Integrating formally gives

$$\int_{n_0}^{n} \frac{e^{-bn}}{n^{\gamma}} dn = -\nu t e^{-E_0/kT}, \tag{3}$$

where $b=\alpha/kT$. The integrals of (3) are expressible in terms of the exponential integral for any integer γ. At a fixed value of n one may write

$$t = K' e^{E_0/kT} e^{-\alpha'/kT}, \tag{4}$$

where K' is a constant and $\alpha'=\alpha n$. Thus, a plot of log$t$ *versus* $1/T$ obtained at a fixed n is linear just as in the simple case of constant activation energy but the effective activation energy, $E_{\mathrm{eff}}=E_0-\alpha'$, depends on the value chosen for n; i.e., the phenomenologically determined activation energy[4] varies systematically with n.

The influence of a variable activation energy on the nature of the annealing curve is best discussed on the basis of an actual example. n *versus* t curves were calculated at a fixed temperature (250°C) for $n_0=0.10$; $E_0/k=15\,000$ with $\alpha/k=25\,000$; $E_0/k=12\,500$ with $\alpha=0$; $\nu=10^{13}$, for $\gamma=2$ and 3. For $\gamma=2$, Eq. (3) integrates to

$$\frac{e^{-bn_0}}{n_0} - \frac{e^{-bn}}{n} + b[\mathrm{Ei}(-bn_0) - \mathrm{Ei}(-bn)] = -\nu t e^{-E_0/kT} \tag{5}$$

(using the notation of Jahnke and Emde). By simple integration by parts the integral for any γ may be obtained. The results obtained are shown in Fig. 1. (The semi-log plot is used for convenience only.) From the curves of Fig. 1 it is clear that a variable activation energy which increases with decreasing n has the effect of lengthening the time scale for any given isothermal annealing curve. A rather small variation of E with n leads to a very large change in the time scale. Figure 1 also shows that an increase in the order of reaction (from $\gamma=2$ to $\gamma=3$ in Fig. 1) has a qualitatively similar effect to that of an E increasing with decreasing n. Computations showed that an annealing curve characterized by a given γ and a variable E can often be fitted by a constant E but a higher and not necessarily integer value of γ. This conclusion is in accord with Overhauser's experimental results.[1]

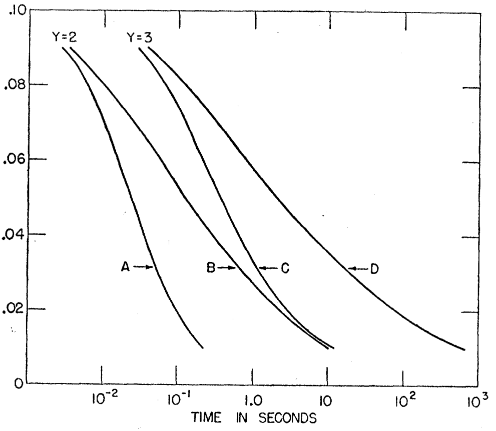

FIG. 1. Relaxation of n at 250°C. Comparison of constant and variable activation energy. A and C: $E_0/k=12\,500$, $\alpha/k=0$. B and D: $E_0/k=15\,000$, $\alpha/k=25\,000$.

The results of this study may be summarized as follows:

1. A variable activation energy leaves the nature of the plot of log$t$ *vs* $1/T$, taken at any fixed n, unchanged but manifests itself in a systematic change of the slopes of these lines as a function of the amount of anneal at which E is being determined.
2. A rather small variation of E with n leads to a very large change in the time scale for the relaxation of n.
3. An annealing curve of low γ and variable E is very similar in character to a reaction of higher order (not necessarily integral) and of constant E.
4. More complex expressions of $E=f(n)$ may be required in specific cases. As long as E is a decreasing function of n the above conclusions are qualitatively valid.

