A Possible Mechanism for $1/f$ Noise Generation in Semiconductor Filaments*

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In describing the mechanism to be proposed here for $1/f$ noise generation in semiconductor filaments, it is first of all assumed that the low-frequency noise is caused by random fluctuations in the number of mobile charges (i.e., holes and electrons). This explains the fact that noise power seems to vary as the square of the current through the semiconductor. The fluctuations of mobile charges are, in turn, assumed to be due to the random creation and annihilation of trapping centers.[1] For reasons that will appear later, it will be assumed that these trapping centers are all on the surface of the material. The physical model which describes this random creation and annihilation process will be as follows. A lattice atom or impurity atom at the surface of the material is knocked out of its equilibrium position (by a source which will not be specified) and starts to diffuse throughout the surface. The motion will then be a sort of Brownian movement about the equilibrium position O. It is assumed that the diffusion must be confined to the surface (i.e., it takes place only over two dimensions). From the theory of Brownian motion the diffusion atom at any time t after its release will on the average be within a circle whose center is at O and whose radius $\bar{r}$ is determined by

$$\bar{r}^2=4D(t+t_0). \tag{1}$$

D is the diffusion constant. The vacancy O is assumed to act as a trapping center for a hole or electron until its diffusing atom or another diffusing atom gets captured in it. At the instant of atom capture the trapped hole or electron is released to become a mobile charge again. Thus, during the time O was unoccupied there was one less mobile charge. With a constant current through the sample, this would have produced a square pulse at the output voltage, whose length is the time O was unoccupied.

It is assumed that the trapping centers are being created randomly at a more or less constant average rate. Their rate of annihilation is dependent upon the intensity of diffusing atoms (and, therefore, the number of trapping centers present); thus, after a while there will be an equilibrium condition where the average density of diffusing atoms and trapping centers will be constant. This is the condition with which we will be concerned here.

The output voltage will then be a sum of randomly occurring square pulses of varying lengths. If the probability of any one pulse width being between s and $s+ds$ is $g(s)ds$, then it can be shown that the correlation function $R(t)$ of the output voltage will be

$$\begin{aligned} R(t)&=K\int_t^\infty (s-t)g(s)ds \\ &=K\int_t^\infty \rho(s)ds, \end{aligned} \tag{2}$$

where

$$\rho(s)\equiv\int_s^\infty g(u)du,$$

and K is the average number of pulses per unit length of time.

The power spectrum $G(\omega)$ for output will then be

$$\begin{aligned} G(\omega)&=K\int_0^\infty\left[\int_t^\infty \rho(s)ds\right]\cos\omega t dt \\ &=(K/\omega)\int_0^\infty \rho(t)\sin\omega t dt. \end{aligned} \tag{3}$$

Now $\rho(t)$ can also be shown to be the probability that an unoccupied trapping center is still unoccupied after a time t, after having been created at $t=0$. $\rho(t)$ is to be determined from the following relation:

$$-(d\rho/dt)=[(\bar{N}\sigma_c\bar{v})+((\sigma_c\bar{v})/(\pi\delta\bar{r}^2))]\rho, \tag{4}$$

where σ_c is the capture cross section, $\bar{v}$ is the average velocity of the diffusing atom, δ is the thickness of the surface layer, and $\bar{N}$ is the average density of the diffusing atoms from other vacancies at the position of a given vacancy. Equation (4) expresses the fact that a trapping center can be annihilated in two ways. The first way (given by the second term on the r.h.s. of the equation) is by having its own diffusing atom come back and be recaptured. The second way (denoted by the first term) is by capturing a diffusing atom from another trapping center.

If (1) is substituted in (4) and the boundary condition $\rho(0)=1$ is used, $\rho(t)$ can be obtained specifically and has the form,

$$\rho(t)=[t_0/(t+t_0)]^n\exp(-t/T), \tag{5}$$

where

$$T\equiv(\bar{N}\sigma_c\bar{v})^{-1}, \tag{6a}$$

$$n\equiv(\sigma_c\bar{v})/(4\pi\delta D). \tag{6b}$$

$G(\omega)$ can be obtained by substituting (5) into (3), with the result that

$$\begin{aligned} G(\omega)&=\frac{K\pi t_0{}^nF(n,\theta)}{[\omega^2+(1/T)^2]^{(2-n)/2}} \quad \text{when} \quad \omega<1/t_0, \\ &=1/[\omega^2+(n/t_0)^2] \quad \text{when} \quad \omega>1/t_0, \end{aligned} \tag{7a}$$

where $\theta\equiv\tan^{-1}(\omega T)$ and

$$F(n,\theta)\equiv\frac{\sin(1-n)\theta}{\Gamma(n)[\sin n\pi][\sin\theta]}. \tag{7b}$$

When $1/T<\omega<1/t_0$, (7a) shows that $G(\omega)$ varies as $1/\omega^m$, where $m=2-n$. Thus $G(\omega)$ has the observed $1/\omega$ dependence if $n=1$. Experimental data indicate that m may vary from 0.8 to 1.5, so n can vary from 0.5 to 1.2.

