Cross Terms in Allowed Shape β Spectra*

J. P. DAVIDSON AND D. C. PEASLEE
Columbia University, New York, New York

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A statistical analysis is made of allowed shape β spectra to determine the magnitude of any $1/W$ terms that may be present. Both allowed and first forbidden spectra are analyzed; only the allowed spectra are discussed here. It is concluded that the (T, A) mixture in β decay is $\lesssim 4$ percent. Unfortunately only one measurement can be used for the (S, V) mixture, and it does not limit the mixture to less than ~ 10 percent. For neither (T, A) nor (S, V) is the mixture significantly different from zero.

I. INTRODUCTION

THE object of the present note is to determine the maximum admixture of β-decay interactions $(S+V)$ or $(T+A)$ that is consistent with the observed absence of $1/W$ terms in allowed spectrum shapes.

In allowed β decay, the spectrum shape is given by a unique function,

$$N(W, W_0)dW = F_0(Z, W)(W_0-W)^2 p dW, \quad (1)$$

where p, W are the electron momentum and energy in mc^2 units, W_0 is the spectrum end-point energy, and $F_0(Z, W)$ is a specific function[1] of charge and energy. It is customary to plot the quantity $n=(N/pF)^{\frac{1}{2}}$ as a function of W, obtaining a straight line that extrapolates to the energy W_0.

If the β-decay interaction contains an arbitrary mixture of $SVTA$ terms, Eq. (1) must be modified by writing[2]

$$n=C(W_0-W)\left\{\left|\int 1\right|^2(1+R/W)+\left|\int \sigma\right|^2(1+R'/W)\right\}^{\frac{1}{2}}$$
$$=C'(W-W_0)[1+r/W]^{\frac{1}{2}}, \quad (2)$$

where $\int 1$ and $\int \sigma$ are the Fermi and Gamow-Teller matrix elements, including the respective coupling constants. Here

$$R=-2(1-\alpha^2Z^2)^{\frac{1}{2}}\lambda_S\lambda_V[\lambda_S^2+\lambda_V^2]^{-1},$$
$$R'=-2(1-\alpha^2Z^2)^{\frac{1}{2}}\lambda_A\lambda_T[\lambda_A^2+\lambda_T^2]^{-1}, \quad (3)$$

where the λ's are coupling constants.

II. NUMERICAL ANALYSIS

For the analysis of spectrum shapes, the standard plot of n is rather insensitive because of the dominant (W_0-W) term. When this factor is divided out, deviations from linearity as well as the experimental fluctuations are greatly magnified. The experimental quantity used for the analysis is therefore

$$m(W)=[n/(W_0-W)]^2=(a+b/W). \quad (4)$$

A particular spectrum is fitted by minimizing $\sum_j(a+b/w_j-m_j)^2$ with respect to a and b, whence

$$r=b/a=(MK-NJ)/(NK-ML),$$
$$J=\sum_j 1, \quad K=\sum_j 1/W_j, \quad L=\sum_j 1/W_j^2, \quad (5)$$
$$M=\sum_j m_j, \quad N=\sum_j m_j/W_j.$$

The summation in (5) over j includes all the experimental points considered in a given spectrum. Usually m_j deviate badly at both ends of the spectrum for experimental reasons, and some criterion is necessary for selecting the points to be used. The following procedure is adopted here: after discarding by inspection values of m_j differing by an order of magnitude from the average of any spectrum, the mean value $\langle m_j\rangle_{\text{Av}}$ of the remaining points is computed. From the center of the spectrum, which is experimentally most reliable, is selected the largest continuous range in which no individual point m_j deviates by more than a prescribed amount x from $\langle m_j\rangle_{\text{Av}}$:

$$(1-x)\langle m_j\rangle_{\text{Av}} \leq m_j \leq (1+x)\langle m_j\rangle_{\text{Av}}. \quad (6)$$

The quantity x is always taken so that the adopted range includes the majority of the spectrum. For most of the spectra analyzed, this could be achieved with $x=10$ percent, which was taken as a standard figure; for one or two exceptional cases, however, larger values of x were necessary. Such preselection improves the consistency of the data used at the expense of the total number of independent points in $\sum_j$, thereby reducing the statistical reliability of the fit (5).

The error in the estimate (5) for n comes from the errors Δ_j in the original points n_j, and the error Δ_0 in the choice of W_0 to convert the data from n_j to m_j. Considering these errors as all independent and assuming a constant fractional error for the points $\Delta_j=\gamma n_j$, we have

$$\Delta_r^2=Q^2\{\gamma^2\sum_j m_j^2(N-M/W_j)^2$$
$$+(\Delta_0/W_0)^2[\sum_j m_j'(N-M/W_j)]^2\}$$
$$Q=(K^2-JL)/(NK-ML)^2, \quad (7)$$
$$m_j'=-m_jW_0/(W_0-W_j).$$

The calculations for a number of allowed-shape spectra are summarized in Table I. About half the cases considered are presumably 1st forbidden tran-

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[1] E. Fermi, Z. Physik **88**, 161 (1934).

[2] M. Fierz, Z. Physik **104**, 553 (1937).

sitions.† The column Δ_r in Table I is computed by assuming $\gamma=2$ percent, $\Delta_0/W_0=1$ percent; the column Δ_r' has $\gamma=0.5x$, $\Delta_0/W_0=0.2x$.

