TABLE I. Positrons from Ga^{65}.

Energy Mev	Abundance percent	logft
2.52 ±0.05	10	6.0
2.1 ±0.1	90	4.7

The author wishes to express his sincere thanks to Professor A. C. Helmholz for suggesting this work and for his advice. Thanks are also due Mr. G. B. Rossi, Mr. W. B. Jones, and the staff of the Crocker Laboratory cyclotron for making the bombardments.

[1] J. H. Buck, Phys. Rev. **54**, 1025 (1938).
[2] A. Mukerji and P. Preiswerk, Helv. Phys. Acta **25**, 387 (1952).
[3] It has now come to the author's attention that similar results have been obtained at the Oak Ridge National Laboratory (B. L. Cohen, private communication to J. M. Hollander, I. Perlman, and G. T. Seaborg).
[4] Positrons from Ga^{65} have also recently been observed at the Instituut voor Kernphysisch Onderzoek, Amsterdam. [A. H. W. Aten, Jr., and M. Boelhouwer, Physica **18**, 1032 (1952).]

Differential Cross Sections for the Scattering of 58-Mev π^+ Mesons in Hydrogen*

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USING counter techniques similar to those used by Anderson, Fermi, Nagle, and Yodh,[1] the scattering of 58-Mev positive mesons in hydrogen has been investigated. Figure 1 indicates the experimental arrangement. The mesons pass through an opening in the shielding wall and are deflected by a double focusing magnet. The incident beam, defined by the stilbene counters, 1 and 2, strikes the liquid hydrogen target,[2] and the scattered mesons are detected in the large rectangular liquid counters, 3 and 4. Both on entering and on leaving the target, the beam must traverse the vacuum jacket (0.007 inch of aluminum), the radiation shield (0.002 inch of aluminum), and the liquid container (0.003 inch of iron). Measurements are taken at each point alternately with liquid hydrogen in and out of the target cup.

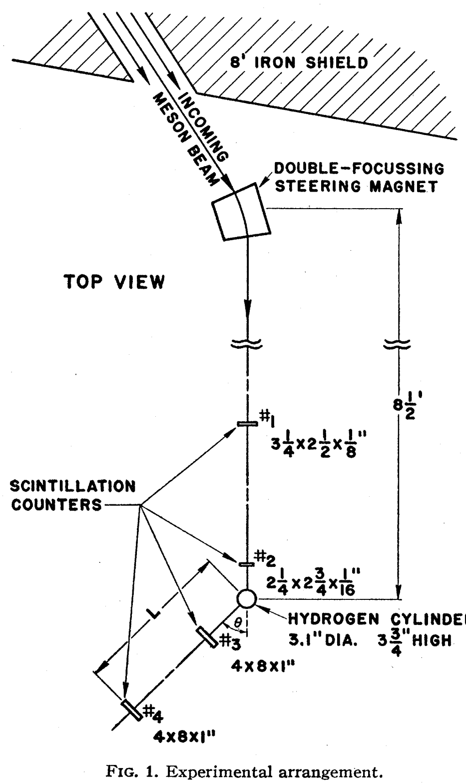

FIG. 1. Experimental arrangement.

From range measurements in carbon, the average energy of the mesons in the hydrogen is determined to be 58 Mev. The width of the energy spread at half-maximum is ±3 Mev, partly due to the initial beam spread and partly due to ionization loss in the hydrogen. The effective angular resolution is measured by rotating the detecting telescope through the main beam. For $L=18.5$ inches, $\Delta\theta=\pm6.6$ deg and for $L=27.5$ inches, $\Delta\theta=\pm4.7$ deg, where $\Delta\theta$ is the half-width at half-maximum.

Table I lists the experimental results for the six laboratory angles.

TABLE I. Experimental data.

θ_{lab} deg	L inches	Counts from hydrogen per 10^6 incident particles	Ratio of counts from hydrogen to background	$\Delta\theta$ deg
30	32.5	1.46±2.36	0.02	±3.7
	27.5[a]	2.06±2.11	0.04	±4.7
40	27.5	5.21±1.26	0.19	±4.7
	18.5	4.15±7.14	0.05	±6.6
55	27.5	4.46±2.44	0.26	±4.7
	18.5	13.98±1.89	0.47	±6.6
90	27.5	11.48±1.71	1.96	±4.7
	18.5	20.39±1.59	1.11	±6.6
120	27.5	11.90±2.08	1.44	±4.7
	18.5	30.46±2.32	1.36	±6.6
150	27.5	12.02±2.26	1.04	±4.7
	18.5	32.23±2.26	0.92	±6.6

[a] One inch of carbon in front of counter 4.

