microwave spectrometer at the Columbia Radiation Laboratory have been in general agreement with the above calculations. As an example, we cite the recent experimental work of Gunther-Mohr and White on an additional fine structure in the ammonia quadrupole spectrum,[8] where the measured total width at half-maximum for NH_3 at dry ice temperature in X-band Stark guide was found to be 68 ± 5 kc/sec. The calculated Doppler broadening and collision broadening for NH_3 under these conditions are, respectively, $2\Delta\nu_{\mathrm{Doppler}}=60$ kc/sec and $2\Delta\nu_{\mathrm{coll}}=25$ kc/sec. Born has tabulated the resultant shape for a Lorentz line broadened by Doppler effect.[9] His tabulated results show that the total line width is given to a good approximation by

$$\Delta\nu\simeq[(\Delta\nu_{\mathrm{Doppler}})^2+(\Delta\nu_{\mathrm{coll}})^2]^{\frac{1}{2}}. \tag{15}$$

Extrapolating from the results of the present paper, one is led to expect that in practice, quite generally (and at least for a rectangular wave guide) the line shape of broadening due to collisions with the wall will be sufficiently close to that of a Lorentz line so that expression (15) can be used. Thus the combined theoretical line width is $2\Delta\nu=(60^2+25^2)^{\frac{1}{2}}=65$ kc/sec in agreement with the observed value.

We wish to thank Professor Townes for his active aid. We also wish to thank Professor Strandberg for an interesting discussion. The help of Mr. George Dousmanis who performed the numerical calculations is gratefully acknowledged.

[8] G. R. Gunther-Mohr and R. White (to be published).
[9] M. Born, *Optik* (J. Springer, Berlin, 1933), Table 38, p. 486, and p. 431 and the following.

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Polarizability of the Deuteron*

N. F. RAMSEY, B. J. MALENKA, AND U. E. KRUSE
Harvard University, Cambridge, Massachusetts
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The theory of the polarizability of the deuteron in a uniform electric field is developed. Using the properties of the deuteron Green's function, we obtain an expression for polarizability of the deuteron which exhibits its dependence upon the spin orientation of the deuteron. This dependence arises from the inclusion of a tensor force in the neutron-proton interaction. In terms of the magnetic quantum number m, with respect to the direction of the electric field, the polarizability α is $\alpha_{SS}+(3m^2-2)\alpha_{SD}+(3m^2-2)^2\alpha_{DD}$. When Hulthén wave functions are used for the deuteron, α_{SS} is found to be approximately 0.56×10^{-39} cm^3 and α_{SD} 0.027×10^{-39} cm^3. The applicability of the theory to intramolecular interaction measurements and deuteron scattering experiments is discussed.

I. INTRODUCTION

THE particles which constitute the deuteron do not all have the same ratio of charge to mass so that, because of reorientation and stretching, the deuteron will exhibit a polarizability with respect to an external electric field.

It has been pointed out earlier by Ramsey[1] that this polarizability of the deuteron should give rise to a measurable departure from Rutherford scattering in certain cases where deuterons are scattered by heavy nuclei. In addition, the dependence of the deuteron polarizability upon its spin orientation should also give rise to a small change in the deuteron quadrupole interaction in D_2 and HD as a result of the difference is amplitude of zero-point vibration in the two molecules and consequently of the oscillating electric field at the deuteron.

In this paper, we calculate the polarizability of the deuteron in an adiabatically applied uniform external electric field. The electric scattering of the deuteron will be examined in a subsequent paper.

The polarizability of a nucleus is related to its polarization energy W_p, in an adiabatically applied, uniform electric field $\mathcal{E}$, by the equation[2]

$$\alpha=-2W_p/\mathcal{E}^2. \tag{1}$$

The W_p is the energy of the second-order Stark effect arising from the perturbation

$$V=-\tfrac{1}{2}ez\mathcal{E}, \tag{2}$$

where z is the component along the direction of the electric field $\mathcal{E}$ of the relative distance $\mathbf{r}=\mathbf{r}_p-\mathbf{r}_n$ of the proton from the neutron. The factor $\frac{1}{2}$ enters because we are concerned with the displacement of the proton with respect to the center of mass of the deuteron. The polarization energy is then

$$W_p=-\tfrac{1}{4}e^2\mathcal{E}^2\sum_{n\neq0}{}'(0|z|n)(n|z|0)/(E_n-E_0), \tag{3}$$

where $\sum'_{n\neq0}$ represents the sum over all the discrete and continuum intermediate states except the ground

