Fast Neutron Cross Sections and Nuclear Level Density*

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The capture cross sections for unmoderated fission neutrons, effective energy one Mev, are determined by the average properties of nuclear levels at excitation energies equal to neutron binding plus one Mev. The level spacings determined from measured cross sections for 61 isotopes are discussed in connection with the statistical and shell nuclear models. Whereas the discontinuity in neutron binding energy at magic numbers accounts for most of the abnormality of the cross sections of the magic-N nuclei, the effect of shell structure is evident in level spacing even at high (5–7 Mev) excitation energy. The level spacings obtained for nonmagic-number nuclei, on the other hand, are in reasonable agreement with the statistical nuclear model, and in addition are the same for levels of widely different spin.

THE results of a survey[1,2] of capture cross sections for one-Mev (effective energy) neutrons showed that nuclei containing 50, 82, and 126 neutrons had cross sections lower by a factor of fifty than neighboring nuclei. The low cross sections for these nuclei were explained in terms of the sudden decrease in neutron binding energy that occurs at completed nuclear shells. The fast neutron absorption cross section is proportional to the average level density in the compound nucleus, which density is a rapidly increasing function of excitation energy. As the excitation energy following capture is given by the neutron binding energy plus one Mev, the low cross sections furnished evidence for the occurrence of "magic-number" nuclei with unusually low binding energy. At the present time, however, neutron binding energies are reasonably well fixed by work with (γ, n), (n, γ) and (d, p) reactions,[3] and the fast capture cross sections can be used to determine the nuclear level densities at known excitation energies. The level densities determined in this way can be used to check the validity of the statistical nuclear model with regard to variation of level density with odd-even character, with atomic weight, and with magic numbers. In the present paper we shall discuss the level densities determined from the experimental cross sections, after a brief description of measurements more recent than those already reported.[1,2]

I. EXPERIMENTAL RESULTS

The fast cross sections of references 1 and 2 were measured at the deuterium-moderated pile of the Argonne National Laboratory, using unmoderated fission neutrons of effective energy one Mev for the capture reactions. The fission neutrons were obtained from a U^{235} plate that was placed in a thermal beam with no shielding, and because of the high fast flux produced, irradiations could be performed only when no other experiments were in progress at the pile. A similar "converter plate," now in use at Brookhaven for the same type of measurements, is shielded in such a way that irradiations can take place without interfering with other pile experiments. The shielding of the fission neutrons is difficult because it is necessary to avoid the production of resonance neutrons by slowing down in the shielding blocks, neutrons which have capture cross sections much higher than the unmoderated neutrons.

The purpose of the present design (Fig. 1) is to obtain a high flux of thermal neutrons (which do not need to be collimated) at the U^{235} plate, with as few resonance neutrons as possible, and to shield the plate without production of resonance neutrons in the shield. The plate is placed near the pile reflector in order to obtain a high thermal neutron flux, even though resonance neutrons from the pile lattice are appreciable at that position. Placing the plate at a thermal column would ensure a low resonance flux, but would entail a serious loss of thermal flux.

The cadmium ratio measured in the position of the plate (the plate being absent) with a $1/v$ detector (a BF_3 proportional counter) is about 200, a surprisingly low value considering the amount of graphite reflector between the point of measurement and the pile lattice. Additional experiments showed that the resonance neutrons causing the poor ratio were those leading out of

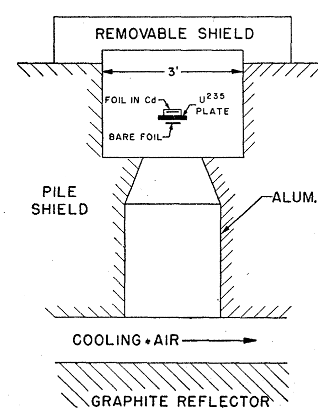

FIG. 1. Experimental arrangement for production of unmoderated fission neutrons inside shield of the Brookhaven pile. The foil inside cadmium is activated by fission neutrons and the bare foil is a monitor of the thermal flux.

* Work carried out under contract with the U. S. Atomic Energy Commission.
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[1] Hughes, Spatz, and Goldstein, Phys. Rev. **75**, 1781 (1949); this paper contains references to older work.
[2] D. J. Hughes and D. Sherman, Phys. Rev. **78**, 632 (1950).
[3] Summarized by J. A. Harvey, Phys. Rev. **81**, 353 (1951).