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Determination of the Neutron-Proton Capture Cross Section

G. VON DARDEL AND A. W. WALTNER*
AB Atomenergi, Department of Physics, Stockholm, Sweden

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A DIRECT determination of the neutron-proton capture cross section has been made by a measurement of the mean life of neutrons in water, as first done by Manley, Haworth, and Luebke.[1] In the present method, which is capable of high precision, the diffusion correction which was the main source of uncertainty in their investigation has been avoided by making the measurements in a large tank, of dimensions 97×97×107 cm, with the modulated neutron source, a stainless steel target bombarded by deuterons from a 150-kv accelerator, in the center. The decay of the neutron intensity at a number of positions was analyzed by a multichannel time analyzer, the detectors being cylindrical brass counters, 1 cm in diameter and filled to 600 mm of BF_3, which extended through the entire depth of the tank and thus automatically performed an integration of the neutron intensity in the vertical direction. By subsequent integration of the data in the radial direction the decay of the total number of neutrons in the tank was obtained and found to be accurately exponential apart from a small constant background.

The period of the exponential decay of the neutrons was found from 3 complete runs to be 206.6 ± 1.5 μsec, the estimated uncertainty being due mostly to the uncertainty of the correction for the constant background, since the statistical accuracy and consistency of the measurements was much better. A small correction of -0.4 μsec for counting losses in the time analyzer has been included in this result. The result has to be corrected for the influence of the counters on the neutron distribution. Since the brass counters used in the experiment have a larger capture cross section than the corresponding amount of water, the decay in the counters will be more rapid than in the water. This effect was investigated by comparing the decay curves obtained at three positions in the tank with the brass counters and with aluminum counters filled to such pressure of BF_3 that they have the same absorption cross section per volume as water. This measurement yielded a correction of $+5.5$ μsec with an uncertainty of 2 μsec. Because of the absence of moderating nuclei in the counter, a depression in neutron density will develop during the first intervals of time when the neutrons are being slowed down. As soon as the neutrons have become thermal this hole will, however, quickly fill up by diffusion. The effect of the absence of scattering in the counter once the neutrons have become thermal should be proportional to the Laplacian of the neutron density and will disappear in the integration over the whole tank. These considerations were verified by comparing the decay curves measured with two aluminum counters of different volume, both having the same absorption as water. The presence of a depression in the larger counter during the first intervals of time after the neutron burst from the source was clearly visible, but after about 50 μsec the decay curves became identical. In calculating the period, the initial part of the decay curve is therefore discarded.

For the corrected mean life in distilled water at 22.6°C, we thus obtain 212 ± 3 μsec, and the corresponding neutron-proton capture cross section for a neutron velocity of 2200 m/sec is 0.321 ± 0.005 barns, in satisfactory agreement with the most recent determinations by quite different methods of Hamermesh, Ringo, and Wexler,[2] 0.329 ± 0.004 barn, and of Muehlhause *et al.*,[3] 0.332

±0.007 barn. The uncertainty is almost entirely the result of the background correction and the correction for counter absorption and should be decreased considerably in future measurements.

The controversial results of recent determinations of the cross section of boron,[4–7] which has been in widespread use as a standard for cross sections, makes a more reliable standard desirable. The knowledge of the neutron-proton capture cross section with considerable accuracy would make water well suited as a primary standard, due to its general availability, well defined nuclear properties, and the ease with which it can be produced in a pure state. For applications where the low absorption and high scattering cross section of water is a disadvantage, secondary standards of boron or other highly absorbing compounds can be prepared by relative measurements.[8] We have prepared such a secondary standard in the form of a solution of 8.83 g borax per liter of water and have for this solution measured a neutron mean life of 74.5 ±0.5 μsec, corresponding to a cross section of 0.0610±0.0004 cm^2 per cm^3 solution. If necessary, the solution may be concentrated by evaporation if account is taken of the change in cross section due to the evaporated water. In the measurements on the borax solution the counters were chosen to have the same absorption as the solution and should thus not give rise to any depression in the neutron density. The major uncertainty is the result of the background correction. The measured value corresponds to a boron absorption cross section of 708±12 barns, where account has been taken in the probable error of the uncertainty of the chemical analysis of the solution, which was about 1 percent. While this value is in good agreement with some of the old values for the cross section,[4] it disagrees with the more recent determinations[6,7] which give much higher values. It is proposed to investigate this problem in more detail by making comparison measurements between our standard solution and samples of the boron used by other investigators.

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* On leave from North Carolina State College, Raleigh, North Carolina.

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The Construction of Potentials in Quantum Field Theory

ABRAHAM KLEIN*
Physics Department, University of Rochester, Rochester, New York

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IN a paper with the above title, Brueckner and Watson[1] have given a method for constructing scattering kernels in quantum field theories by means of an inductive technique starting from the Lippmann-Schwinger integral equation.[2] We wish to show that the same results can be achieved quite simply by a specified method of successive elimination of the components of the state vector from the usual Schrödinger equation.

