value ($n=0$, or $\lambda=0$),

$$E_n=\tfrac{3}{2}(\Sigma\,\zeta^{\frac{1}{2}}-\Sigma_k\,\omega_k).$$

The factor 3 takes account of the three charge states of the π meson in a charge-symmetric theory. As has been shown in I, E_n may be written as a contour integral,

$$E_n=-3(8\pi i)^{-1}\int d\zeta\zeta^{-\frac{1}{2}}\ln\varphi_n(\zeta),$$

where the path of integration in the ζ plane encircles all roots ζ (and all ω_k^2). [The identity of the determinant $\varphi_n(\zeta)$ with the function called $\varphi(\zeta)$ in I, Eqs. (2) and (3), is easily proved: $\varphi_n\Delta_0=\Delta$.] If all nucleons are removed to infinite distances, E_n reduces to nE_1, i.e., the self-energy of the n isolated nucleons. Subtracting this, we get the potential energy proper,

$$U_n=E_n-nE_1=-3(8\pi i)^{-1}\int d\zeta\zeta^{-\frac{1}{2}}\ln[\varphi_n(\zeta)\varphi_1^{-n}(\zeta)]. \quad (2)$$

Note that, according to (1), $\varphi_n\varphi_1^{-n}$ is an $n\times n$ determinant $\|\beta_{st}\|$ with

$$\beta_{ss}=1;\ \beta_{st}=\beta_{ts}=-\lambda\varphi_1^{-1}(\zeta)(2\pi)^{-3}\times\int d^3\mathbf{k}(\zeta-\omega_k^2)^{-1}|u_k|^2e^{i\mathbf{k}\cdot(\mathbf{x}_s-\mathbf{x}_t)}\quad(s\neq t) \quad (3)$$

(in the limit $V\rightarrow\infty$).

As long as all nucleon distances are large compared with the "nucleon radius" a (the cut-off momentum $A\sim a^{-1}$ is assumed $\gg\mu$), three approximations can be made (see I for details):

(a) In the **k**-space integral in (3), $|u_k|^2$ can be replaced by 1.

(b) The factor $\varphi_1^{-1}(\zeta)$ in (3) can be replaced by its value at the point $\zeta=\mu^2$. The error thereby introduced in the contour integral (2) is negligible. The energy U_n then depends only on the "effective coupling strength,"

$$\lambda_A=\lambda\varphi_1^{-1}(\mu^2), \quad (4)$$

which, actually, is smaller than λ and tends to zero in the limit $a\rightarrow0$, or $A\rightarrow\infty$.

(c) The off-diagonal elements of the matrix β are $\ll1$ in magnitude, and the logarithm of the determinant in (2) can be expanded into powers of β_{st} (or λ_A).

Take, as an example, $n=3$,

$$U_3=-3(8\pi i)^{-1}\int d\zeta\zeta^{-\frac{1}{2}}\ln[1-(\beta_{12}^2+\beta_{13}^2+\beta_{23}^2)+2\beta_{12}\beta_{23}\beta_{31}].$$

Neglecting terms $\sim\lambda_A^4$, one sees that U_3 comprises, as leading terms, the two-body potentials of the 3 pairs (1,2), (1,3), (2,3), and, next, the three-body potential

$$W_3=-3(4\pi i)^{-1}\int d\zeta\zeta^{-\frac{1}{2}}\beta_{12}\beta_{23}\beta_{31}. \quad (5)$$

With the approximations (a) and (b), the $\mathbf{k}$ and ζ integrals are easily evaluated,

$$W_3=3(64\pi^4)^{-1}\mu K_1(\mu[r_{12}+r_{23}+r_{31}])\lambda_A^3(r_{12}r_{23}r_{31})^{-1}. \quad (6)$$