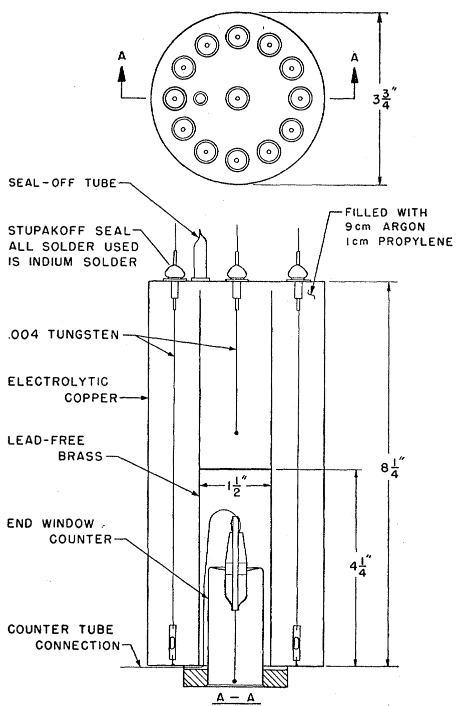

FIG. 2. Anticoincidence counter used for measurement of weak activities. The counter surrounding the central G-M tube removes the cosmic-ray background and thus reduces the background to about 6 counts/min.

the center of the pile through cooling channels (air ducts) and reaching the plate indirectly, rather than coming directly through the graphite itself. In measurements of fast neutron capture cross sections, it has been necessary in many cases to surround the samples by protective layers of the element being irradiated in order to eliminate, by self-protection, the resonance neutrons. Such a "sandwich" arrangement is satisfactory for most isotopes, but for a few with prominent low-lying resonances, such as In and Au, further corrections must be made. It has been found to be quite simple to carry on long irradiations in the arrangement of Fig. 1 without disturbing other experiments at the pile, and this property of the apparatus is very useful for the long-lived activities that have been measured recently.

An anticoincidence counter has been developed[4] for measuring the long-lived samples of low activity often produced in this work. This counter, shown in Fig. 2, removes the cosmic-ray component of the end window counter background, thus lowering the background to about 6 c/m, which is probably contributed by the material of the counter tube itself (lead gaskets, for instance). The reduction in background already obtained is very useful for weak activities, and counters are now being prepared of materials that should have still lower background ratios.

The recent work at Brookhaven[5,6] has been devoted mainly to cross sections at and near magic number nuclei, both proton- and neutron-magic. Some of these cross sections have been much more difficult to measure than the earlier ones because of isomeric states and long half-lives. The results of the Brookhaven measurements will be combined with the earlier work in the subsequent discussion.

In order to illustrate the general behavior of the measured cross sections we present in Fig. 3 the isotopic activation values as a function of atomic weight, A. The cross sections are found to increase steadily up to a value of about 100 millibarns at $A=100$, after which there is no great change, with the marked exceptions of those nuclei for which there is a magic or near-magic number of protons or neutrons. All the target nuclei

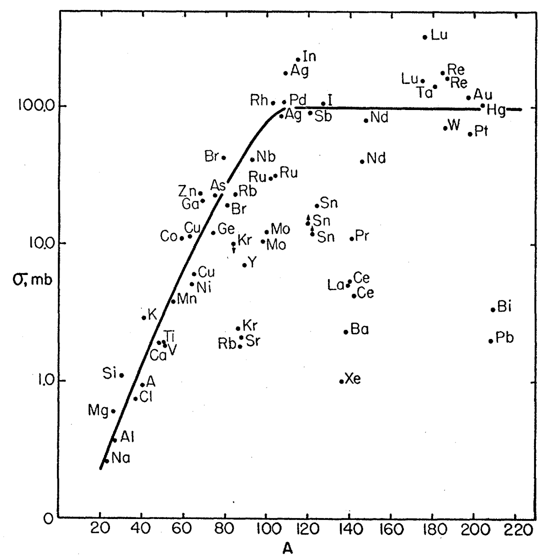

FIG. 3. Isotopic activation cross sections at effective energy one Mev as a function of atomic weight.

investigated have an even number of neutrons, with the single exception of $_{71}Lu^{176}$, which is seen to have a fast cross section over twice as great as that for nuclei of comparable atomic weight. The 61 isotopes studied are approximately equally divided into those of odd and even A. In all cases the residual nuclei after neutron capture are beta emitters, being on the neutron rich side of the valley of stability. The smooth curve is a a graphical average of the experimental points for those isotopes whose cross sections represent what may be called "normal behavior."

The exceptional behavior of nuclei having a magic number of neutrons is brought out more clearly by plotting the cross sections against the number of neutrons as in Fig. 4. Those nuclei containing 50, 82, or 126 neutrons are found to have cross sections that

[4] We wish to express our appreciation to H. Palevsky for the design of this counter and to W. Higinbotham for its construction.

