however are very small and were not entered in the tables.

### 5. CONCLUSION

The foregoing should enable correct treatment of first forbidden transitions with an arbitrary amount of pseudoscalar admixture. This is at present particularly valuable in the case of the decay of RaE; there, however, additional corrections due to the finite size of the nucleus must be calculated.

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# Quantum Theory of a Damped Electrical Oscillator and Noise*

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Field quantization is applied to an electrical oscillating circuit. Damping effects are treated by perturbation theory. Quantum effects occur both in the damping and in the noise, and are discussed in detail. An interpretation is given of the infinite zero point contribution which appears in the theory of Callen and Welton. The average electromagnetic field energy of an oscillator with capacitance $C$, conductance $G$, and natural frequency $\omega$ as a function of time is given by

$$U=\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right][1-e^{-Gt/C}]+U_0e^{-Gt/C}.$$

The mean squared noise voltage which would be measured in an experiment with a damped oscillator is given by

$$\overline{V^2}=\frac{1}{C}\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right].$$

The maximum noise power which a conductance $G$ at temperature $T$ can transfer to a damped oscillator approaches the value

$$\frac{G\hbar\omega}{C[\exp(\hbar\omega/kT)-1]}.$$

The vacuum fluctuations are shown to be observable in certain noise experiments.

## INTRODUCTION

VIRTUALLY all of the phenomena occurring in electric circuits are described classically in a satisfactory way by Maxwell's equations. Application of classical statistics has led to a satisfactory understanding of most electrical fluctuation phenomena. The classical description is usually adequate because ordinary measurements are made at room temperature with circuit currents exceeding noise currents. If measurements were made at low temperatures, with smaller currents, the quantum effects would be significant. No experiments have been carried out under such conditions. The present paper is an effort to provide some basic work for a general quantum theory of circuits and noise.

Recently Callen and Welton[1] presented an elegant quantum theory of noise. Their results showed as one of the quantum effects an infinite zero-point noise contribution for a pure resistance. The theory to be presented here gives insight into the origin of the infinite zero-point contribution and predicts finite quantum effects in certain experiments.

## AN OSCILLATING CIRCUIT WITH NO DISSIPATION

We consider first an electrical oscillator which we imagine made up of perfect conductors with no radiation. One is tempted to treat such a system as Fig. 1 as an ensemble of particles and to discuss its behavior in terms of charges and currents. This procedure leads to difficulties because with perfect conductors there are no tangential electric fields near the conductors. In order to allow currents to change without electric fields, charged particles without mass or an infinite number of carriers with mass would be required. To avoid these difficulties we choose to discuss the fields. The energy is

$$U=\frac{1}{8\pi}\int_V (E^2+H^2)d\tau, \tag{1}$$

where $E$ and $H$ are the electric and magnetic fields, and the integral is throughout space. We represent the magnetic vector potential as the product of a time-dependent and space-dependent part $q(t)A(r)$. In terms of the vector potential,

$$\mathbf{E}=-(1/c)\dot{q}\mathbf{A},\quad \mathbf{H}=q\nabla\times\mathbf{A}. \tag{2}$$

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[1] H. B. Callen and T. H. Welton, Phys. Rev. 83, 34 (1951).

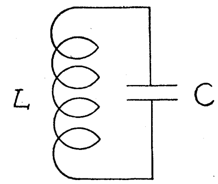


FIG. 1. Electrical oscillator with no dissipation.

From (2) we get, using Maxwell's equations,

$$\nabla\times\mathbf{H}=[\nabla(\nabla\cdot\mathbf{A})-\nabla^2\mathbf{A}]q=-(1/c^2)\ddot{q}\mathbf{A}. \qquad (3)$$