We use the notation of reference 1. For the sake of simplicity, we limit ourselves here to the case of a coupling linear in the meson variables and without nucleon pair production. The Schrödinger equation is

$$a\psi = H'\psi. \tag{1}$$

Let $\psi=\psi_e+\psi_o$, where ψ_e contains only amplitudes for an even number (including zero) of mesons and ψ_o only amplitudes for an odd number. In virtue of the properties of H', we can write in place of Eq. (1),

$$\begin{aligned} a\psi_e &= H'\psi_o, \\ a\psi_o &= H'\psi_e. \end{aligned} \tag{2}$$

Eliminating ψ_o (thinking of the two-nucleon problem for definiteness), we obtain the equation

$$a\psi = H'a^{-1}H'\psi = \Delta_0\psi, \tag{3}$$

having dropped the subscript on ψ.

Again we divide the remaining ψ into an even and an odd part, the even part consisting now of the amplitudes for $4l$ mesons ($l=0, 1, 2\cdots$), the odd part embracing the remaining amplitudes. The effective interaction,[1] Δ_0, has both a diagonal part V_0 and an off-diagonal part U_0. Equation (3) then becomes

$$\begin{aligned} (a-V_0)\psi_e &= U_0\psi_o, \\ (a-V_0)\psi_o &= U_0\psi_e, \end{aligned} \tag{4}$$

or, after elimination of the odd part,

$$(a-V_0)\psi = [U_0(a-V_0)^{-1}U_0]\psi \equiv \Delta_1\psi. \tag{5}$$

This procedure can be repeated any number of times. After the first step the amplitude for no mesons is coupled only to the amplitude for two mesons, after the nth step only to the amplitude for 2^n mesons. In the limit as n increases indefinitely, we obtain an ordinary Schrödinger equation,

$$a\psi = V\psi, \tag{6}$$

where V is the infinite (nonperturbation) series diagonal in occupation numbers defined in reference 1.

The same method can be applied to non-linear interactions and to nucleon-pair production. As soon as one departs from the case of a Hamiltonian linear in creation and absorption operators, however, the resulting solution is only a formal one, since the energy denominators of the "potential" are still nondiagonal in occupation numbers.[1] This corresponds to the fact that in the general case, at any stage of elimination, a given amplitude is coupled to members of both the even and odd set. In any practical calculation, where one includes only a few amplitudes, the most suitable procedure is to work directly with the coupled equations and eliminate alternate components. This yields an effective potential diagonal in occupation numbers which can be obtained from the formal solution only by additional manipulation of the latter.

Finally, we wish to point out that the method of integral equations proposed by Lévy[3] can also be expressed simply by the symbolic means used in this note. The relationship between the different methods will be pursued further in a more complete communication.

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* On leave for the summer of 1953 from the Society of Fellows, Harvard University.

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Deficiency of the Phenomenological Theory of the Lamb Shift

FREDERICK J. BELINFANTE
Department of Physics, Purdue University, Lafayette, Indiana

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DR. W. E. Lamb has kindly drawn my attention to the fact that Eqs. (1) and (2) with (17) of Bethe, Brown, and Stehn[1] do not give the complete dependence of the Lamb shift on the atomic number Z. Instead of

$$\begin{aligned} \delta E_{nlj} = (Z^4\alpha^3 Ry/\pi n^3)\cdot\{c_{lj}/(2l+1) \\ +(8/3)[\ln(Ry/k_{nl})+10.0988\delta_{l,0}]\}+\text{corrections}, \end{aligned} \tag{1}$$

with $c_{lj}=(-1)^{l-j+\frac{1}{2}}/(j+\frac{1}{2})$ and with

$$\ln|Ry/k_{nl}| \approx -2.812\delta_{l,0}, \tag{2}$$

as derived from these equations,[2] the correct quantum-electrodynamical formula for the Lamb shift is

$$\begin{aligned} \delta E_{nlj} = (Z^4\alpha^3 Ry/\pi n^3)\cdot\{c_{lj}/(2l+1)+(8/3)[\ln(Ry/k_{nl}) \\ +(10.0988-2\ln Z)\delta_{l,0}]\}+\text{corrections}. \end{aligned} \tag{3}$$