$\theta=90°$, $\varphi=45°$. Values of z_i at angular intervals of 15° on the boundary are computed with Eqs. (7) and (8); values at the same intervals on the meridians $\varphi=15°$, $\varphi=30°$ are obtained by solution of the cubic Eq. (4).[8] The corresponding mean velocities v_{mi} are computed with the trapezoidal formula, and then v_m with Eq. (2).

[8] A. Zavrotsky, *Tablas para la Resolucion de las Ecuaciones Cubicas* (Editorial Standard, Caracas, 1945), contains the real and complex roots of the reduced cubic [Eq. (9)] to six figures, for $-100<p,q<+100$, with the interval unity.

Röhl obtained by this method the mean Debye temperatures 158°K for gold and 212°K for silver at room temperature.[9] The present method *applied to Röhl's data*, yields the values 157.6°K and 211.3°K, respectively.

In conclusion, the authors gratefully acknowledge their indebtedness to the Watson Scientific Computing Laboratory of Columbia University for an especially prepared, six-place, differenced table of values of $x^{-\frac{3}{2}}$.

[9] H. Röhl, Ann. phys. **16**, 887 (1933).

PHYSICAL REVIEW VOLUME 91, NUMBER 5 SEPTEMBER 1, 1953

Deflection of High-Energy Electrons in Magnetized Iron*

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(Received March 23, 1953)

High-energy electrons are scattered in magnetized iron. From a shift in the multiple scattering curves due to the reversal of the magnetic field, the effective field b_{eff} acting on electrons traversing ferromagnetic media is computed. Such an effective field is dependent on the short range forces between beam electrons and spin-aligned ferromagnetic electrons. The measurements show an effective field equal to the macroscopically measured flux density B.

THE question as to the effective magnetic field acting on charged particles traversing a ferromagnetic medium was first raised in connection with cosmic-ray deflection experiments.[1] The most complete theoretical discussion of the problem is given by Wannier,[2] who translated into quantum-mechanical language the objections, first raised by Swann[3] in classical terms, against the supposition that the effective field is necessarily equal to the macroscopically defined flux density B. Wannier's theoretical conclusions can be summed up as follows.

Since the magnetization in a ferromagnet is due to the electron spin, the macroscopically defined flux density B is the result of an average over all elementary dipoles (spin-aligned electrons). If a fast charged particle traverses the magnet, it is influenced at each point by a force due to the "true" field at this point. This true field, however, varies over a very wide range of magnitudes within regions of the order of a Compton wavelength around the spin-aligned electrons. The effective field b_{eff} is defined as an average field acting on the particle along its path. It can be shown that, although rare, close range interactions (corresponding to classical "head-on collisions") between the beam particle and the ferromagnetic electrons are decisive in determining this average. Only if all points in the magnet can be given equal statistical weight, will $b_{\text{eff}}=B$. Should short-range forces exist between beam particle and electrons, the effective field will be changed accordingly. This effect is described by introducing a "coincidence probability,"

$$p(r)=\frac{(\text{chance of finding the electron at } r \text{ if beam particle is also at } r)}{(\text{chance of finding the electron at } r \text{ if beam particle is far away})}.$$

The average of the magnetization along the path of the beam particle is taken by first weighting the true magnetization at each point with this coincidence probability $p(r)$, a quantity dependent on the force between beam particle and electrons. Wannier has computed $p(r)$ for the case of Coulomb forces. The final result is expressed by $b_{\text{eff}}=B+2\pi M(p-1)$.

In this formula $p>1$ means attractive forces (for example positrons) $p<1$, replusive ones. It is seen that b_{eff} can be larger than B for the attractive case. The deviation of p from 1 occurs, however, at such low beam energies that it would be hardly verifiable experimentally. Should, however, short-range forces exist, the deviation of b_{eff} from B could be more pronounced than for the pure Coulomb case. An experiment on the effect-

* Supported by the Office of Ordnance Research, U. S. Army.
[1] B. Rossi, Atti accad. Lincei **11**, 478 (1930); L. M. Mott-Smith, Phys. Rev. **39**, 403 (1932); B. Rossi, Nature **128**, 300 (1931).
[2] G. H. Wannier, Phys. Rev. **72**, 304 (1947).
[3] W. F. G. Swann, Phys. Rev. **49**, 574 (1936).