[5] Hughes, Garth, and Eggler, Phys. Rev. **83**, 234 (1951).

[6] Garth, Hughes, and Levin, Phys. Rev. **87**, 222 (1952).

range from one-hundredth to one-tenth of those of normal nuclei. The two Mo and the two Ru isotopes indicate an anomalously low cross section in the region 56–60 neutrons but the cross sections of Rh and Ag are normal. It is perhaps significant that the latter have even, and the former odd Z. When the cross sections are plotted against the atomic number Z (Fig. 5), it is found that there is a less sensitive dependence upon the number of protons in the target nuclei. The low values of cross section for tin isotopes having 50 protons and for lead having 82 protons are noteworthy, although for $_{82}Pb^{208}$ the magic number of neutrons may obscure the effect of a magic number of protons. For $_{20}Ca^{48}$, however, there is no marked difference in behavior from that of neighboring nuclei. The points are plotted with

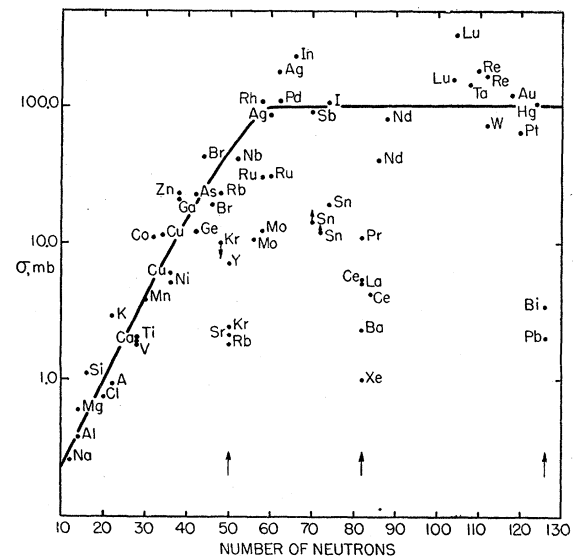

FIG. 4. The cross sections of Fig. 3 as a function of neutron number of target nucleus.

open circles for even Z and solid dots for odd Z to show that there is no obvious regularity in the pattern that may be ascribed to the even-odd character of Z. Quantitatively, it is found that on the average the nuclei of odd Z have a fast cross section that is slightly (35 percent) higher than the nuclei of even Z, if all nuclei that contain magic or near-magic numbers of neutrons or protons are ignored. The 56–60 neutron isotopes just discussed are also omitted in this calculation.

The regular pattern of cross sections as well as the exceptional behavior at magic numbers that is illustrated in Figs. 3, 4, and 5 is directly related to the level densities in the compound nuclei involved. These level densities can be calculated from the observed cross sections and compared with various nuclear models, as well as with other experimental results on level densities.

II. DETERMINATION OF LEVEL DENSITIES FROM CROSS SECTIONS

The fission neutrons have an effective energy[1] for the capture reaction of 1 Mev, and an energy spread sufficiently wide that the cross section is determined by resonance properties averaged over many levels. The cross section in this situation is given[7] by

$$\sigma_l = \pi\lambda\!\!\!^{-2}(2l+1)\frac{2\pi(\Gamma_{nl}\Gamma_{\gamma l})_{\text{Av}}}{D_l[(\Gamma_{nl})_{\text{Av}}+(\Gamma_{\gamma l})_{\text{Av}}]}, \qquad (1)$$

where σ_l is the contribution to the cross section of neutrons with angular momentum $l\hbar$ and wavelength $\lambda\!\!\!^-$. Γ_{nl} and $\Gamma_{\gamma l}$ are the neutron and radiation widths respectively of the many levels involved, with "Av" signifying average values, and D_l is the spacing of levels formed by l neutrons.[8] Considering only $l=0$, it is seen that where $\Gamma_n > \Gamma_\gamma$ (neutron energy >10 kev) the variation of cross section with energy will be as $1/E$. As the neutron energy increases further, however, the capture cross section falls off less rapidly as higher l's become important.