The fields are entirely outside of the perfectly conducting boundaries, and the most general[2] solution of Maxwell's equations can be expressed in terms of potentials such that the divergence of $\mathbf{A}$ is zero, and the scalar potential is also zero. If $q$ oscillates harmonically with time, with angular frequency $\omega$, (3) becomes

$$\nabla^2\mathbf{A}+(\omega^2\mathbf{A}/c^2)=0. \qquad (4)$$

We normalize $A$ so that

$$\int_V A^2 d\tau=4\pi c^2.$$

We introduce a variable $p$ canonically conjugate to $q$ by letting $p=\dot{q}$; from (2), $\mathbf{E}=-(1/c)p\mathbf{A}$. The total energy in the electric field becomes

$$\frac{1}{8\pi}\int_V E^2 d\tau=\frac{p^2}{8\pi c^2}\int A^2 d\tau=\frac{p^2}{2}. \qquad (5)$$

The magnetic energy is

$$\frac{1}{8\pi}\int_V H^2 d\tau=\frac{q^2}{8\pi}\int_V(\nabla\times A)^2 d\tau=\frac{q^2}{8\pi}\left[\int_S \bar{A}\times\nabla\times\bar{A}\cdot d\bar{s}+\int\bar{A}\cdot[\nabla(\nabla\cdot\bar{A})-\nabla^2\bar{A}]d\tau\right]. \qquad (6)$$

Substitution of (2) into the first term on the right side of (6) reduces it to

$$(qc/8\pi\omega)\int_S \bar{E}\times\bar{H}\cdot d\bar{s},$$

where the surface integral is over a closed surface surrounding the circuit. From Poynting's theorem this term is proportional to the radiated power which is postulated to be zero. Equation (6) becomes

$$\frac{1}{8\pi}\int_V H^2 d\tau=\frac{q^2\omega^2}{2}. \qquad (7)$$

The Hamiltonian (1) is now

$$H=\tfrac{1}{2}(p^2+\omega^2q^2). \qquad (8)$$

Our variables $p$ and $q$ must be noncommutable operators; otherwise it would be possible[3] to measure in the same region simultaneously the electric field and the magnetic field with arbitrarily great precision. We therefore adopt the commutation rule:

$$pq-qp=-i\hbar. \qquad (9)$$

Expressions (8) and (9) make the problem of the undamped electrical oscillator formally identical with the harmonic oscillator. The wave functions for $q$ are the well-known harmonic oscillator wave functions. The allowed values for the energy are $U=(n+\frac{1}{2})\hbar\omega$.

## HARMONIC OSCILLATOR WITH DISSIPATION

We represent a damped oscillator by an oscillator of the type discussed above, coupled to a resistance, as shown in Fig. 2. We follow the general method of Callen and Welton.[1] The Hamiltonian can be written as

$$H=\tfrac{1}{2}(p^2+\omega^2q^2)+H_R+Q\int\mathbf{E}\cdot d\mathbf{l}. \qquad (10)$$

Here $H_R$ is the Hamiltonian of the ensemble of particles making up the resistance; $Q$ is a function of the coordinates and momenta of the particles of the resistance, and the line integral is over the length of the resistance. In terms of the capacity $C$ we can write

$$\frac{1}{8\pi}\int_V E^2 d\tau=\tfrac{1}{2}C\left[\int\mathbf{E}\cdot d\mathbf{l}\right]^2=\tfrac{1}{2}p^2. \qquad (11)$$

Making use of (11) the Hamiltonian becomes

$$H=\tfrac{1}{2}(p^2+\omega^2q^2)+H_R+(pQ/\sqrt{C}). \qquad (12)$$

We treat the last term as an interaction term which will cause transitions with exchange of energy between the LC circuit and the resistance. We assume that the oscillator is weakly damped; if we use first order perturbation theory, the transition probability can be shown to be:

$$W_r=\frac{2\pi}{\hbar}[\rho(E_R+\hbar\omega)\langle E_R|Q|E_R+\hbar\omega\rangle^2\times\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2+\rho(E_R-\hbar\omega)\times\langle E_R|Q|E_R-\hbar\omega\rangle^2\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2]. \qquad (13)$$

The symbol $\langle E_R|Q|E_R+\hbar\omega\rangle$ indicates the matrix element of the operator corresponding to $Q$ between the quantum states of the resistance with eigenvalues $E_R$ and $E_R+\hbar\omega$, $\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle$ has the corresponding meaning for the quantum states of the field. $\rho(E_R+\hbar\omega)$ is the density in energy of the quantum states of the resistance in the vicinity of the energy $E_R+\hbar\omega$. Expression (13) gives the total transition probability from

[2] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 265.