Similarly, for $n=4$, one finds the sum of the 6 pair potentials and of the 4 triplet potentials, supplemented by various terms $\sim\lambda_A^4$. Among these one recognizes the 4-body potential

$$W_4=3(4\pi i)^{-1}\int d\zeta\zeta^{-\frac{1}{2}}[\beta_{12}\beta_{23}\beta_{34}\beta_{41}+\beta_{13}\beta_{34}\beta_{42}\beta_{21}+\beta_{14}\beta_{42}\beta_{23}\beta_{31}]. \quad (7)$$

In addition, there are terms involving products like β_{12}^4 and $\beta_{12}^2\beta_{13}^2$ which represent corrections to the two- and three-body potentials.[3] (Terms like $\beta_{12}^2\beta_{34}^2$ cancel.) Note that a product $\Pi_i\,\beta_{s_it_i}$ in the expansion of the logarithm in (2) gives rise to the potential

$$3(2\pi)^{-1}\mu K_1(\mu\,\Sigma_i\,r_{s_it_i})\,\Pi_i(\lambda_A/4\pi r_{s_it_i}). \quad (8)$$

It is then a mere combinatorial problem to write down the general n-nucleon potential; the answer need not be restated here [see I, Eqs. (22), (23); also Drell and Huang, reference 1, Eq. (7)[4]]. But a final remark concerning volume integrals may be of interest. Using the definition (3) and the approximations (a), (b), one finds immediately

$$\int d^3\mathbf{x}_3\beta_{13}\beta_{23}=\lambda_A\partial\beta_{12}/\partial\zeta=-\lambda_A\partial\beta_{12}/\partial(\mu^2).$$

[The regions where $r_{13}\lesssim a$, or $r_{23}\lesssim a$, do not contribute appreciably to this integral.] Hence

$$\int d^3\mathbf{x}_3W_3=\lambda_A\partial W_2(\mathbf{x}_1,\mathbf{x}_2)/\partial(\mu^2),$$
$$\int d^3\mathbf{x}_4W_4=\lambda_A\partial W_3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)/\partial(\mu^2), \quad (9)$$

and so forth. After $n-1$ volume integrations (and ζ integration by Cauchy's theorem)

$$\int d^3\mathbf{x}_2\cdots\int d^3\mathbf{x}_nW_n=(-1)^{n-1}\tfrac{3}{4}[\tfrac{1}{2}\times\tfrac{3}{2}\times\cdots\times(n-\tfrac{3}{2})]\times\lambda_A^n(2\pi)^{-3}\int d^3\mathbf{k}\omega_k^{-2n+1}\quad(n\geqslant3). \quad (10)$$

These formulas prove useful in computations of nuclear energies, including surface energies resulting from many-body forces.

[1] G. Wentzel, Helv. Phys. Acta **25**, 569 (1952); S. D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953).

[2] G. Wentzel, Helv. Phys. Acta **15**, 111 (1942), to be quoted as I.

[3] These terms also appear in the energy of a nucleon lattice, as calculated in I, Eq. (22); there they are formally listed among the 4-body terms. The two-body term (from β_{12}^4) is listed by A. Klein, Phys. Rev. **90**, 1101 (1953), Eq. (48). The three-body terms (like $\beta_{12}^2\beta_{13}^2$), which have the effect of weakening the repulsive potential W_3, may well be equally important.

[4] Drell and Huang use the symbol λ for the pure number $\lambda\mu/8\pi$ (our notation). They ignore higher-order effects, in particular those which reduce λ to λ_A.

The Energy Spectrum of Particles from Stars*

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The range distribution of particles from stars was observed in Ilford G5 plates, 200 microns thick, exposed to a high-energy neutron beam (about 90 Mev) by methods already described.[1]

In previous work, the range distributions have been inferred from the distributions of the projected range. In our case the actual range for each prong was determined. The critical factor entering this determination is the shrinkage factor. Several plates of the same batch were exposed to a well-collimated alpha beam of thorium C' particles entering the emulsion at a predetermined angle of 45°. Measurements were also carried out with extremely thin x-ray beams which penetrated through the whole emulsion and gave information as to the uniformity of the shrinkage. The shrinkage factor was then determined by the change in inclination. Changes in the shrinkage factor due to the aging of the plates and varying humidity conditions of the scanning room were also