The effect of higher l interactions can be illustrated by summing Eq. (1) over all significant l's, remembering that the highest effective l is given by $l\lambda\!\!\!^- = R$ (the nuclear radius), and that $\sum_0^l (2l+1) = (l+1)^2$. We thus obtain

$$\sigma(n,\gamma) = 2\pi^2(\lambda\!\!\!^- + R)^2\Gamma_\gamma/D, \qquad (2)$$

if we assume that D_l (here written as D) is the same for all l's, a form already given by Bethe,[9] which shows that

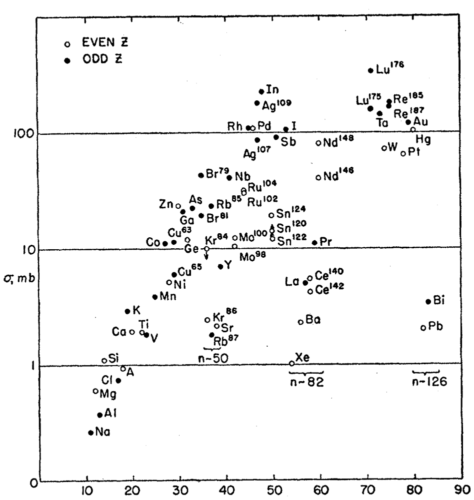

FIG. 5. The cross sections of Fig. 3 as a function of atomic number of target nucleus.

[7] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[8] The spin factor that appears for the cross section of a single resonance becomes unity when averaged over many resonances. The level spacing D_l refers to levels of a single spin and parity value of the compound nucleus, formed by neutrons of angular momentum $l\hbar$.
[9] H. A. Bethe, Phys. Rev. **57**, 1125 (1940).

σ is proportional to $1/E$ if $\lambda\!\!\!^{-} > R$, that is, for $l=0$. For this case we have

$$\sigma(n,\gamma)=2\pi^2\lambda\!\!\!^{-2}\Gamma_\gamma/D_0, \tag{3}$$

which gives the cross section as a simple function of the radiation width and level spacing. Measured cross-section[1] curves near 1 Mev show a definite $1/E$ behavior and hence indicate that $l=0$ interactions predominate, in spite of the fact that $\lambda\!\!\!^{-}$ and R are comparable for 1-Mev neutrons at $A=100$.

When the measured values of the capture cross sections (for the effective energy 1 Mev) are substituted in Eq. (3), together with values of the radiation width, Γ_γ, the average spacing of levels, D_0, can be found. Fortunately the radiation width is known reasonably well, at least for heavy elements. For atomic weights greater than 100, Γ_γ is about 0.1 ev, and is not much affected by changes in excitation energy. Its value increases to several ev for atomic weights near 20. Heidman and Bethe[10] have made an estimate of Γ_γ as a function of A for excitation by neutron capture, and their results are shown in Fig. 6. The level separation thus obtained by use of Eq. (3) refers to levels formed by neutrons for which $l=0$, and to an excitation energy, E^*, of the compound nucleus equal to 1 Mev plus the neutron binding energy, E_B. The neutron binding energies for most nuclei are known to a few hundred kilovolts, and can be computed by means of Fermi's empirical mass formula,[11] with corrections obtained from the summary of experimental data by Harvey.[3]

The values of D_0 calculated from Eq. (3) are listed in Table I, which also includes the isotopic capture cross sections for neutrons of thermal energy that are used in the measurements[1,2,5,6] for neutrons of 1 Mev effective energy. The excitation energy, E^*, the spin of the target nucleus, and the particular activity measured are also tabulated. In a few cases isomeric activities

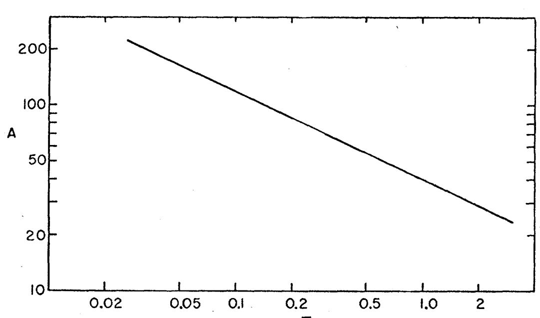

FIG. 6. The radiation width as a function of atomic weight for levels excited by slow neutrons, as given by Heidman and Bethe (see reference 10).

[10] J. Heidman and H. A. Bethe, Phys. Rev. **84**, 274 (1951); our use of the radiation width estimated by these authors is not legitimate for $A<40$ because of their use of the fast capture cross section data itself in that region in determination of the widths.

[11] N. Metropolis and G. Reitwiesner, *Table of Atomic Masses* (Argonne National Laboratory, Lemont, Illinois, 1950).

