

carefully calibrated against the magnetic temperature of the salt which was measured using the ballistic method.

With the bath  $\text{He}^4$  held at  $1^\circ\text{K}$  and the copper cavity filled with pure  $\text{He}^4$ , the warm-up period for the apparatus after demagnetization was a little over twenty-two minutes. It should be added that during this warm-up period the transmitter was pulsed about seventy times. With mixtures of  $\text{He}^3$  and  $\text{He}^4$  in the cavity the warm-up period was as long as fifty minutes. Since the major heat leak can be attributed to evaporation and recondensation of the creeping film in the filling tube of the cavity, this observed increase in the warm-up time clearly indicates a reduced creep rate for the mixtures.

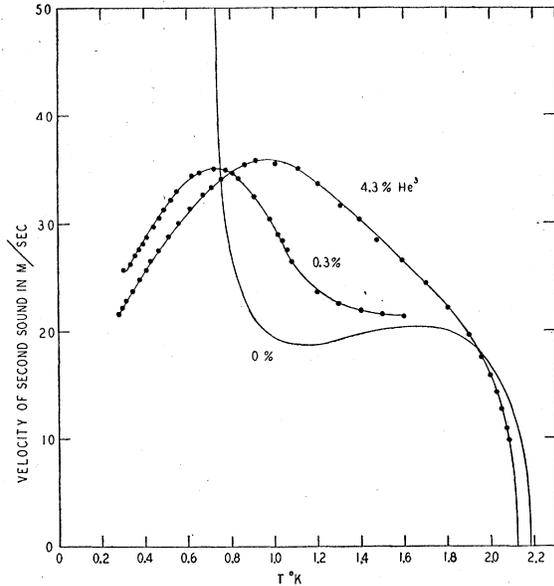


FIG. 1. Second-sound velocity versus temperature for various concentrations of  $\text{He}^3$ . The experimental points plotted below  $1^\circ\text{K}$  are approximately half those obtained from a single demagnetization.

Several concentrations of the  $\text{He}^3$  were used, ranging between 0.017 percent to 4.3 percent.<sup>6</sup> Representative results are shown in Fig. 1. The points on the 0.3 percent curve above  $1.1^\circ\text{K}$  were taken from previous measurements.<sup>1</sup> The points for the 4.3 percent curve above  $1^\circ\text{K}$ <sup>7</sup> are in rough agreement with the results of Weinstock and Pellam<sup>8</sup> using the thermal Rayleigh disk technique. The precision of this method was apparently insufficient to show the maximum which occurs at about  $1^\circ\text{K}$  for this concentration.

Significant results of the experiment are as follows:

(a) As the temperature nears  $0^\circ\text{K}$ , the second-sound velocity  $u_2$  for the mixture does indeed reach a maximum and decreases toward a low finite value in agreement with Pomeranchuk's theory.

(b)  $u_2$  for pure  $\text{He}^4$  reaches a value of 215 m/sec below  $0.25^\circ\text{K}$  and, as previously observed,<sup>9,10</sup> the pulse shape is greatly distorted indicating large dispersion.

(c) For the mixtures no such pulse distortion occurs even at the lowest temperatures, indicating little if any dispersion. This effect can be understood most readily if we consider Landau's<sup>11</sup> theoretical prediction that  $\text{He}^4$  below  $0.6^\circ\text{K}$  is essentially a phonon gas consisting of elementary Debye excitations. In this temperature region near  $0^\circ\text{K}$ , the mean free path for a phonon becomes so large<sup>12,13</sup> that it may travel from the transmitting resistor to the receiver by any number of paths without collision with other phonons, thereby giving rise to a gradual build-up of the received signal. With the addition of a small quantity of  $\text{He}^3$  the situation is radically changed in this low-temperature region. Here the excitation energy is almost entirely associated with the  $\text{He}^3$  com-

ponent. Since the mean free path of these excitations is quite small, no such pulse distortion would be expected for the mixtures.

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## Energy Absorption in Ferromagnetic Resonance

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RECENT results of measurements made in this laboratory show that the usual interpretation of ferromagnetic resonance in terms of a complex permeability coefficient<sup>1-3</sup> is inadequate. An interpretation based on the concept of a permeability tensor<sup>4</sup> is in closer agreement with the observations. We find also that it is possible to distinguish between the Bloch type of damping<sup>5</sup> and the Landau-Lifshitz (LL) type.<sup>6</sup> The experimental results favor the Bloch type.

The outstanding discrepancy which has to be resolved concerned the ratio of energy absorption in a cavity for the two cases of constant magnetic field parallel and perpendicular to the face of a thin ferromagnetic sample constituting one wall of that cavity. Denoting by  $Q_{\parallel}$  and  $Q_{\perp}$  the contribution which the ferromagnetic material makes to the quality factor of the cavity in these two cases, we list in Table I the experimental and theoretical values of the ratio  $R \equiv Q_{\perp}/Q_{\parallel}$ .