A crude estimate of n will now be made in terms of the physical parameters which are involved. From the theory of diffusion, it can be shown that D and $\bar{v}$ can have the following forms:[2,3]

$$D=\tfrac{1}{4}\bar{f}a^2, \tag{8a}$$

$$\bar{v}=\bar{f}a. \tag{8b}$$

In the above equation $\bar{f}$ is the average jump frequency and a is the separation between adjacent potential wells. It should also be noted at this point that $\sigma_c=\delta\lambda_c$, where λ_c is a mean capture length. Thus with the aid of (8a) and (8b), n from (6b) can be brought into the following simple form:

$$n=\lambda_c/(\pi a). \tag{9}$$

If n is to be of the order of unity, λ_c must be of the order of πa ($=10^{-7}$ cm), a value which is not too unreasonable physically.

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[1] The general picture up to this point is similar to noise models of other investigators. See, for example, G. G. Macfarlane, Proc. Phys. Soc. (London) **B63**, 807 (1950); J. M. Richardson, Bell System Tech. J. **29**, 117 (1950).

[2] See, for example, A. H. Cottrell, *Theoretical Structural Metallurgy* (Longmans Green, New York, 1948), pp. 190–5.

[3] Equation (8a) was derived on the assumption that the jumps can take place in any direction.

Infrared Absorption Induced by Static Electric Fields

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Condon[1] showed theoretically in 1932 that vibrational modes normally infrared inactive could be rendered active by a static electric field. He estimated the magnitude of the effect and concluded that, although the absorption would be small, it should be observable under suitable experimental conditions.

The rotation-vibration spectrum of hydrogen induced by a static electric field has been observed and initial results are reported in this letter.

The absorption cell was that used in the investigation of pressure-induced absorption.[2] The light guide also served as a parallel

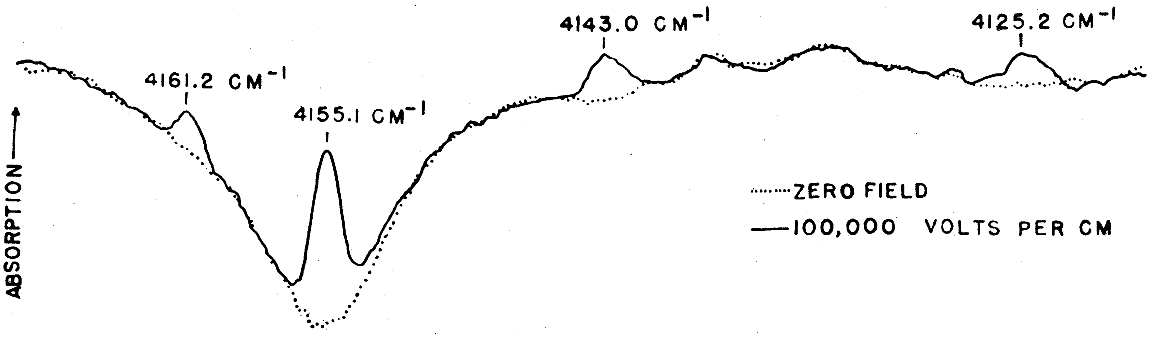

FIG. 1. Field-induced components of the Q-branch of the $1\leftarrow 0$ vibrational transition of H_2.

plate condenser. The two side walls were polished strips of aluminum 85 cm long by 1.75 cm wide; the top and bottom were strips of Plexiglass which separated the metal walls by 1.4 mm. One plate of this condenser was connected to an insulated terminal in the wall of the absorption cell. The other plate was in electrical contact with the wall. Hydrogen pressures up to 2000 lb per square inch were used. The absorption was measured with a Littrow type grating spectrometer using a PbS photoconductive cell as a detector.

