× .4

neglect of dissipation. We are here concerned with wave propagation in a good conducting medium. In the case of perpendicular magnetization one obtains two contributions to  $Q^{-1}$  arising separately from the two directions of circular polarization into which the cavity mode can be decomposed. In the case of parallel magnetization the final solution of the resonance problem is identical to that given by the usual theories, although demagnetizing factors for the oscillating fields are not specifically introduced. The results for  $R^2$  are

$$R^{2} = 2 \frac{\eta + (\eta^{2} + 1)^{\frac{1}{2}}}{1 + (\eta \omega T_{2})^{-\frac{1}{2}} (\eta + 1)^{\frac{1}{2}}}, \quad \text{(Bloch)}$$

$$R^{2} = 2 \frac{1 + \eta (\eta^{2} + 1)^{-\frac{1}{2}}}{1 + (\alpha / \eta)^{\frac{1}{2}} (\eta + 1)^{\frac{1}{2}}}, \quad \text{(LL)}$$

where  $T_2$  is the Bloch time constant and  $\alpha$  is the coefficient of LL damping. We have used  $T_2^{-1} = 3.5$  and  $1.5 \times 10^9$  for nickel and Supermalloy, respectively, as determined by line width.

The agreement between measured and calculated values of  $R^2$ , where calculations of the latter are based on wave propagation using Bloch damping, is good. There is also an improvement in line shape. Further details will be published in a subsequent paper.

We wish to thank Dr. S. O. Morgan and Mr. R. A. Chegwidden, of the Bell Telephone Laboratories for supplying the Supermalloy samples.

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## Theory of Polarized Particles in Nuclear Reactions

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 $\mathbf{C}^{\mathrm{ONSIDER}}$  the most general two-body nuclear reaction,

 $a + X \rightarrow Y + b$ ,

where a and b have spins i and i', respectively, and I and I' are the spins of X and Y. The density matrix for the initial state of acan always be written as a linear sum of its irreducible spin tensor moments  $T_{\kappa}^{(q)}$ , where q is the rank of the tensor and  $\kappa$  its component  $(|\kappa| \le q \le 2i)$ . Similarly, the final states of b are determined by its spin tensor moments  $T_{\kappa'}(q')$   $(|\kappa'| \le q' \le 2i')$ . In a previous communication<sup>1</sup> an expression was given for the spin tensor moments arising from a reaction initiated by an unpolarized beam; i.e., one in which all initial tensor moments other than  $q = \kappa = 0$  vanish. This expression can now be generalized to the case of an arbitrarily polarized initial beam.

Let  $T_{\kappa'}{}^{(q')}$  be the expectation value of a spin tensor moment of b, (in channel  $\alpha'$ ) measured with respect to the coordinate system with z axis along  $\mathbf{k}'$  and y axis along  $\mathbf{k} \times \mathbf{k}'$ , the normal to the scattering plane. The contribution to this quantity of an initial spin tensor moment  $T_{\kappa}^{(q)}$  in channel  $\alpha$  (measured relative to the incident beam direction) can then be written (since the effects of several initial tensor moments are additive):

$$T_{\kappa'}{}^{(q')} = \frac{\lambda_{\alpha}^{2}(2i)![(2i'-q')!(2i'+q'+1)!]^{\frac{1}{2}}(2q+1)P_{q'}([i'/i'+1]^{\frac{1}{2}})}{4[(2i-q)!(2i+q+1)!]^{\frac{1}{2}}(2I+1)P_{q}([i/i+1]^{\frac{1}{2}})(2i')!} \times \Sigma R^{*}(\alpha l_{1}s_{1}, \alpha' l_{1}'s_{1}'; J_{1}\pi_{1})R(\alpha l_{2}s_{2}, \alpha' l_{2}'s_{2}'; J_{2}\pi_{2})} \times W(iIqs_{1}; s_{2}i)W(i'Iq's_{1}'; s_{2}'i')(-1)^{\kappa'}D_{\kappa,\kappa'}{}^{(L)}(\phi, \theta, 0) \times G_{\kappa}^{*}(J_{1}l_{1}s_{1}; J_{2}l_{2}s_{2}; Lq)G_{\kappa}(J_{1}l_{1}'s_{1}'; J_{2}l_{2}'s_{2}'; Lq')T_{\kappa}{}^{(q)}, \quad (1)$$

where  $\theta$  and  $\phi$  are the direction angles of **k**' relative to **k**, D is the usual representation of the rotation group, W is the Racah function, and the sum is over L as well as all dynamical variables

 $l_1 l_2 s_1 s_2 J_1 J_2 \pi_1$  and  $\pi_2$ . The reaction matrix R is related to the scattering matrix S by  $R \equiv 1 - S$ .