Uncertainties in the experimental results involve both experimental errors and variations among different samples. The theoretical results given in the third and fourth columns are obtained from the usual theory of ferromagnetic resonance phenomena derived by Kittel<sup>1</sup> and employed by Bloembergen<sup>3</sup> with Bloch type damping and by Yager<sup>6</sup> with LL damping. With the Bloch type damping,

$$R^2 = \eta + (\eta^2 + 1)^{\frac{1}{2}},$$

where  $\eta = 2\pi\gamma M_0/\omega$ . With LL damping there is an additional factor in  $R^2$  of  $(\eta^2 + 1)^{-\frac{1}{2}}$ .  $M_0$  is the saturation magnetization,

TABLE I. Values of  $R^2$  at 9200 Mc/sec.

Material	Measured values	Predicted values using			
		Permeability coefficient		Permeability tensor	
		LL	Bloch	LL	Bloch
Nickel	3-4	1.7	2.5	2.6	3.7
Supermalloy	4.5-5.5	1.8	2.8	2.9	4.6

$\gamma = ge/2mc$  is the gyromagnetic ratio, and  $\omega$  is the circular frequency of excitation. On the basis of our other data we have chosen  $4\pi M_0$  to be 5890 and 7690 gauss for nickel and Supermalloy, respectively, and  $g$  to be 2.28 and 2.07.

The predicted values of  $R^2$  given in the fifth and sixth columns of the table are based on a theory which treats ferromagnetic resonance in the cavity walls as an electromagnetic wave propagation rather than in terms of demagnetizing factors. The problem has been considered by Polder<sup>4</sup> with neglect of dissipation, by Hogan<sup>6</sup> for propagation in ferrites along the direction of the constant field and by Weiss and Fox<sup>7</sup> again for ferrites and with

neglect of dissipation. We are here concerned with wave propagation in a good conducting medium. In the case of perpendicular magnetization one obtains two contributions to  $Q^{-1}$  arising separately from the two directions of circular polarization into which the cavity mode can be decomposed. In the case of parallel magnetization the final solution of the resonance problem is identical to that given by the usual theories, although demagnetizing factors for the oscillating fields are not specifically introduced. The results for  $R^2$  are

$$R^2 = 2 \frac{\eta + (\eta^2 + 1)^{\frac{1}{2}}}{1 + (\eta \omega T_2)^{-1} (\eta + 1)^{\frac{1}{2}}}, \quad (\text{Bloch})$$

$$R^2 = 2 \frac{1 + \eta(\eta^2 + 1)^{-\frac{1}{2}}}{1 + (\alpha/\eta)^{\frac{1}{2}} (\eta + 1)^{\frac{1}{2}}}, \quad (\text{LL})$$

where  $T_2$  is the Bloch time constant and  $\alpha$  is the coefficient of LL damping. We have used  $T_2^{-1} = 3.5$  and  $1.5 \times 10^9$  for nickel and Supermalloy, respectively, as determined by line width.

The agreement between measured and calculated values of  $R^2$ , where calculations of the latter are based on wave propagation using Bloch damping, is good. There is also an improvement in line shape. Further details will be published in a subsequent paper.

We wish to thank Dr. S. O. Morgan and Mr. R. A. Chegwidzen, of the Bell Telephone Laboratories for supplying the Supermalloy samples.

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## Theory of Polarized Particles in Nuclear Reactions

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CONSIDER the most general two-body nuclear reaction,

$$a + X \rightarrow Y + b,$$

where  $a$  and  $b$  have spins  $i$  and  $i'$ , respectively, and  $I$  and  $I'$  are the spins of  $X$  and  $Y$ . The density matrix for the initial state of  $a$  can always be written as a linear sum of its irreducible spin tensor moments  $T_{\kappa}^{(a)}$ , where  $q$  is the rank of the tensor and  $\kappa$  its component ( $|\kappa| \leq q \leq 2i$ ). Similarly, the final states of  $b$  are determined by its spin tensor moments  $T_{\kappa'}^{(b)}$  ( $|\kappa'| \leq q' \leq 2i'$ ). In a previous communication<sup>1</sup> an expression was given for the spin tensor moments arising from a reaction initiated by an unpolarized beam; i.e., one in which all initial tensor moments other than  $q = \kappa = 0$  vanish. This expression can now be generalized to the case of an arbitrarily polarized initial beam.