The absorption induced by the static electric field is observed superimposed on the pressure-induced absorption,[3] but the two effects are readily distinguished from each other because their half-widths are radically different. The dotted curve of Fig. 1 shows a contour of a portion of the Q branch of the pressure-induced absorption of hydrogen. The sharp maximum of transmission at 4155 cm^{-1} arises from the splitting of the Q branch. The continuous curve of Fig. 1 shows the contour obtained when the potential difference between the plates of the light guide was 14 000 volts, other conditions remaining the same. It is apparent that the application of the electric field has produced four sharp absorption lines. These are components of the Q branch of the fundamental rotation-vibration band. A comparison of the frequencies of these lines with the frequencies of the Q branch components of the Raman spectrum of hydrogen[4] is given in Table I.

TABLE I. Observed transitions.

Transition $\Delta v=1-0$ $\Delta J=0$	Frequency, (cm^{-1}); static field (density $=82.9$ amagat[a]	Frequency, (cm^{-1}); Raman spectrum (density $=5$ amagats)
0–0	4161.2	4161.28
1–1	4155.1	4155.48
2–2	4143.0	4143.55
3–3	4125.2	4125.88

[a] The density in amagats is defined as the ratio of the density of the gas under experimental conditions to the density of the gas at one atmosphere pressure and zero °C.

The static field frequencies are slightly smaller; the differences apparently are significant since they increase with J and with density.

In principle the matrix element of the mean value of the polarizability for the 1–0 vibrational transition, α_{01}, and the matrix element of the anisotropy γ_{01} can be evaluated from the integrated absorption coefficients. The integrated absorption coefficients as yet are not sufficiently accurate to warrant an estimate of γ_{01}. However, the coefficient of the 1–1 component of the Q branch has been measured sufficiently well to allow a first determination of α_{01}. It has been established experimentally that the absorption coefficient is proportional to the square of the field strength. Thus a specific absorption coefficient can be defined and for the 1–1 component is 5.53×10^{-29} cm^{-1}, per cm path, per molecule per cm^3, per (esu of potential per cm)^2. The matrix element determined from this value is $\alpha_{01}=1.2(5)\times10^{-25}$ cm^3.[5] Corrections have been applied for polarization produced in the incident light flux by the quartz prism of the monochromator and for the small effect of the matrix element of the anisotropy. The result is intermediate to the value obtained from the ratio of Raman to Rayleigh scattering, 1.05×10^{-25} cm^3,[6] and the theoretical value, 1.393×10^{-25} cm^3.[7] By using polarized light and a more efficient grating, more accurate determinations of α_{01} and γ_{01} should be possible.

These results definitely establish the field-induced absorption predicted by Condon. The effect has several interesting and potentially important applications.

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[1] E. U. Condon, Phys. Rev. **41**, 759 (1932).
[2] Welsh, Crawford, and Locke, Phys. Rev. **76**, 580 (1949).
[3] Crawford, Welsh, MacDonald, and Locke, Phys. Rev. **80**, 469 (1950); Chisholm, MacDonald, Crawford, and Welsh, Phys. Rev. **88**, 957 (1952).
[4] C. Cumming, Ph.D. thesis, University of Toronto (unpublished).
[5] The calculation will be given in a subsequent publication.
[6] Stansbury, Crawford, and Welsh, Can. J. Phys. (to be published).
[7] Ishiguro, Arai, Mizushima, and Kotani, Proc. Phys. Soc. (London) **A65**, 178 (1952).

Bleaching of CaF_2 Crystals Colored by X-Rays*

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SYNTHETIC and natural CaF_2 crystals colored by x-rays exhibit absorption bands: 5800, 4000, 3350, and 2250A.[1] The increase of absorption below 2100A indicates at least one more band. The same absorption is produced by weak, additive coloration.[2] A theoretical explanation of the nature of these bands is still lacking.

A study of thermal and light bleaching of colored alkali halides contributed much to the understanding of color centers in those

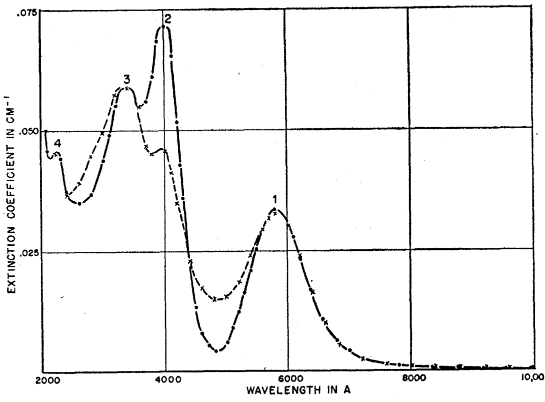

FIG. 1. Bleaching of colored CaF_2 crystal by light absorbed by band 2; —·— before bleaching, - - X - - after bleaching.