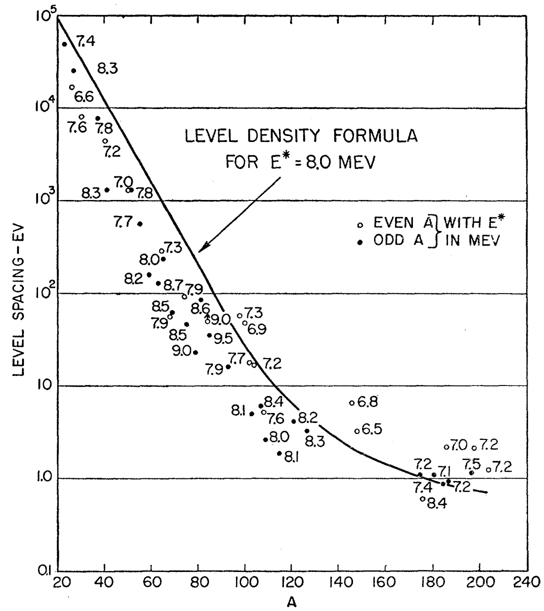

FIG. 7. Level spacings for "normal" (nonmagic-number) nuclei calculated from experimental cross sections, compared with the prediction of the statistical model for 8-Mev excitation. The actual excitation energies of the individual isotopes are also given.

result and it is necessary to add these activities to obtain the complete (n,γ) cross section. For Sn^{120} and Sn^{122} only lower limits for the latter were obtained because the long-lived isomeric states have not been measured, while in several other cases the thermal isomeric ratio was assumed to hold for the fast cross sections.

III. COMPARISON OF LEVEL DENSITIES WITH NUCLEAR MODELS

It is of interest to compare the average level spacings obtained from the measured fast cross section with the predictions of various theoretical models. In particular, it is of great interest to see to what extent the statistical model is evidenced as contrasted to the shell model that seems to be valid for ground and low-lying levels. The statistical model of the excited nucleus leads to the familiar formula:

$$W(E)=c\exp[2(aE^*)^{\frac{1}{2}}]=10^6/D_0, \tag{4}$$

where $W(E)$ is the level density, or the average number of levels per Mev while a and b are parameters. The parameters, which vary with atomic weight, are determined[12] from experimental values of D_0 for levels excited by neutrons slightly above thermal energy ($E^*=E_B$) and for levels near ground ($E^*\sim 1$ Mev).

The predicted level spacing may be compared with the experimental values for *normal* (nonmagic-number) nuclei very easily because the neutron binding energy of each nucleus measured is about 7 Mev. Using the

[12] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, New York, 1952), pp. 371–374.

TABLE I. Measured isotopic activation cross sections for fission neutrons (effective energy 1 Mev). Also listed are the thermal activation cross sections[a] and level spacings (D_0) at excitation energy E^*.