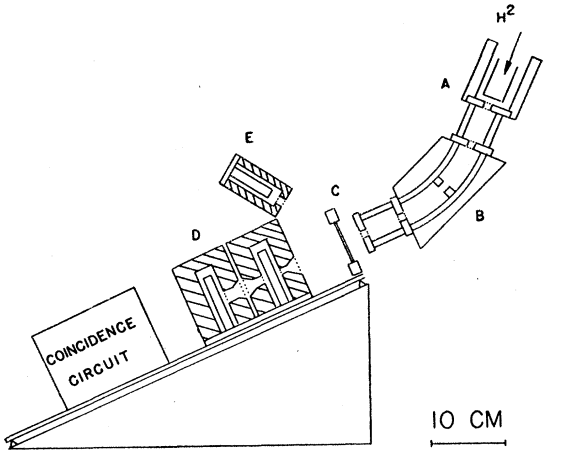

FIG. 1. Experimental setup. A=boron target; B=magnetic analyzer; C=iron scatterer; D=counter telescope; E=monitor. Shaded areas represent lead shielding.

ive field can therefore be thought of as a detection measurement of possible short-range interactions.

Successful experiments on cosmic-ray mu mesons[4] have shown the effective field to be equal to the macroscopically measured B. These experiments use calculated corrections for multiple scattering effects. A more recent experiment[5] eliminates these calculated corrections by measuring directly the change in the multiple scattering distribution of cosmic-ray mesons traversing a block of iron, due to an applied magnetic field. The results verify $b_{\mathrm{eff}}=B$.

The only existing experiment on electron deflection was that of Alvarez,[6] the results of which are inconclusive, showing, however, that at best b_{eff} is much smaller than B. This was most probably due to the great extent multiple scattering competes with the magnetic deflection, and to the way these multiple scattering effects were corrected.[7] Calculations using Wannier's results show that, in order to be able to differentiate the magnetic deflection from Coulomb scattering, higher energies than those used by Alvarez are needed.

EXPERIMENTAL METHOD

The method of the present experiment is indicated in Fig. 1. A small Van de Graaff generator was used to produce high energy electrons from the $B^{11}(d, p)B^{12}$-$(\beta^-)C^{12}$ reaction.[8] The generator was operated at 900 kv and furnished approximately 25 microamperes of deuteron current. The electrons emerging from the boron target (a) were analyzed magnetically and collimated by the analyzer (b) which had a 17.5-cm radius of curvature. The monoenergetic beam of electrons emerging from (b) struck an iron sheet (c) perpendicular to it, which was part of a closed magnetic circuit. The electrons were multiply scattered by the 0.06-cm iron sheet and were counted by the G–M counter telescope (d). The telescope consisted of two thin wall (25 mg/cm²) G–M counters surrounded by lead cylinders with $\frac{1}{4}$-in. collimating slits. It could be rotated in steps of 3° in a plane containing the incident beam about an axis going through the iron scatterer. Counting rates were taken by a coincidence circuit having 1.6×10^{-6} sec resolving time.

In order to calibrate the energy of the incident electrons, a momentum distribution of the B^{12} beta spectrum was first measured with the telescope at 0° and with the scatterer (c) removed. This momentum distribution was then fitted to existing curves[8] and in this way an energy *vs* magnetic analyzer current calibration curve was found. This curve fitted well the one computed from magnetic field measurements and geometric considerations. An angular distribution curve without the iron scatterer was measured and the spread in the collimated beam was found to be less than 3°. Finally, the background counting rate was taken by reversing the field in the analyzer and also stopping the beam with a lead block placed in front of the collimator. It amounted to less than 20 percent of the total counting rate and was not subtracted from the final data since it did not alter the effective magnetic shift.