All selection rules are contained in the "geometrical" coefficient G, which is related to the X coefficient of Fano<sup>2</sup> by the relation

 $G_{\kappa}(J_1l_1s_1; J_2l_2s_2; L_q)$  $= [2l_1+1)(2l_2+1)(2s_1+1)(2s_2+1)(2J_1+1)(2J_2+1)]^{\frac{1}{2}}i^{l_1+l_2}$  $\times \Sigma_r (2r+1)^{\frac{1}{2}} (l_1 l_2 00 | l_1 l_2 r 0) (qr \kappa 0 | qr L \kappa)$ 

 $\times X(J_1 l_1 s_1; J_2 l_2 s_2; Lqr).$  (2)

Note that the elements  $(J_{1s_{1}l_{1}})$ ,  $(J_{2s_{2}l_{2}})$ ,  $(J_{1}J_{2}L)$ , and  $(s_{1s_{2}q})$  of G must form a possible vector "triad." In addition, the elements  $(l_1 l_2 Lq)$  must form a possible vector "tetrad." The G coefficient is the generalization of the Z coefficient of Blatt and Biedenharn.<sup>3</sup>

The maximum complexity of the angular dependence is given by the largest value of L. This is given by the simultaneous conditions

$$L \leq \begin{cases} 2l_{\max} + q \\ 2l_{\max} + q - 1 \\ 2J_{\max} \\ 2l'_{\max} + q' \\ 2l'_{\max} + q' \\ 2l'_{\max} + q' - 1 \\ 2l'_{\max} + q' - 1 \end{cases}$$
(3)

along with the condition that L must be even if  $\kappa(\kappa')$  is zero, q(q') is even and the interfering levels have the same parity. These rules are more restrictive than those given previously.<sup>4</sup>

A more detailed paper, which is in preparation, will give the derivation of the above results as well as applications to arbitrarily polarized target nuclei, reactions involving gamma-rays, and the detection of polarized particles. In addition, an alternative expression for G which is very convenient for tabulation will be given. It is a pleasure to thank Dr. T. A. Welton for many stimulating discussions.

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## Cyclic Adiabatic Demagnetization\*

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YCLIC demagnetization apparatus has been devised and used for maintaining the temperature of a paramagnetic salt sample at 0.73°K while the heat sink temperature was 1.13°K. The refrigerant is a second salt sample 12 cm distant, movable within an evacuated case and capable of being forced into contact either with the surrounding vacuum case which is immersed in pumped liquid helium or the first-named salt sample which is to be cooled.

A sectional view of the apparatus is shown in Fig. 1. The two salt samples are placed at a distance from each other so that a magnetic field may be applied to one sample only at a time. Temperatures are estimated from susceptibility measurements. The top sample can be brought into thermal contact with the case at its upper end or with a tubular extension of the bottom sample at its lower end. Iron ammonium alum is the salt used in this experiment. It is sealed in a brass capsule with a small amount of helium as the heat transfer agent.

In operation the poles of a permanent magnet (1850 gauss) are brought up to the top sample while it is in contact with the helium bath and kept there for a period of two minutes. The contact is broken and the magnet removed. The demagnetized sample, which is now cold, is placed in contact with the lower sample. At the end of four or five minutes substantial equalization of temperatures has been achieved, and a new cycle is started. After several cycles the lower sample acquires an equilibrium



bined into the five types  $(\alpha+\beta)$ ,  $(\alpha'+\beta')$ ,  $(\gamma+\gamma')$ ,  $(\delta+\epsilon)$ ,  $(\delta'+\epsilon')$ . The symmetric and antisymmetric properties of the wave functions for a translation to an adjacent atomic cell polyhedron following an inversion operation are indicated by the suffixes S and A. Function subspaces of the space group for k=0 are split into the types  $(\alpha+\beta)_s$ ,  $(\alpha+\beta)_A$ ,  $\cdots$ . For instance, the  $(\alpha+\beta)_s$  type wave function is expanded in terms of  $\alpha$ - and  $\beta$ -type harmonics, and has the same form in the adjacent polyhedron referring to inverted coordinate axes. As boundary conditions, the expansion coefficients are calculated according to the symmetry property and Bloch periodicity, and fitting is performed at the center and on a circle which is an intersection with the equal volume sphere, on the hexagonal face of the atomic polyhedron, just as in the case treated by Von der Lage and Bethe.