* This work was partially supported by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.
[1] N. F. Ramsey, Phys. Rev. **83**, 659 (1951).
[2] D. Bohm, *Quantum Theory* (Prentice Hall, Inc., New York, 1951) p. 461.

state, $n=0$, so that

$$\alpha=\tfrac{1}{2}e^2(2\mu/\hbar^2)\sum_{n\neq 0}{}'(0|z|n)(n|z|0)/(k_n^2+\gamma^2), \qquad (4)$$

where $k_n^2=(2\mu/\hbar^2)E_n$ and $\gamma^2=(2\mu/\hbar^2)\epsilon$. As usual, $\epsilon=-E_0$ denotes the binding energy and μ the reduced mass of the deuteron.

In this paper, we will take the ground state of the deuteron to be the usual admixture of $^3S_1$ and approximately 4 percent of $^3D_1$ states. We note that for the deuteron, as described in terms of the customary central and tensor interactions, parity is a good quantum number. The ground state then being of a definite (even) parity, we have $(0|z|0)=0$. If we now consider ϵ to be slightly different from the binding energy, we can then extend the summation in Eq. (4) over the complete set of deuteron wave functions. Then writing the wave function explicitly,[3] we can express (4) in the form

$$\alpha=\tfrac{1}{2}e^2(2\mu/\hbar^2)\int\int\Psi_0^*(\mathbf{r})zG_d(\mathbf{r},\mathbf{r}')z'\Psi_0(\mathbf{r}')(d\tau)(d\tau'), \qquad (5)$$

where

$$G_d(\mathbf{r},\mathbf{r}')=\sum_n{}'\frac{\Psi_n(\mathbf{r})\Psi_n^*(\mathbf{r}')}{k_n^2+\gamma^2}. \qquad (6)$$

This sum of the bilinear product over the complete set of deuteron wave functions will be recognized to be the Green's function satisfying the Schrödinger equation for the deuteron,

$$[-\nabla^2+\gamma^2+(2\mu/\hbar^2)V_d(\mathbf{r})]G_d(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'). \qquad (7)$$

Here $V_d(\mathbf{r})$ is the relative proton-neutron potential in the deuteron.

To simplify the evaluation of α in Eq. (5), we note that G_d is related to the free particle Green's function G by the equation

$$G_d(\mathbf{r},\mathbf{r}')=G(\mathbf{r},\mathbf{r}')-(2\mu/\hbar^2)\int G(\mathbf{r},\mathbf{r}'')V_d(\mathbf{r}'')G_d(\mathbf{r}'',\mathbf{r}')(d\tau''). \qquad (8)$$

This relation can be easily verified by operating on both sides with $-\nabla^2+\gamma^2$. If we put this expression for G_d into Eq. (5), we observe that the contribution of the second term in the right-hand side of Eq. (8) to α must be small since the short-range potential $V_d(\mathbf{r})$ contributes mainly for S states but the z matrices vanish for states of even parity and hence annul any such contribution. We can then make the approximation

$$\alpha\approx\tfrac{1}{2}e^2(2\mu/\hbar^2)\int\int\Psi_0^*(\mathbf{r})zG(\mathbf{r},\mathbf{r}')z'\times\Psi_0(\mathbf{r}')(d\tau)(d\tau'). \qquad (9)$$

Explicitly including the effect of spin, the wave function with magnetic quantum number m for the ground state of the deuteron can be written as[4]

$$\Psi_0(r)\rightarrow\psi_{1,m}(\mathbf{r},\sigma)=(4\pi)^{-\frac{1}{2}}[u(r)/r+2^{-\frac{3}{2}}S_{12}w(r)/r]\chi_{1,m}(\sigma), \qquad (10)$$

where $S_{12}=[3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})/r^2]-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ and $\chi_{1,m}(\sigma)$ is the spin wave function. The free particle Green's function is

$$G(r,r')\rightarrow G_{\sigma,\sigma'}(\mathbf{r},\mathbf{r}')=(2\pi)^{-3}\int(k^2+\gamma^2)^{-1}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}(d\mathbf{k})\times\sum_{m'}\chi_{1,m'}(\sigma)\chi_{1,m'}^*(\sigma'), \qquad (11)$$

where the usual form[5] of G is somewhat modified by the inclusion of the spin wave functions.