The actual data were obtained by taking the counting rate at different angles, with the magnetic field in the scatterer in both directions. A single thin wall G–M counter with a lead collimator (e) placed at 0° behind the scatterer and 45° above the telescope was used to monitor the electron flux. Thus, two multiple scattering curves were obtained, displaced from the symmetry position by the magnetic deflection due to the effective field. It was this displacement which was used to

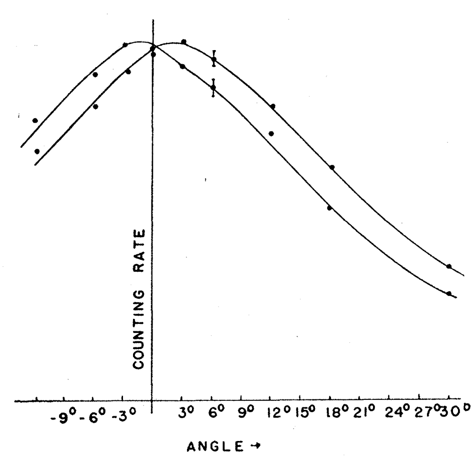

FIG. 2. Magnetically shifted multiple scattering distribution curves $f_+(\theta)$ and $f_-(\theta)$ for incident electron momentum $P_0=7.9$ Mev/c and magnetic field $B=17\,500$ gauss.

[4] F. Rasetti, Phys. Rev. **66**, 1 (1944); G. Bernardini *et al.*, Phys. Rev. **68**, 109 (1945).
[5] S. Berko, Phys. Rev. **86**, 598 (1952).
[6] L. Alvarez, Phys. Rev. **45**, 225 (1934).
[7] Professor L. Alvarez (private communication).
[8] See for example Hornyak, Dougherty, and Lauritsen, Phys. Rev. **74**, 1727 (1948).

compute b_{eff}. As a secondary effect, the displacement of the scattering curves due to hysteresis in the iron scatterer was measured. This was achieved by taking the counting rate at a given angle setting of the telescope as a function of the current in the magnetizing coil of the scatterer.

DISCUSSION OF DATA

As mentioned, it was necessary to use high-energy electrons in order to differentiate between the magnetic deflection and Coulomb multiple scattering. A complete description of the multiple scattering plus magnetic scattering phenomena would have required the solution of a generalized diffusion equation. Such an equation was set up for the case of constant magnetic field by Scott,[9] in connection with a discussion of cloud-chamber experiments. His conclusion was that for small angles the magnetic curvature is simply to be added to the scattering produced deflections. The evaluation of our results rely therefore on the fact that the magnetic deflection is, in a first approximation, independent of multiple scattering. We therefore expect, as a main effect, a total shift of the multiple scattering angular distribution curve by an amount equal to the magnetic deflection. The constancy of the shift with angle was, in first order, verified by our experimental data.

Letting $f_+(\theta)$ be the experimentally measured angular distribution function with the field in the scatterer up, and $f_-(\theta)$ the distribution with the field down, we have

$$\Delta(\theta)=f_+(\theta)-f_-(\theta).$$

In order to be able to detect such a $\Delta(\theta)$, the following conditions had to be satisfied:

(1) The ratio $\Phi/\langle\theta\rangle$, where Φ is the magnetic deflection and $\langle\theta\rangle$ the root-mean-square angle of multiple scattering, had to be as large as possible.

(2) Since we wanted to maximize the measured quantity,

$$\Delta(\theta)=f_+(\theta)-f_-(\theta)=g(\theta+\Phi)-g(\theta-\Phi), \qquad (1)$$

where $g(\theta)$ is the multiple scattering curve without field on, $dg/d\theta$ had to be large, and therefore $\langle\theta\rangle$ small.

(3) In order to have clear-cut effects, the energy of the electron beam had to be smaller than 10 Mev, so that radiation losses in iron would be small.