Target nucleus	Neutron number	Spin	Half-life	σ thermal (b)	σ fast (mb)	E^* (Mev)	D_0 (ev)
$_{11}Na^{23}$	12	3/2	15 hr	0.49	0.26	7.39	4.8×10^4
$_{12}Mg^{26}$	14	0	9.6 min	0.049	0.6	6.61	1.7×10^4
$_{13}Al^{27}$	14	5/2	2.3 min	0.215	0.37	8.3	2.5×10^4
$_{14}Si^{30}$	16	0	2.7 hr	0.12	1.1	7.6	8.0×10^3
$_{17}Cl^{37}$	20	3/2	38 min	0.56	0.74	7.8	6.6×10^3
$_{18}A^{40}$	22	0	1.8 hr	1.2	0.93	7.22	4.4×10^3
$_{19}K^{41}$	22	3/2	12.4 hr	1.0	2.9	8.27	1.3×10^3
$_{20}Ca^{48}$	28	0	8.5 min	1.1	1.9	5.35	1.5×10^3
$_{22}Ti^{50}$	28	0	6 min	0.14	1.9	7.01	1.3×10^3
$_{23}V^{51}$	28	7/2	3.9 min	4.5	1.8	7.81	1.3×10^3
$_{25}Mn^{55}$	30	5/2	2.6 hr	12.6	3.82	7.73	5.4×10^2
$_{27}Co^{59}$	32	7/2	11 min	14	4.6 } 11.0[b]	8.22	1.6×10^2
			5 yr	20	...		
$_{28}Ni^{64}$	36	0	2.6 hr	2.6	5.1	7.34	2.9×10^2
$_{29}Cu^{63}$	34	3/2	12.9 hr	4.0	11.4	8.74	1.3×10^2
$_{29}Cu^{65}$	36	3/2	4.3 min	2.0	6.0	8.00	2.4×10^2
$_{30}Zn^{68}$	38	0	52 min	1.0	8.0 } 23.2	7.90	56
			14 hr	0.1	15.2		
$_{31}Ga^{69}$	38	3/2	20 min	1.4	20.9	8.51	63
$_{32}Ge^{74}$	42	0	82 min	0.45	12	7.93	92
$_{33}As^{75}$	42	3/2	27 hr	4.1	22.5	8.51	47
$_{35}Br^{79}$	44	3/2	18 min	8.5	29 } 42.5	9.03	23
			4.4 hr	2.9	13.5		
$_{35}Br^{81}$	46	3/2	36 hr	2.5	17	8.62	85
$_{36}Kr^{84}$	48	0	4 hr	0.1	1.9 } <10	9.01	>50
			10 yr	0.06	<8		
$_{36}Kr^{86}$	50	0	78 min	0.06	2.4	6.17	3.1×10^2
$_{37}Rb^{85}$	48	5/2	19.5 day	0.72	23.1	9.50	35
$_{37}Rb^{87}$	50	3/2	17.5 min	0.12	1.8	6.67	4.1×10^2
$_{38}Sr^{88}$	50	0	53 day	0.005	2.1	7.12	3.8×10^2
$_{39}Y^{89}$	50	1/2	61 hr	1.38	7.0	7.63	1.1×10^2
$_{41}Nb^{93}$	52	9/2	6.6 min	1.1	41	7.94	16
$_{42}Mo^{98}$	56	0	67 hr	0.13	10.4	7.32	58
$_{42}Mo^{100}$	58	0	15 min	0.18	12.3	6.87	47
$_{44}Ru^{102}$	58	0	42 day	1.2	30	7.67	18
$_{44}Ru^{104}$	60	0	4 hr	0.7	31	7.21	17
$_{45}Rh^{103}$	58	1/2	44 sec	137	94 } 109	8.12	5
			4.2 min	12	15.4		
$_{46}Pd^{108}$	62	0	13 hr	7.7	108	7.60	5.3
$_{47}Ag^{107}$	60	1/2	2.3 min	32	85	8.41	6
$_{47}Ag^{109}$	62	1/2	25 sec	84	174 } 178*[b]	8.03	2.7
			225 day	2.2	...		
$_{49}In^{115}$	66	9/2	13 sec	52	57 } 223	8.09	1.9
			54 min	145	166		
$_{50}Sn^{120}$	70	1/2	27.5 hr	0.13	14 } >14	7.82	<29
			>400 day	...	...		
$_{50}Sn^{122}$	72	5/2	40 min	0.16	12 } >12	7.63	<32
			126 day	...	...		
$_{50}Sn^{124}$	74	7/2	9.5 min	0.15	15 } 19	7.38	19
			9.4 day	0.005	4		
$_{51}Sb^{121}$	70	5/2	2.8 day	6.8	90	8.18	4.2
$_{53}I^{127}$	74	5/2	27 min	6.7	105	8.33	3.3
$_{54}Xe^{136}$	82	0	3.9 min	0.15	1.0	5.65	3.02×10^2
$_{56}Ba^{138}$	82	0	86 min	0.5	2.3	6.26	1.35×10^2
$_{57}La^{139}$	82	7/2	40 hr	8.4	5.0	6.55	63
$_{58}Ce^{140}$	82	0	28 day	0.27	5.4	6.85	52
$_{58}Ce^{142}$	84	0	33 hr	0.85	4.2	6.54	65
$_{59}Pr^{141}$	82	5/2	19 hr	11.2	11.0	7.15	26
$_{60}Nd^{146}$	86	0	11 day	1.8	40	6.78	6.7
$_{60}Nd^{148}$	88	0	1.7 hr	3.7	80	6.46	3.3
$_{71}Lu^{175}$	104	7/2	3.7 hr	25	158 } 158[b]	7.19	1.1
			10^{10} yr	(small)	...		
$_{71}Lu^{176}$	105	7/2	6.7 day	4000	330	8.42	0.5
$_{73}Ta^{181}$	108	7/2	16 min	0.02	... } 142[b]	7.12	1.13
			117 day	21	142		
$_{74}W^{186}$	112	0	25 hr	40	71	7.01	2.18
$_{75}Re^{185}$	110	5/2	90 hr	101	180	7.40	0.87
$_{75}Re^{187}$	112	5/2	18 hr	75	165	7.23	0.93
$_{78}Pt^{198}$	120	0	31 min	3.9	64	7.16	2.14
$_{79}Au^{197}$	118	3/2	2.7 day	95	120	7.52	1.15
$_{80}Hg^{204}$	124	0	5.5 min	0.43	102	7.19	1.26
$_{82}Pb^{208}$	126	0	3.3 hr	0.0006	2.0	5.06	61
$_{83}Bi^{209}$	126	9/2	5 day	0.017	3.4	5.27	35

[a] U. S. Atomic Energy Commission Report AECU-2040 (Office of Technical Services, Dept. of Commerce, Washington, D. C., 1952).
[b] It is assumed that the isomeric ratio is the same for both thermal and fast activations.