We then evaluate Eq. (9), making use of the relations

$$\sum_\sigma\chi_{1,m}^*(\sigma)\chi_{1,m'}(\sigma)=\delta_{m,m'},$$

$$\sum_\sigma S_{12}^*\chi_{1,m}^*(\sigma)\chi_{1,m'}(\sigma)=\sum_\sigma\chi_{1,m}^*(\sigma)S_{12}\chi_{1,m}(\sigma)\delta_{m,m'}=(3m^2-2)(3\cos^2\theta-1)\delta_{m,m'}, \qquad (12)$$

where θ is the angle between r and this z axis. We find

$$\alpha=\alpha_{SS}+(3m^2-2)\alpha_{SD}+(3m^2-2)^2\alpha_{DD}, \qquad (13)$$

where

$$\alpha_{SS}=(e^2/64\pi^4)(2\mu/\hbar^2)\int(k^2+\gamma^2)^{-1}\times\left|\int u(r)\cos\theta e^{i\mathbf{k}\cdot\mathbf{r}}(d\tau)\right|^2(d\mathbf{k}), \qquad (14)$$

$$\alpha_{SD}=(e^2/64\pi^4\sqrt{2})(2\mu/\hbar^2)\int(k^2+\gamma^2)^{-1}\times\left[\int u(r)\cos\theta e^{i\mathbf{k}\cdot\mathbf{r}}(d\tau)\int w(r')\cos\theta'\times(3\cos^2\theta'-1)e^{-i\mathbf{k}\cdot\mathbf{r}'}(d\tau')\right](d\mathbf{k}), \qquad (15)$$

$$\alpha_{DD}=(e^2/512\pi^4)(2\mu/\hbar^2)\int(k^2+\gamma^2)^{-1}\times\left|\int w(r)\cos\theta(3\cos^2\theta-1)e^{i\mathbf{k}\cdot\mathbf{r}}(d\tau)\right|^2(d\mathbf{k}). \qquad (16)$$

Since we are assuming the admixture of the deuteron ground state to be approximately 96 percent triplet S state, most of the contribution to α comes from the α_{SS} term.

To evaluate the α's we take $u(r)$ to the Hulthén[4] wave function,

$$u(r)=Ne^{-\gamma r}(1-e^{-r/R})=N(e^{-\gamma r}-e^{-\Gamma r}), \qquad (17)$$

where R is the Hulthén range of the nuclear force and $\Gamma=\gamma+1/R$.

For $w(r)$, we use the form that is appropriate outside the range nuclear forces,

$$w(r)\approx N'e^{-\gamma r}[3(\gamma r)^{-2}+3(\gamma r)^{-1}+1], \qquad (18)$$

which although incorrect for small r, will only slightly alter the value of the already small α_{SD} and still smaller α_{DD} terms since the r and r' integrals contain factors of

[3] The appropriate spin indices and summations are understood.

[4] H. Feshback and J. Schwinger, Phys. Rev. 84, 194 (1951).

[5] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), pp. 160–161.

r^2 and r'^2, respectively. The N and N' are normalization factors. In carrying out the indicated integration, it is convenient to do the angular and radial integrals first. The final k integration can then be evaluated by the method of residues.

Neglecting the small α_{DD} term, we find

$$\alpha_{SS}=A\gamma^{-3}(1+3\gamma R+2(\gamma R)^2)\{1-[1+(\Gamma/\gamma)]^{-4} \times(32-(8/3)(\Gamma/\gamma)^{-3}[1+4(\Gamma/\gamma)+(\Gamma/\gamma)^2])\}, \quad (19)$$

where $A=(32)^{-1}(1-P_D)(e^2/\hbar c)(2\mu c/\hbar)\gamma^{-1}$ and P_D is the ${}^3D_1$ state probability. Similarly we find

$$\alpha_{SD}=NN'2^{-9.2}(e^2/\hbar c)(2\mu c/\hbar)\gamma^{-5} \times\{1-(2^4/5)[1+(\Gamma/\gamma)]^{-4}[3+2(\Gamma/\gamma)]\} \approx 2^{-3}Q\gamma^{-2}(e^2/\hbar c)(2\mu c/\hbar)[1+(6/5)\gamma R], \quad (20)$$

to first order in γR, where Q is the deuteron quadrupole moment.[6] Taking $\Gamma/\gamma\approx 7$, and hence $\gamma R\approx 0.17$,[7] we find

$$\alpha_{SS}\approx 0.56\times 10^{-39}\text{ cm}^3, \quad (21)$$

$$\alpha_{SD}\approx 0.027\times 10^{-39}\text{ cm}^3. \quad (22)$$

We see that, as expected, the α_{SD} term is considerably smaller than the α_{SS} term; also, making a rough calculation, we find that neglecting the second term in Eq. (8) for the Green's function, apparently alters the value of α_{SS} by only about 2 percent so that the free particle Green's function approximation is apparently valid for this calculation.

