

provided $(k_0/4\pi i) \int g(\mathbf{k}, \mathbf{k}') d\Omega' > 0$. Putting the value of $C(\mathbf{k}, \mathbf{k}_0)$ in (2), we get

$$f^{(n+1)}(\mathbf{k}, \mathbf{k}_0) = \frac{\left[g(\mathbf{k}, \mathbf{k}_0) + \frac{k_0}{4\pi i} f^{(n)}(\mathbf{k}, \mathbf{k}_0) \int g(\mathbf{k}, \mathbf{k}') d\Omega' - \frac{k_0}{4\pi i} \int g(\mathbf{k}, \mathbf{k}') f^{(n)}(\mathbf{k}', \mathbf{k}_0) d\Omega' \right]}{1 + \frac{k_0}{4\pi i} \int g(\mathbf{k}, \mathbf{k}') d\Omega'} \quad (7)$$

Assuming $f^{(n)}(k, k_0)$ not dependent on angles, Eq. (7) gives, as first iteration:

$$C^{(n+1)}(\mathbf{k}, \mathbf{k}_0) = g(\mathbf{k}, \mathbf{k}_0) \left/ \left[1 + \frac{k_0}{4\pi i} \int g(\mathbf{k}, \mathbf{k}') d\Omega' \right] \right. \quad (8)$$

The criterion of convergence² may be very easily deduced. We simply state that the absolute value of the correction amplitude does not exceed the maximum amount of deviation of the foregoing approximate amplitude from the exact amplitude; i.e.,

$$|K^{(n+1)}(\mathbf{k}, \mathbf{k}_0)| \leq \epsilon^{(n)}_{\max} \text{ if } g(\mathbf{k}, \mathbf{k}') > 0 \text{ and } |\mathbf{k}_0| > 0,$$

where $\epsilon^{(n)}_{\max}$ is the maximum of all deviations of the approximate solutions from the true solution.

Further, the average amounts of the deviations $\epsilon^{(n)}(\mathbf{k}, \mathbf{k}_0)$, $\epsilon^{(n+1)}(\mathbf{k}, \mathbf{k}_0)$, ..., of the approximate amplitudes from the true amplitudes converge towards zero if

$$1 + |C(\mathbf{k}, \mathbf{k}_0)| + \frac{k_0}{4\pi i} \int |C(\mathbf{k}, \mathbf{k}')| |g(\mathbf{k}, \mathbf{k}')| d\Omega' \leq \theta,$$

where $0 < \theta \leq 1$.

The general reliability of this method may be shown by considering the scattering of positive mesons by neutrons. It will be shown that the solution (8) gives the exact result as obtained by Ma³ and Goldberger⁴ using variational methods.

The relevant matrix element for this process may be written as

$$\langle \mathbf{p}, \mathbf{k} | \mathcal{G} | \mathbf{p}_0, \mathbf{k}_0 \rangle = \bar{u}(a_1 + a_2 \gamma_4) u_0 / 2\epsilon_0 (W_0^2 - M^2),$$

where \bar{u} and u_0 are the usual Dirac spinors corresponding to four-momenta \mathbf{p} and \mathbf{p}_0 , respectively. \mathbf{k} and \mathbf{k}_0 are the final and initial four-momenta of the meson, and $\epsilon_0 = (\mu^2 + q^2)^{1/2}$, where q is the magnitude of the momentum of either particle and μ the meson mass. W_0 is the total energy. $a_1 = f^2 M$ and $a_2 = f^2 W_0$ in the case of pseudoscalar coupling.