From analysis of the range curve, the incident beam is estimated to consist of 89 percent π-mesons. The remaining 11 percent of the particles are μ-mesons and possibly some electrons, and are assumed to undergo negligible scattering in the hydrogen. The effective target thickness is determined to be 0.460 g/cm^2 of hydrogen, based on a scanning of the lateral distribution of the beam by the use of a ⅛-inch wide counter.

By placing counters 3 and 4 between the smaller counters 1 and 2, the efficiency of the detecting telescope is found to be 91 percent. However, 3 percent of the scattered mesons are lost through nuclear collisions in counter 3, and an additional 5 percent are lost in those 30 deg runs where one inch of carbon is placed in front of counter 4 to reduce the background.

A correction is made to the 150-deg data to account for the 10±5 percent of the scattered mesons with insufficient range. Also, 0.08±0.06 millibarn per steradian is added to the uncorrected laboratory cross sections at all angles to account for the fraction of the background which is not detected because of the extra ionization loss in the hydrogen.

TABLE II. Corrected differential cross sections.

Laboratory system		Center-of-mass system	
θ (deg)	$d\sigma/d\Omega$ (mb/sterad)	θ (deg)	$d\sigma/d\Omega$ (mb/sterad)
30	0.33±0.15	36	0.24±0.11
40	0.64±0.10	47	0.48±0.08
55	0.81±0.08	64	0.66±0.06
90	1.21±0.06	101	1.24±0.07
120	1.68±0.09	129	2.10±0.11
150	1.95±0.10	155	2.79±0.15

The corrected differential cross sections in the laboratory system and in the center-of-mass system are given in Table II. The errors quoted combine statistical probable errors with the estimated errors in the two corrections listed above.

Fowler, Fowler, Shutt, Thorndike, and Whittemore[3] have reported a distribution more nearly symmetric about 90 deg, on the basis of 20 events. On the other hand, Minguzzi, Puppi, and Ranzi[4] find scattering almost entirely in the backward direction, on the basis of 10 events in photographic emulsions. Their combined results, although still statistically poor, are not in disagreement with the results in Table II.

The cross section integrated between 25 and 180 deg (in laboratory system) is 15.3±1.0 mb. This is not inconsistent with the 20±10 mb reported by Chicago[5] and 20±4 mb reported by Brookhaven,[3] but it is lower than the 27.8±2.5 mb reported by our group[6] from attenuation measurements in polyethylene and carbon. The discrepancy may be due to systematic errors in the attenuation measurements, and experiments are under way to resolve it. Pending the outcome, we consider the result reported here more valid.

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[1] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952).
[2] Lindenfeld, Sachs, and Steinberger, Phys. Rev. **89**, 531 (1953).
[3] Fowler, Fowler, Shutt, Thorndike, and Whittemore, Phys. Rev. **86**, 1053 (1952).
[4] Minguzzi, Puppi, and Ranzi (unpublished). We are grateful to Dr. G. Goldhaber for calling these results to our attention.
[5] Anderson, Fermi, Long, and Nagle, Phys. Rev. **85**, 936 (1952).
[6] Isaacs, Sachs, and Steinberger, Phys. Rev. **85**, 803 (1952).

Phase-Shift Analysis of the Scattering of Positive Mesons at 58 Mev*

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The data reported in the previous letter are neither sufficiently extensive nor precise to permit definitive phase-shift reduction without arbitrary assumptions. In pseudoscalar meson theory with pseudovector coupling[1] but possibly also in the pseudoscalar coupling case,[2] the bulk of the scattering is expected in the p state. The strong asymmetry about 90°, however, must be attributed to interference between even and odd angular momentum states. It is simplest to assume the even contribution pure s, but this assumption is dangerous because of the rapid increase of even scattering with energy. This rather suggests[3] that the even scattering is velocity-dependent and probably a recoil effect, in which case d and s contributions of the same order may be expected.[4] Nevertheless, following the Chicago group[5] we will analyze on the premise that the d contribution is zero. It is possible, however, to obtain good agreement using only p and d waves: e.g., $\delta(p_{3/2})=8.4°$, $\delta(p_{1/2})=-2.4°$, $\delta(d_{5/2})=-2.3°$, $\delta(d_{3/2})=1.0°$.