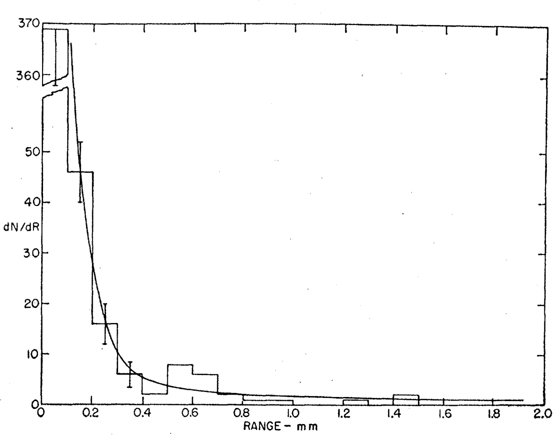

Fig. 1. Plate 11 G5 at 0°. Range histogram of all prong events (0–2 mm).

checked. A reasonable estimate of the variation of the shrinkage factor attributable mainly to changes in humidity was taken as 20 percent which is probably an overestimate. Thus the percentage error involved in the range determination δR, assuming the shrinkage factor as the predominant source of error, can be determined from the following equation:

$$\delta R/R=(d^2S^2/R)\cdot(\delta S/S),$$

where d is the vertical component of the track, S is the shrinkage factor, and $\delta S/S$ is 20 percent. This results, for instance, in an uncertainty of roughly 20 percent at the peak of the range distribution.

Figure 1 shows the range spectrum of the particles from all types of stars (1-prong to 5-prong events; total number of stars equal to 400) from the plate exposed at 0°.[1,2]

The detailed spectrum between 5 and 200 microns is given in Fig. 2 (particles with range under 5 microns were not included).

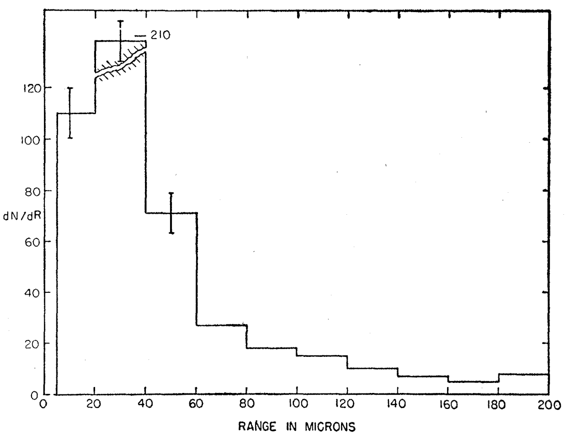

FIG. 2. Plate 11 G5 at 0°. Range histogram of all prong events (0–200 microns).

The spectra have been corrected for escape in the conventional manner.[3]

In recently developed plates in which the shrinkage was eliminated with various techniques, the shape of the spectrum was essentially unaffected. In Figs. 1 and 2 no discrimination is introduced between the different types of particles. We carried out some additional work in the discrimination of protons from alpha particles (deuterons and tritons could not be discriminated from protons). The peak of the range distribution of the alpha particles was found to coincide approximately with the one of the proton distribution. The alpha particles amounted to about 12 percent of the total number of prongs. The peak would correspond in the case of the protons to an energy of about 2.5 Mev and in the case of the alpha particles to about 6 Mev. These values are definitely much lower than the average Coulomb barrier of the heavy nuclei component of the emulsion (approximately 8 Mev for protons and 16 Mev for alphas); however, they would correspond to the average Coulomb barrier of the light nuclei component. Assuming no unusual behavior of the nucleus, one would surmise that the majority of our events originate in the light nuclei of the emulsion. From consideration of the composition and the geometrical cross section alone, about 70 percent of the events should be ascribed to the heavy nuclei. Blau's measurement at 300 Mev favors the heavy nuclei even more (82 percent heavy nuclei and 18 percent light nuclei for stars with two or more prongs).[4]