III. APPLICATIONS TO DEUTERON QUADRUPOLE MEASUREMENTS AND TO ELECTRIC SCATTERING EXPERIMENTS

A possible manifestation of the deuteron polarizability might be the molecular beam experiments which measure the deuteron electric quadrupole moment.[8,9] When the deuteron is in a molecule, such as D_2, it is subject to zero-point oscillations and consequently is acted upon by an electric field which has a nonvanishing expectation value for $\langle\mathcal{E}^2\rangle$ even though $\langle\mathcal{E}\rangle$ vanishes. As a result and by Eq. (1), the deuteron will possess a polarization energy. Since, by Eq. (13), the polarizability depends upon the orientation of the deuteron spin, the polarizability energy will vary with the orientation. Furthermore, the polarizability relations in Eq. (13) depends upon m^2 and not upon m and in this respect is similar to the deuteron electric quadrupole interactions. Consequently, the deuteron polarizability will give rise to a change in the apparent magnitude of the deuteron quadrupole moment.

At first sight, it might be thought that this polarizability effect could not be distinguished from the deuteron quadrupole moment. However, the apparent deuteron quadrupole moment can be measured both in HD and D_2. Since the amplitudes of zero-point vibration in these two molecules are different, the values of $\langle\mathcal{E}^2\rangle$ and of polarization energy will differ. Consequently, the anisotropy of the deuteron polarizability can, at least in principle, be measured by the variation in the apparent deuteron quadrupole moment when measured in D_2 and HD.

The above analysis can be made quantitative, as follows. From Eqs. (1) and (13), if $W_{p,m}$ is the polarization energy in nuclear orientation state m,

$$W_{p,1}-W_{p,0}=-\tfrac{1}{2}(3\alpha_{SD}-3\alpha_{DD})\langle\mathcal{E}^2\rangle. \quad (23)$$

On the other hand, as discussed in the literature,[10] one can easily show that the energy difference for the same transition for an effective nuclear electric quadrupole moment Q' is

$$W_{q,1}-W_{q,0}=\tfrac{3}{4}eQ'(\partial^2V^e/\partial z_0^2). \quad (24)$$

Consequently, from Eqs. (23) and (24), the apparent quadrupole interaction caused by the nuclear polarizability becomes

$$h^{-1}eQ'(\partial^2V^e/\partial z_0^2) = h^{-1}\langle\mathcal{E}^2\rangle[-2\alpha_{SD}+2\alpha_{DD}] = (hc/e^2)^3 8\pi^3(\mu/m)^2\omega_e^4 c(R_e/a_0)^2Z^{-2} \times[(B_e/\omega_e)+(B_e/\omega_e)^2(aR_e)^2][-2\alpha_{SD}+2\alpha_{DD}], \quad (25)$$

where the value for $\langle\mathcal{E}^2\rangle$ which can be calculated by normal procedures[11] from the zero-point vibration of the molecules has been substituted in the last form of the equation, and where the symbols used have their conventional meaning.[11] If numerical values are substituted in Eq. (25), the apparent quadrupole interaction arising from the deuteron polarizability becomes 0.0075 cps for HD and 2.0060 cps for D_2, which is unfortunately much too small to be detected with the present experimental accuracy.

Another and more favorable way in which the deuteron polarizability should be observable is in the electric scattering of deuterons by highly charged nuclei. If the deuteron is polarized by the electric field of the scattering nucleus, it will be less deflected than if the deuteron were not polarizable. It can be shown that 8-Mev deuterons with the above calculated polarizability, when scattered by Bi, should have their scattering cross section in the backward direction reduced below the Rutherford scattering by 3 percent as a result of the deuteron polarizability. At such energies and with such a highly charged *a* nucleus, the effects of nuclear interaction, deuteron disintegration, etc., should be small compared to the 3 percent polarizability effect on the scattering. A detailed discussion of this application will be given in an accompanying paper.[12]

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[6] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 105.

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