Assuming then only s and p scattering and, in addition, that the Coulomb field is a perturbation on the meson-nucleon interaction at distances smaller than its range, the cross section in the center-of-mass system may be written:[6]

$$\frac{d\sigma}{d\Omega}=\frac{\hbar^2}{4k^2}\left\{\left|\frac{-i\alpha}{\sin^2(\frac{1}{2}\theta)}\exp(-i\alpha\ln\sin^2\tfrac{1}{2}\theta)+P+Q\mu\right|^2+|R|^2(1-\mu^2)\right\}, \qquad (1)$$

where $k=$ meson momentum, $\alpha=e^2/\hbar v$, and $P=\exp[2i\delta(s)]-1$, $Q=[(1+i\alpha)/(1-i\alpha)]\{\exp[2i\delta(p_{3/2})]+\exp[2i\delta(p_{1/2})]-3\}$,

$$R=\exp[2i\delta(p_{3/2})]-\exp[2i\delta(p_{1/2})].$$

Table I. S and p wave phase shifts for 58-Mev π^+-p scattering.

	Phase shifts (deg) (δs)	$\delta(p_{\frac{1}{2}})$	$\delta(p_{\frac{3}{2}})$	Deviation ω
Best fits:				
positive $\delta(s)$	+7.4	+2.2	−6.5	9
negative $\delta(s)$	−4.9	−1.8	+7.6	2
50 percent fits:				
only $\delta(s)$ varied	−4.0	−1.8	+7.6	12
	−5.7	−1.8	+7.6	12
only $\delta(p_{\frac{1}{2}})$ varied	−4.9	−0.8	+7.6	12
	−4.9	−2.7	+7.6	12
only $\delta(p_{\frac{3}{2}})$ varied	−4.9	−1.8	+7.1	12
	−4.9	−1.8	+8.1	12
all varied[a]	−3.6	−1.5	+8.1	12
	−10.8	+2.5	+2.5	12

[a] To give maximum variation of $\delta(s)$.

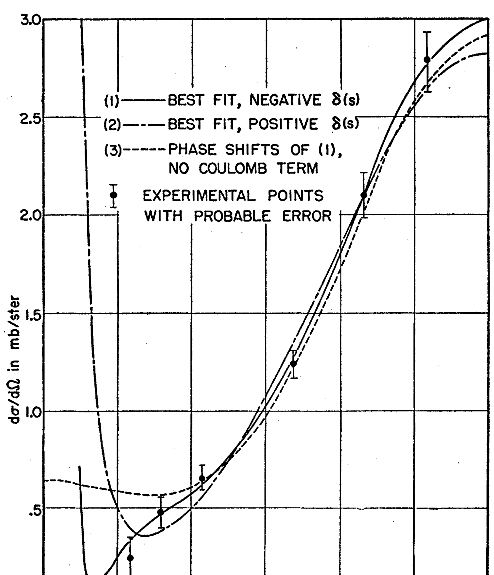

Fig. 1. Differential cross section for 58-Mev π^+-p scattering.

Although the precise determination of v requires solution of the relativistic two-particle problem, it is very nearly the relative velocity in the center-of-mass system.

It can be seen from Eq. (1) that the sign of R is free, so that the solutions of Eq. (1) will occur in pairs of "Fermi" and "Yang" types. We give only the former [large $\delta(p_{3/2})$]. For the small phase shifts the "Yang" type can be obtained from the "Fermi" type by

$$\delta(s)^{(\mathrm{Y})}=\delta(s)^{(\mathrm{F})},$$

$$\delta(p_{1/2})^{(\mathrm{Y})}=\tfrac{1}{3}[4\delta(p_{3/2})^{(\mathrm{F})}-\delta(p_{1/2})^{(\mathrm{F})}],$$

$$\delta(p_{3/2})^{(\mathrm{Y})}=\tfrac{1}{3}[\delta(p_{3/2})^{(\mathrm{F})}+2\delta(p_{1/2})^{(\mathrm{F})}].$$

The observed differential cross section permits either positive or negative $\delta(s)$ but favors negative $\delta(s)$. The best fits for the two cases are compared with the experimental points in Fig. 1.

"Best fits" are those which minimize the quantity

$$\omega=\sum_{i=1}^{6}\left(\frac{\epsilon_i}{\mathrm{PE}_i}\right)^2,$$

where ϵ_1 is the difference between the calculated and observed cross sections at the ith angle and PE_i is the probable error of the cross section at the ith angle. On comparing the observed data with the (unknown) true cross section one would expect that 50 percent of the time ω would be less than 11.8.

The uncertainty in the determination of the phase shifts may be investigated in terms of sets of phase shifts which give $\omega=11.8$. Four illustrative sets, each for negative $\delta(s)$, are listed in Table I. In three of these sets only one of the phase shifts is different from the best fit values. In the fourth set the values of $\delta(p_{1/2})$ and $\delta(p_{3/2})$ have been chosen to minimize ω for the indicated $\delta(s)$, thus allowing a maximum variation of $\delta(s)$.

In the absence of the Coulomb field, the two "best fit" solutions differ only in the reversal of signs in the phase shifts. However, the Coulomb field significantly alters the angular distribution