It has been shown[2] that for $\Phi/\langle\theta\rangle$ to be maximum, a thick target is necessary, such that

$$E\cong\tfrac{1}{4}E_0,$$

where E= energy of electrons when leaving scatterer, E_0= energy of electrons when entering scatterer. Such a scatterer would have had, for $E_0<10$ Mev, a thickness above the electron diffusion limit, and would have led to hardly detectable $\Delta(\theta)$.

In order to satisfy all conditions, a compromise value for the thickness of the scatterer and initial energy of electron beam had to be selected.

[9] W. T. Scott, Phys Rev. **76**, 212 (1949).

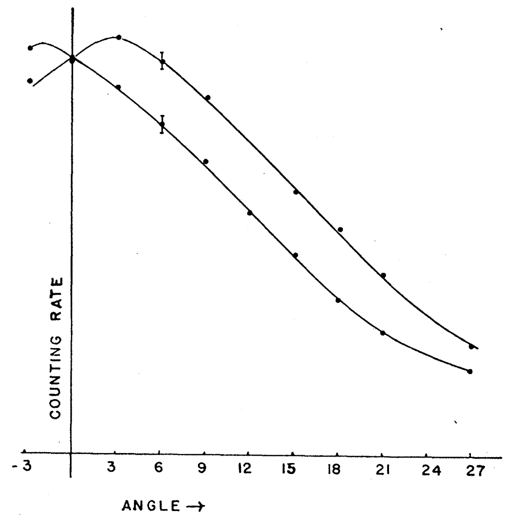

FIG. 3. Magnetically shifted multiple scattering curves $f_+(\theta)$ and $f_-(\theta)$ for incident electron momentum $P_0=8.8$ Mev/c and magnetic field $B=18\,800$ gauss. These curves were used to compute b_{eff} in text.

Figure 2 shows one set of data, used for qualitatively establishing the effect. An improvement in the experimental techniques led to Fig. 3 which is used for the quantitative discussions. The improvement consisted in an increase of the applied magnetic field, an increase in the initial electron energy, and better current normalization.

The 0.06-cm thick scatterer was a Westinghouse iron-cobalt alloy (Hyperco) capable of high magnetic saturation values.

The experimental histograms were approximated by two multiple scattering curves such as to yield the best fit for the condition:

$$f_+(\theta)-f_-(\theta)=g(\theta+\Phi)-g(\theta-\Phi).$$

The angle Φ was obtained by shifting the two scattering curves until they over-lapped, the shift being 2Φ.

Figure 3 yields $\Phi=0.040$ radian for the incident electron momentum of $P_0=8.8$ Mev/c. It was estimated that the error involved in getting Φ by curve matching was ± 3 percent.

Since energy loss is not negligible it had to be corrected for, before b_{eff} could be calculated.

Given the multiple scattering formula, the expression for magnetic deflection and the range energy relation, Wannier computed the magnetic shift Φ and the multiple scattering angle $\langle\theta\rangle$ in terms of the initial and final energy of the electron beam (see formulas 36 and 37 of reference 2).

His formulas express Φ as the difference of two large terms; the precise knowledge of the energy loss in the iron is therefore imperative. As a simpler approximation to these formulas, we can assume for our case linear energy-range dependence.

Taking the known relativistic form of magnetic

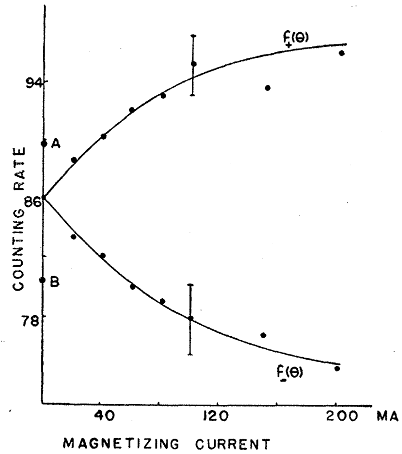

FIG. 4. Magnetization curve and hysteresis effect. $f_+(\theta)$ and $f_-(\theta)$ at $\theta=6°$ as function of magnetizing current. Points A and B are the counting rates after the current was turned off and before demagnetization, and represent the effect due to the retentivity of the iron scatterer.

deflection as

$$d\Phi/dx=300\, b_{\rm eff}/P, \tag{2}$$

and assuming

$$dP/dx=C=\text{const}(\text{ev}/c\cdot\text{cm}),$$

we get for the magnetic deflection due to $b_{\rm eff}$,

$$\Phi=\frac{300\, b_{\rm eff}}{C}\ln\frac{P_0}{P}. \tag{3}$$

In this expression P_0 and P are the initial and final momenta of the electron beam in ev/c, and $b_{\rm eff}$ is given in gauss.