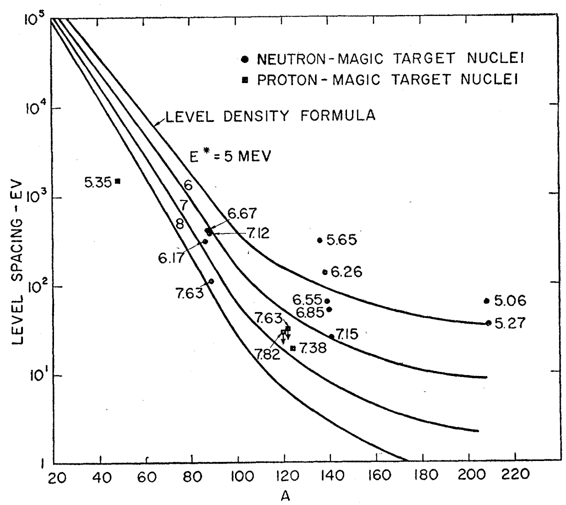

FIG. 8. Level spacings for magic and near-magic number nuclei calculated from experimental cross sections, compared with predictions of statistical model for several excitation energies. Actual excitation energies for individual isotopes are given by number. ●—Magic-N target nuclei. ■—Magic-Z target nuclei.

average value for E^* of $7+1$ Mev in Eq. (4), we obtain a curve that shows the variation of D_0, the average level spacing for this excitation energy, with atomic weight. This curve is shown in Fig. 7, together with the level spacings obtained from Eq. (3) for the individual isotopes. Beside each point is listed its individual excitation energy, which is usually near 8 Mev. The points are seen to follow the pattern of the computed curve with good general agreement, especially when allowance is made for the deviation of the actual values of E^* from the 8-Mev average represented by the curve. As the experimental error is of the order of 50 percent, no significance can be attached to variations of this magnitude for individual cross sections. The theoretical level spacing is about right for $A>150$ but rises above the experimental points with decreasing A, reaching a factor of about five at $A=20$. Actually, considering the approximate nature of the level density formula[12] and the fact that it was adjusted to fit levels in a lower energy region, we see that the agreement exhibited in Fig. 7 is satisfactory, especially at high A.

Before discussing the magic-number nuclei, which were omitted from Fig. 7, it is of value to consider several other characteristics of the spacings of the "normal" nuclei. From the observed cross sections for nuclei of different spins, we can ascertain if the level density increases with spin of the levels. The high-spin target nuclei of Table I however, three of spin 9/2 and seven of spin 7/2, show the same level spacing as the low-spin nuclei. This finding cannot be explained by a large Γ_γ that cancels a large D_0 because slow neutron resonances of these isotopes have normal values of Γ_γ ($\sim$0.1 ev). The equality of level spacing for different spins is not in agreement with the suggestion[13,14] that the level density is proportional to $(2J+1)$, where J is the spin of the compound nucleus.

As the nuclei of Table I are approximately equally divided between even and odd Z, it is possible to obtain a reasonably accurate estimate of the effect of even-odd Z on level density. The neutron binding energy (hence E^*) is not affected by the odd-even nature of Z, hence any difference in cross section depending on Z would indicate a corresponding difference in level spacing at the same excitation energy. The cross sections of Fig. 5 do not indicate a large difference between even and odd Z, omitting magic-number nuclei, and calculation shows that the odd-Z nuclei are higher on the average by only 35 percent, an amount of doubtful significance. A more sensitive test of Z-dependence of level spacing is furnished by the magic-Z nuclei to be discussed. As the binding energy of a neutron to an odd-*neutron* nucleus is higher than to a even-neutron nucleus, a higher excitation energy and cross section is expected. The only example studied is Lu^{176}, which has a cross section at least twice normal, in agreement with its high neutron binding energy.

In the case of magic-N target nuclei, it is necessary to consider their anomalously low neutron binding energy, which accounts for most of the decrease in cross section for these nuclei relative to their normal neighbors. As magic-Z nuclei exhibit normal neutron binding energies, no cross-section decrease for this reason, analogous to that for magic-N nuclei, is expected. In Fig. 8 are shown curves calculated from Eq. (4) for values of E^* ranging from 5 to 8 Mev, together with experimental level spacings for nuclei resulting from target nuclei containing a magic number of neutrons or protons. In the majority of cases it is noted that the level spacing calculated from the experimental cross sections for nuclei with a magic number of neutrons exceeds the value predicted by the statistical model by as much as a factor of two or three, and in some instances by as much as six. Thus, whereas the low binding energy of magic-N nuclei accounts for a large part of their decrease in cross section, the remaining discrepancy of Fig. 8 implies that the level spacing in the compound nuclei (magic plus one neutron) is larger than predicted by Eq. (4) by a factor of about four. This anomaly would be even larger if the theoretical level spacings were reduced to conform better to the experimental spacing observed for normal nuclei.