Let  $T_{\kappa}^{(a)}$  be the expectation value of a spin tensor moment of  $b$ , (in channel  $\alpha'$ ) measured with respect to the coordinate system with  $z$  axis along  $\mathbf{k}'$  and  $y$  axis along  $\mathbf{k} \times \mathbf{k}'$ , the normal to the scattering plane. The contribution to this quantity of an initial spin tensor moment  $T_{\kappa}^{(a)}$  in channel  $\alpha$  (measured relative to the incident beam direction) can then be written (since the effects of several initial tensor moments are additive):

$$T_{\kappa}^{(a')} = \frac{\lambda_{\alpha}^2(2i) [(2i' - q')! (2i' + q' + 1)!]^{\frac{1}{2}} (2q + 1) P_{q'}([\mathbf{i}'/i' + 1]^{\frac{1}{2}})}{4 [(2i - q)! (2i + q + 1)!]^{\frac{1}{2}} (2I + 1) P_q([\mathbf{i}/i + 1]^{\frac{1}{2}}) (2i')!} \\ \times \sum R^*(\alpha l_1 s_1, \alpha' l_1' s_1'; J_1 \pi_1) R(\alpha l_2 s_2, \alpha' l_2' s_2'; J_2 \pi_2) \\ \times W(i I q s_1; s_2 i) W(i' I' q' s_1'; s_2' i') (-1)^{\kappa'} D_{\kappa, \kappa'}^{(L)}(\phi, \theta, 0) \\ \times G_{\kappa}^*(J_1 l_1 s_1; J_2 l_2 s_2; L q) G_{\kappa'}(J_1 l_1' s_1'; J_2 l_2' s_2'; L q') T_{\kappa}^{(a)}, \quad (1)$$

where  $\theta$  and  $\phi$  are the direction angles of  $\mathbf{k}'$  relative to  $\mathbf{k}$ ,  $D$  is the usual representation of the rotation group,  $W$  is the Racah function, and the sum is over  $L$  as well as all dynamical variables

$l_1 l_2 s_1 s_2 J_1 J_2 \pi_1$  and  $\pi_2$ . The reaction matrix  $R$  is related to the scattering matrix  $S$  by  $R = 1 - S$ .

All selection rules are contained in the "geometrical" coefficient  $G$ , which is related to the  $X$  coefficient of Fano<sup>2</sup> by the relation

$$G_{\kappa}(J_1 l_1 s_1; J_2 l_2 s_2; L q) \\ = [2l_1 + 1] (2l_2 + 1) (2s_1 + 1) (2s_2 + 1) (2J_1 + 1) (2J_2 + 1) [i_1^{l_1 + l_2} \\ \times \sum_r (2r + 1)^{\frac{1}{2}} (l_1 l_2 0 0 | l_1 l_2 r 0) (q r \kappa 0 | q r L \kappa) \\ \times X(J_1 l_1 s_1; J_2 l_2 s_2; L q r)]. \quad (2)$$

Note that the elements  $(J_1 s_1 l_1)$ ,  $(J_2 s_2 l_2)$ ,  $(J_1 J_2 L)$ , and  $(s_1 s_2 q)$  of  $G$  must form a possible vector "triad." In addition, the elements  $(l_1 l_2 L q)$  must form a possible vector "tetrad." The  $G$  coefficient is the generalization of the  $Z$  coefficient of Blatt and Biedenharn.<sup>3</sup>

The maximum complexity of the angular dependence is given by the largest value of  $L$ . This is given by the simultaneous conditions

$$L \leq \begin{cases} 2l_{\max} + q \\ 2l_{\max} + q - 1 & (\kappa' = 0, q' \text{ even and } q \text{ odd}) \\ 2J_{\max} \\ 2l'_{\max} + q' \\ 2l'_{\max} + q' - 1 & (\kappa = 0, q \text{ even and } q' \text{ odd}) \end{cases} \quad (3)$$

along with the condition that  $L$  must be even if  $\kappa(\kappa')$  is zero,  $q(q')$  is even and the interfering levels have the same parity. These rules are more restrictive than those given previously.<sup>4</sup>

A more detailed paper, which is in preparation, will give the derivation of the above results as well as applications to arbitrarily polarized target nuclei, reactions involving gamma-rays, and the detection of polarized particles. In addition, an alternative expression for  $G$  which is very convenient for tabulation will be given. It is a pleasure to thank Dr. T. A. Welton for many stimulating discussions.

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## Cyclic Adiabatic Demagnetization\*

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CYCLIC demagnetization apparatus has been devised and used for maintaining the temperature of a paramagnetic salt sample at 0.73°K while the heat sink temperature was 1.13°K. The refrigerant is a second salt sample 12 cm distant, movable within an evacuated case and capable of being forced into contact either with the surrounding vacuum case which is immersed in pumped liquid helium or the first-named salt sample which is to be cooled.

A sectional view of the apparatus is shown in Fig. 1. The two salt samples are placed at a distance from each other so that a magnetic field may be applied to one sample only at a time. Temperatures are estimated from susceptibility measurements. The top sample can be brought into thermal contact with the case at its upper end or with a tubular extension of the bottom sample at its lower end. Iron ammonium alum is the salt used in this experiment. It is sealed in a brass capsule with a small amount of helium as the heat transfer agent.

In operation the poles of a permanent magnet (1850 gauss) are brought up to the top sample while it is in contact with the helium bath and kept there for a period of two minutes. The contact is broken and the magnet removed. The demagnetized sample, which is now cold, is placed in contact with the lower sample. At the end of four or five minutes substantial equalization of temperatures has been achieved, and a new cycle is started. After several cycles the lower sample acquires an equilibrium