Writing $\langle \mathbf{p}, \mathbf{k} | \mathcal{G} | \mathbf{p}_0, \mathbf{k}_0 \rangle = (\frac{1}{2}\epsilon_0) (\bar{u} g(\mathbf{p}, \mathbf{p}_0) u)$, the Heitler equation reduces to

$$f(\mathbf{p}, \mathbf{p}_0) = g(\mathbf{p}, \mathbf{p}_0) + \frac{iq}{32\pi^2 W_0} \int d\Omega' g(\mathbf{p}, \mathbf{p}') (-i\gamma \cdot \mathbf{p}' + M) f(\mathbf{p}', \mathbf{p}_0).$$

Using the solution (7), we have

$$f(\mathbf{p}, \mathbf{p}_0) = g(\mathbf{p}, \mathbf{p}_0) \left/ \left[1 - \frac{iq}{32\pi^2 W} \int d\Omega' g(\mathbf{p}, \mathbf{p}') (-i\gamma \cdot \mathbf{p}' + M) \right] \right. \\ = g(\mathbf{p}, \mathbf{p}_0) / (1 - i\lambda), \quad (9)$$

where

$$\lambda = (q/8\pi W_0) g(\mathbf{p}, \mathbf{p}_0) (E_0 \gamma_4 + M),$$

M being the nucleon mass and E_0 the nucleon energy.

The same result has been obtained by Goldberger⁴ and Ma.³

That (9) is the exact solution may be verified by iterating this result by means of (7). The second iteration will give the same result (9).

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¹ W. Heitler, Proc. Cambridge Phil. Soc. **37**, 291 (1941).

² C. Wagner, J. Math. and Phys. **30**, 23 (1951).

³ S. T. Ma, Proc. Cambridge Phil. Soc. **39**, 168 (1943).

⁴ M. L. Goldberger, Phys. Rev. **84**, 929 (1952).

Gamma Radiation Following Decay of $I^{131}\dagger$

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IN view of the long time which elapsed between the first measurements of I^{131} gamma radiation with the curved crystal gamma-ray spectrometer¹ and the most recent calibration of the instrument,² these radiations have been remeasured. Three gamma-ray lines, of approximately 364, 284, and 80 keV, were detected and measured at this time. The wavelengths, in both x units (Siegbahn scale) and milliangstroms, and the energies of these lines are given in Table I. The stated uncertainties are

TABLE I. Wavelengths and energies of radiation following decay of I^{131} .

Wavelength in x units (Siegbahn scale)	Wavelength in milliangstroms	Energy in keV
33.946 ± 0.0045	34.016 ± 0.0047	364.467 ± 0.050
43.517 ± 0.0073	43.607 ± 0.0075	284.307 ± 0.049
154.336 ± 0.0161	154.656 ± 0.0170	80.164 ± 0.0088

standard deviations. As reported in reference 1, these three gamma-ray lines form a Ritz combination. Equation (1) shows the excellent agreement of the present results when expressed in such a form:

$$(284.307 \pm 0.049) + (80.164 \pm 0.0088) - (364.467 \pm 0.050) \\ = 0.004 \pm 0.071. \quad (1)$$

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¹ Lind, Brown, Klein, Muller, and DuMond, Phys. Rev. **75**, 1544 (1949).

² Muller, Hoyt, Klein, and DuMond, Phys. Rev. **88**, 775 (1952).

Cohesive Energy of Alkali Metals

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RECENTLY Kuhn and Van Vleck¹ have given a method for calculating the ground-state energy ϵ_0 and the Fermi energy ϵ_F of an alkali metal in the Wigner-Seitz sphere approximation² without explicit knowledge of the ion-core potential or numerical integration of the radial wave equation. The spectroscopic term values of the free atom were the only empirical data used in the calculation. The Kuhn-Van Vleck method appeared to give reliable results for the ground state energy, but the values for the Fermi energy deviated so drastically from the free electron values, at least for potassium and rubidium, that doubt was thrown on the validity of the method.

It is the purpose of the present note to point out a number of simplifications of the Kuhn-Van Vleck method, which, when applied, not only greatly reduce the numerical labor of a calculation, but also give more reasonable values of the Fermi energy, and permit, for the first time, reasonable theoretical predictions of the cohesive energy of all the alkali metals using only the properties of the free-atom spectra. The chief remaining uncertainties in the calculation are in the exchange and correlation energies.

Kuhn and Van Vleck make use of the fact that in the alkali metals the potential is Coulombic at the surface of the unit cell and that therefore the wave functions in this region may be expressed as linear combinations of confluent hypergeometric functions. The improvement in the present work is that we have found an explicit analytical form of the wave function in terms