It is not feasible to distinguish between stars originating in light and heavy nuclei, unless one uses special techniques. Experiments along this line are now under way. One can, however, set a lower limit to the number of stars originating in light nuclei by selecting stars with at least one prong having an energy below the Coulomb barrier of the heavy nuclei. In carrying out this analysis we found that for 1- and 2-prong stars at least 80 percent of the stars are attributable to light nuclei, whereas for 3-prong stars the fraction increases to about 95 percent. The 4- and 5-prong stars are almost completely the result of light nuclei. More conclusive results are expected upon completion of our experiments with wire-embedded emulsions.

The range histogram has also been plotted for different spatial angles (Fig. 3). There was no appreciable variation in the position

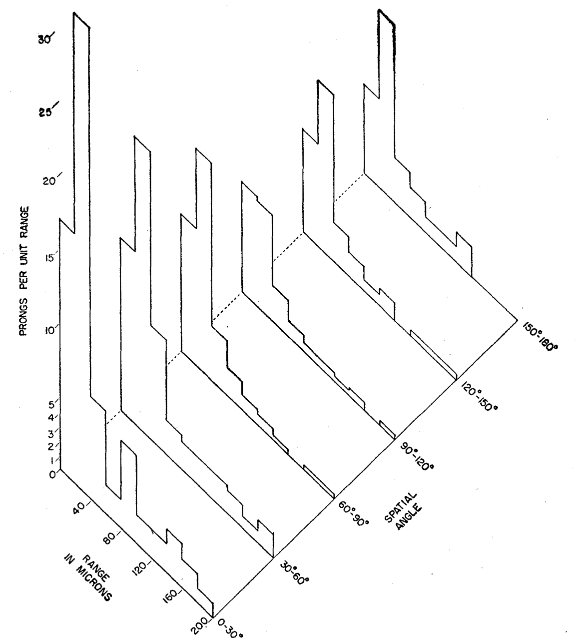

FIG. 3. Plate 11 G5 at 0°. Range histogram at different spatial angles (30° intervals) from all prong events.

of the peak of the range spectrum at different angles. However, the intensity of the peak varies in such a fashion as to exhibit a forward asymmetry.

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1 R. W. Waniek and T. Ohtsuka, Phys. Rev. **89**, 882 (1953).
2 R. W. Waniek and T. Ohtsuka, Phys. Rev. **89**, 1307 (1953).
3 F. C. Champion and C. F. Powell, Proc. Roy. Soc. (London) **A183**, 64 (1944).
4 M. Blau and A. R. Oliver, Phys. Rev. **87**, 182 (1952).

The Reaction $Li^6(\alpha,\gamma)B^{10}$

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We have investigated the reaction $Li^6(\alpha,\gamma)B^{10}$ with alpha particles of up to 1.5 Mev. Within the range of excitation in B^{10} so produced (4.45–5.35 Mev), states are known at 4.77, 5.11, and 5.16 Mev[1] from the reactions $Be^9(d,n)B^{10}$ [2] and $B^{10}(p,p')B^{10}$.[3] We observe the lowest state ($\omega\Gamma\sim 0.15$ ev, where Γ is the smaller of Γ_α and Γ_γ) at 4.75±0.02 Mev and the highest at 5.162×0.008 Mev ($\omega\Gamma\sim 0.2$ ev); the latter measurement agrees well with those of Bonner and Butler[2] (5.165±0.006 Mev) and Bockelman *et al.*[3] (5.159±0.010 Mev). We do not observe the middle level ($\omega\Gamma<\sim 0.007$ ev); we believe it to be (2−) and its absence due to an isotopic spin discouragement of the $E1$ transitions.[4] [Ajzenberg[2] has suggested (1−) or (2−) for one or other element of the 5.11–5.16-Mev doublet.]