As a first approximation for the multiple scattering angle one gets, for constant dP/dx and $\beta=1$,

$$\langle\theta^2\rangle=(E_s^2/PP_0)(x/X_0) \tag{4}$$

where $E_s=21\times10^6$ ev. X_0 is the radiation length for iron, P_0 and P, are the same as in Eq. (3), and x is the thickness of the iron sheet.

Instead of using the maximum range-energy relations in the above formulas, an average range-energy value was introduced,[10] yielding for iron $C=17.94$ Mev/$c\cdot$cm and $P=7.72$ Mev/c for $P_0=8.8$ Mev/c.

Introducing, then, these values in the approximate formula (3), we finally get $b_{\rm eff}=18\,400$ gauss. The macroscopic magnetic flux density B has been measured with a ballistic galvanometer setup, calibrated with a standard mutual inductance. $B=18\,800$ gauss was found. The effective field, therefore, is within 5 percent of the macroscopic B. We expect a ±5 percent discrepancy because of the uncertain energy-range values, the error introduced in finding Φ and the intrinsic approximations involved in the formulas used. Within these approximations $b_{\rm eff}=B$.

The approximate formula for multiple scattering [Eq. (4)] was checked only in first order; however, no better fit is to be expected, because of the background, the finite width of the incident electron beam and the large thickness of iron involved.

Figure 4 shows the results of the magnetization curve and hysteresis effect measurement. In the part of the experiment, we have plotted $f_+(\theta)$ and $f_-(\theta)$ for $\theta=6°$ as a function of the magnetizing current in the scatterer. Had $g(\theta)$ been a linear function of θ, $\Delta(\theta)$ would have been a direct measure of Φ, and therefore of $b_{\rm eff}$ *vs* current. This is, as a first approximation, the case for $g(6°\pm3°)$ in which region we are interested. The plot can be thought of as a magnetization curve for $b_{\rm eff}$ *vs* magnetizing current, with a corresponding change in ordinate. The saturation shape of the magnetization curve checks with the macroscopically measured B *vs* magnetizing current curve. Points A and B on the plot are the counting rates observed after turning the magnetizing current off and before demagnetizing the scatterer with a decreasing alternating current. They represent the effect due to the retentivity of the iron scatterer and also check quantitatively with the macroscopically measured retentivity.

An investigation with a pickup coil of the magnetic field of the magnet outside the scatterer shows negligible effects due to stray magnetic field. This was also checked by doing an experiment with a copper sheet replacing the iron scatterer in the magnet. The copper had a thickness giving about equal multiple scattering curve as the iron sheet (equal radiation lengths). No detectable shift of the scattering curves was observed by changing the direction of the magnetizing field.

CONCLUSION

Within experimental errors, and within the errors introduced by the approximations correcting for energy loss, it has been demonstrated that $b_{\rm eff}=B$. The experiment shows, therefore, no evidence for strong short-range interaction forces between the beam electron and the spin-aligned ferromagnetic electrons. A thorough theoretical investigation of the strength of the short-range forces needed to alter the $b_{\rm eff}=B$ result would be necessary, in order to decide, whether a similar experiment with high-energy positrons instead of electrons would show up quantitatively the existence of electron-positron annihilation forces. Such an experiment is being contemplated in the near future.

The authors would like to thank the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, and in particular Dr. Norman P. Heydenburg, for the use of their Van de Graaff generator.

[10] Fowler, Lauritsen, and Lauritsen, Rev. Modern Phys. **20**, 267 (1948)