The few magic-Z nuclei ($Z=50$) of Fig. 8, with approximately normal values of E^*, have level spacings greater than expected from the theoretical level density formula (again allowing for a reduction in theoretical spacing to conform with normal nuclei). This result indicates that the level spacing in these target magic-Z nuclei is greater than predicted by the statistical model, as for the magic-N case. The spacing for ${}_{20}Ca^{48}$

[13] L. Wolfenstein, Phys. Rev. **82**, 690 (1951).
[14] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

is actually less than expected; hence $Z=20$ does not seem to exhibit magic behavior. Since the number of neutrons is twenty-eight in this nucleus, it is of interest to note that there is no evidence from this result that twenty-eight is a magic number, as is also shown by ${}_{22}Ti^{50}$ and ${}_{23}V^{51}$.

It is thus seen from the results with magic-number nuclei that the level spacing is greater than predicted by the statistical model for those nuclei containing one neutron in addition to a closed shell, whether the shell is composed of protons or neutrons. A wide level spacing also exists for these particular nuclei for excitation energies about one Mev lower, as shown by the sparse resonance level structure measured[15] with slow neutrons. The difference in behavior between these and the normal nuclei is evidence for the effect of shell structure on the level spacing, which spacing we have interpreted in terms of the statistical model for normal nuclei. The simultaneous appearance of features of the statistical and the shell model, which models seem contradictory in their main features, has been the subject of several recent studies that have attempted to correlate the features of the two models.[16–20]

There are several methods, more or less equivalent, of reconciling the shell and statistical models with regard to the level spacing at high excitation that is our concern. Perhaps the simplest approach is to consider that the ground state energy of a nucleus possesses the shell characteristics; this approach explains why the binding energy of a neutron to a magic-N nucleus is low. The anomalous level density at the appropriate excitation energy of the nucleus formed, however, requires additional explanation. The observed low density of levels can be explained by assuming that energy is necessary to break the shell structure, with the result that the normal level density is not reached until several Mev higher excitation energy. This effect can be taken into account crudely by substituting the expression E^*-E_s in the level density formula for E^*, where E_s is the energy that must be added to the nucleus to break the shell. The present results for the magic-Z and magic-N target nuclei make it appear that E_s is very roughly the same for proton and neutron shells, and in magnitude about 1–2 Mev.

Another way of looking at the combination of the shell and statistical models is to assume that at high excitation energy the statistical model is valid, and that all similar weight nuclei will have the same spacing at an energy measured relative to a fictitious ground state energy,[20] whose location corrects for the shell effects, rather than the actual ground state. At first approximation, this energy state would be a smooth function of N and Z and most simply can be considered just the energy value given by the semiempirical mass formula.[11] According to this picture, magic-number nuclei have a lower ground-state energy than the formula predicts because of shell effects. The actual mass formula is somewhat arbitrary, of course, and as usually expressed,[11] individual nuclei are sometimes below and sometimes above the formula prediction; thus certain nuclei would have their ground-state energies raised, others depressed, by shell effects. The two models we have described here of course consist of equivalent views of the interpolation between the first excited states, which exhibit shell properties, to higher excitation energies where the statistical variation of level density with energy seems to be correct.

The view that the ground state is depressed for magic-number nuclei is applied by Blatt and Weisskopf[21] to the level spacings appropriate for neutron capture. The results obtained from their diagram (p. 766), however, differ in several respects from the present experimental level spacings. Thus the high spacing in a compound nucleus consisting of a magic-N nucleus plus one neutron is ascribed to the binding energy shift alone, whereas an additional change in level spacing is indicated by the experimental cross sections. Also, neutron capture in a magic-N *or* magic-Z nucleus is predicted to give the same level spacing (hence cross section); experimentally the former results in a much larger spacing, however.

The level spacings obtained in this work from the fast capture cross sections will be useful in a comparison with the directly measured spacings now becoming available with recently improved velocity selectors. These instruments are able to resolve the individual resonances for slow neutrons in sufficient numbers to give directly measured level spacings at excitation energies one million volts below those used in the present work. The comparison will give a valuable check on the assumed radiation widths in the present study and also help to clarify the effect of higher l's in the Mev work because only $l=0$ collisions are possible at the lower energy. Comparison of the same nuclei at one Mev and low energy will also be useful in the case of certain isotopes that seem to have unusually great spacings as measured with velocity selectors.

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[21] Reference 10, pp. 763–767.