III. DISCUSSION

We discuss only the allowed transitions, which form the first part of Table I. Using the error Δ_r', we have

$$\Delta_r' \gtrsim |r| \tag{8}$$

for all members of the table. That is, by assuming generous but not unreasonable errors (the points used were all read off published graphs) we conclude that none of the values of r is significantly different from zero.

Consider the $\Delta I=1$ transitions, which contain only the Gamow-Teller terms T and A and comprise all but the first case in Table I. Deviations in the spectrum shape caused by the finite size of the nucleus[3] contribute an effective $r\approx\pm0.01$ in the cases considered and can be neglected here. Screening corrections[4] are of the same order of magnitude for the β^- emitters but are an order of magnitude larger for the β^+ spectrum of Cu^{64}. The Gamow-Teller cases in Table I that do not require substantial corrections and have small straggling of the experimental points ($x=0.10$) are P^{32}, $Cu^{64\,d,f}(\beta^-)$. The mean of these cases weighted with Δ_r is

$$r=0.00\pm0.08. \tag{9}$$

The lower limit of our knowledge of r is set by the error term in (9); comparing with (3), we may say that the (T, A) mixture in the β-decay interaction is at most about 4 percent.

To obtain similar information on the mixture of S and V, we must use transitions with $\Delta I=0$, $\Delta T=0$, where T is the total isotopic spin of the nucleus. This second condition rules out cases like S^{35} and in fact restricts us to mirror level transitions. These are generally high-energy positron emitters of short lifetime, which are therefore difficult to measure. Only the N^{13} spectrum is available, from which

$$r\approx0.1\pm0.1. \tag{10}$$

This r combines that of the Fermi term and the Gamow-Teller term, which is also present. For this $p_{\frac{1}{2}}$ transition, however, the Gamow-Teller contribution is considerably smaller than the Fermi part, so that the (S, V) mixture is specified by (10).‡

Measurements of additional mirror level transitions in odd A nuclei would be helpful in reducing this upper limit for the (S, V) mixture. High precision is essential, especially in determining W_0. The most favorable transitions are those with small values of the Gamow-Teller matrix element $\int\boldsymbol{\sigma}$, as discussed in relation to the Fermi term in $\beta-\nu$ correlation.[5]

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TABLE I. Analysis of $1/W$ terms in allowed shape spectra.

Decay	E_0 (Mev)†	x	r	Δ_r	Δ_r'
(A) Allowed transitions					
N^{13} [a]	+1.200	0.10	0.25	0.27	0.53
He^6 [b]	3.500	0.25	0.66	0.24	1.24
P^{32} [c]	1.698	0.10	−0.07	0.09	0.19
Cu^{64} [d]	0.582	0.10	0.14	0.17	0.34
Cu^{64} [e]	0.582	0.15	−0.07	0.03	0.12
Cu^{64} [f]	0.571	0.10	0.18	0.24	0.49
Cu^{64} [d]	+0.661	0.15	−0.31	0.24	0.74
Cu^{64} [f]	+0.657	0.10	−0.08	0.16	0.36
(B) First forbidden transitions					
Ba^{139} [g]	2.270	0.10	0.22	0.11	0.24
Pr^{143} [h]	0.932	0.10	0.01	0.11	0.23
Pm^{147} [i]	0.227	0.10	−0.22	0.47	0.96
Hf^{181} [j]	0.404	0.10	0.38	0.36	0.71
Hf^{181} [k]	0.412	0.10	−0.21	0.19	0.43
Re^{186} [l]	1.073	0.15	0.13	0.15	0.46
Re^{186} [c]	1.063	0.10	0.07	0.16	0.32
Au^{198} [m]	0.975	0.10	0.38	0.48	0.97
Au^{198} [e]	0.956	0.10	−0.10	0.08	0.17

† Energies with a + sign indicate β^+ decay.
[a] W. F. Hornyak and T. Lauritsen, Phys. Rev. **77**, 160 (1950).
[b] Wu, Rustad, Perez-Mendez, and Lidofsky, Phys. Rev. **87**, 1140 (1952).
[c] L. M. Langer and H. C. Price, Jr., Phys. Rev. **76**, 641 (1949).
[d] Langer, Moffat, and Price, Phys. Rev. **76**, 1725 (1949).
[e] Same as d, but with thicker source.
[f] G. E. Owen and C. S. Cook, Phys. Rev. **76**, 1726 (1949).
[g] L. R. Shepard and J. M. Hill, Nature **162**, 566 (1948).
[h] Feldman, Lidofsky, Macklin, and Wu, Phys. Rev. **76**, 1888 (1949).
[i] Lidofsky, Macklin, and Wu, Phys. Rev. **76**, 1888 (1949).
[j] K. Y. Chu and M. L. Wiedenbeck, Phys. Rev. **75**, 226 (1949).
[k] C. Y. Fan, Phys. Rev. **87**, 252 (1952).
[l] Beach, Peacock, and Wilkinson, Phys. Rev. **76**, 1585 (1949).
[m] Steffen, Huber, and Humbel, Helv. Phys. Acta **22**, 167 (1949).

† These are discussed in a separate publication, D. C. Peaslee, Phys. Rev. (to be published).
[3] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953).
[4] J. R. Reitz, Phys. Rev. **77**, 10 (1950).

‡ The quantity r expresses directly the mixture of interactions, according to (3). If the square of this quantity is used as a measure of the mixture [H. M. Mahmoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952)], the mixture parameter becomes $\lesssim0.2$ percent for (T, A) and is of order 1 percent for (S,V).
[5] D. C. Peaslee, Phys. Rev. **89**, 1148 (1953).