It seems from our results that the more precise method gives better agreement with the experimental data. (See Table II.)

TABLE II. Comparison of results of calculations with experimental data.

	Our results	Mullaney	Holmes	Experiment
Energy gap of band in ev Band width in ev	1.08 15.1	very large 15.5	9.96 	1.1 19.2

[In fact, the  $(\delta + \epsilon)_s$  type wave function includes contributions from l=3 and =4 of the same order as those from l=1 and =2.] Therefore, from this agreement, the highest point of the filled band and lowest point of the conduction band presumably occur at the center of the Brillouin zone.

<sup>1</sup> F. C. Von der Lage and H. A. Bethe, Phys. Rev. **71**, 612 (1947).
 <sup>2</sup> J. F. Mullaney, Phys. Rev. **66**, 326 (1944).
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temperature which is determined principally by the magnitude of the heat leaks and the strength of the magnet.

Heat generated in the system by mechanical vibration seems to be much more important than thermal conduction of the supporting members. It is also more elusive with respect to control measures.

\* Supported by the U. S. Office of Naval Research and the U. S. Atomic lergy Commission. Energy

## **Energy Band Structure in Silicon Crystal**

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(Received February 6, 1953)

HE energy band structure in silicon has been calculated in a way quite similar to that used by Von der Lage and Bethe<sup>1</sup> for sodium. The levels evaluated are the lowest three having the wave vector k=0. Angular functions were constructed from the Von der Lage-Bethe cubic harmonics, and radial functions were computed numerically up to l=6, making use of the effective nuclear charge given in Mullaney's paper.<sup>2</sup> Results are shown in Table I, with the results obtained by Mullaney<sup>2</sup> and Holmes.<sup>3</sup>

TABLE I. Energy values for the lowest three levels at k = 0, in ev.

Type of wave	Our results	Mullaney's	Holmes'
function		results	results
$\begin{array}{c} (\alpha + \beta)_{A^{\mathbf{a}}} \\ (\delta + \epsilon)_{S^{\mathbf{b}}} \\ (\alpha + \beta)_{A^{\mathbf{c}}} \end{array}$	$0.405 \\ -0.675 \\ -15.8$	very high -3.24 -18.71	4.24 - 5.72

Bottom of conduction band.
Top of filled band.
Bottom of filled band.

The notation  $\alpha$ ,  $\alpha'$ ,  $\beta$ ,  $\beta'$ ,  $\cdots$  denotes the type of cubic harmonics after Von der Lage and Bethe.<sup>1</sup> The point group for the diamond type lattice is the tetrahedral group which is a subgroup of the cubic group. Therefore, function subspaces of this group are com-

## The Phosphorescence of Thoria\*

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WHILE observing luminescent effects resulting from the irradiation of various metallic oxides with nuclear particles, the writers chanced to note a continuous light emission from thoria. This luminescence was detected prior to application of any primary excitants such as alpha-particles, beta-rays, gamma-rays, or ultraviolet light. The property is exhibited by both Norton's reddish arc-melted coarsely crystalline thoria and white pieces of thoria sintered from powdered ThO2.1.2 The measurements of light intensity were performed with the use of phototubes RCA-5819 and RCA-1P28 as detectors.

Owing to the natural radioactivity of Th<sup>232</sup> and its daughter elements, ThO<sub>2</sub> emits  $\sim 22\,000\,\alpha$ 's per gram per second,  $\sim 15\,000\,$  $\beta$ 's per gram per sec, and  $\sim 20\,000\,\gamma$ 's per gram per sec. These internally emitted nuclear radiations give rise through selfabsorption in the thoria to the observed luminescence which has apparently attained a nearly constant intensity, decaying with the enormous half-period of Th<sup>232</sup>. When a small piece of crystalline thoria was placed before the slit of a small Hilger quartz spectrograph, no noticeable darkening of the photographic plate was produced after an exposure of two weeks. However, when the



FIG. 1. Luminescent emission of thoria under alpha-irradiation.