop. All other prescriptions remain unchanged. If the interaction takes place between different particles, every closed loop appears twice, once for $m_1$ and once for $m_2$.

By these results an exact description of the interaction operator $V$ is given, being valid for all graphs and orders in the coupling constant. It gives the Tamm-Dancoff equation with the following exceptions: first, it does not contain the vacuum graphs, and second, the excitation energies $E_{\text{ex}}$ differ in the case in which the two fermion lines are in the initial or final state. Further, this derivation gives us a direct possibility to carry through the renormalization of the remaining divergent parts; a paper on this matter is in preparation.

[1] J. Tamm, J. phys. (U.S.S.R.) **9**, 449 (1945).
[2] S. M. Dancoff, Phys. Rev. **78**, 382 (1950).
[3] M. Lévy, Phys. Rev. **88**, 72 (1952).
[4] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951).
[5] W. Macke, Z. Naturforsch. (to be published).

## Hyperfine Structure of the Metastable Triplet State Helium-3†

GABRIEL WEINREICH* AND GERARD M. GROSOF,
*Columbia University, New York, New York*
AND
VERNON W. HUGHES, *University of Pennsylvania, Philadelphia, Pennsylvania*
(Received May 18, 1953)

AN experimental determination has been made of the hyperfine structure of the $^3S_1$ metastable state of helium-3. The atomic beam magnetic resonance method was used in a manner similar to that used in experiments on the magnetic moment of the same state in helium-4.[1]