[3] W. Heisenberg, *Physical Principles of the Quantum Theory* (Dover Publications, New York, 1950), p. 50.

an eigenstate of the unperturbed system in which the field has the eigenvalue $E_F$, and the resistance has the eigenvalue $E_R$. We may assume that initially the circuit is in an eigenstate (before being coupled to the resistance). There will never be enough information about the resistance to say that it is in an eigenstate, but its state will be partially specified in that its temperature will be known. It is therefore necessary to average (13) over an ensemble of similar systems, the result is

$$W_r=\frac{2\pi}{\hbar}\bigg[\left\langle E_F\left|\frac{p}{\sqrt{C}}\right|E_F-\hbar\omega\right\rangle^2\int_0^\infty \rho(E_r+\hbar\omega)\times\langle E_r|Q|E_r+\hbar\omega\rangle^2\rho(E_r)f(E_r)dE_r+\left\langle E_F\left|\frac{p}{\sqrt{C}}\right|E_F+\hbar\omega\right\rangle^2\int_{\hbar\omega}^\infty \rho(E_r-\hbar\omega)\times\langle E_r|Q|E_r-\hbar\omega\rangle^2\rho(E_r)f(E_r)dE_r\bigg]. \quad (14)$$

Consider $f(E)$ the statistical weighting factor, and $f(E+\hbar\omega)/f(E)=\exp-(\hbar\omega/kT)$. The second integral has $\hbar\omega$ as a lower limit because energy is conserved in these transitions, and no resistance in the ensemble can undergo transitions which reduce its energy if its energy is less than $\hbar\omega$. We introduce the quantity

$$S=\frac{2\pi}{\hbar}\int_0^\infty \rho(E_r+\hbar\omega)\langle E_r|Q|E_r+\hbar\omega\rangle^2\times\rho(E_r)f(E_r)dE_r. \quad (15)$$

By making a change of variable in the second integral of (14) and making use of (15) we can put (14) into the form:

$$W_r=S[\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2+\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\exp-(\hbar\omega/kT)]. \quad (16)$$

The first term of (16) is the probability per unit time that the circuit and fields will lose a quantum, and the second term is the probability per unit time that the circuit and fields will gain a quantum. The net probability that a quantum will be lost will be the difference of the two terms. The average rate of change of energy of a circuit in the ensemble will be

$$dU/dt=S\hbar\omega[\langle E_F|p/\sqrt{C}|E_F+\hbar\omega\rangle^2\exp-(\hbar\omega/kT)-\langle E_F|p/\sqrt{C}|E_F-\hbar\omega\rangle^2]. \quad (17)$$

If we insert the well-known harmonic oscillator matrix elements into (17) the result is

$$\frac{dU}{dt}=\frac{S(\hbar\omega)^2}{2C}[(n+1)\exp-(\hbar\omega/kT)-n], \quad (18)$$

where $n$ is the quantum number of the circuit. Equation (18) states that the circuit may either gain or lose energy, depending upon $n$. It is interesting that the equilibrium value of $n$ [for which (18) is zero] is

$$n=1/[\exp(\hbar\omega/kT)-1].$$

The principal quantum effects are evident in (18); classically the rate of energy loss would be proportional to the energy at time $t$. This is only true in (18) if $n$ is large.

If the relation for the energy, $U=(n+\frac{1}{2})\hbar\omega$, is inserted into (18) and the resulting equation integrated,† we obtain

$$U=\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right]\times\left[1-\exp\left\{-\frac{\hbar\omega S}{2C}[1-\exp-(\hbar\omega/kT)]t\right\}\right]+U_0\exp\left\{-\frac{\hbar\omega S}{2C}[1-\exp-(\hbar\omega/kT)]t\right\}. \quad (19)$$

In Eq. (19), $U$ is the average energy of a circuit in the ensemble as a function of time and the initial energy $U_0$; $C$ is the capacity, and $S$ is defined by (15). For large energy (classical limit) the second term is the only significant one. Comparing this with the known classical solution $U=U_0e^{-Gt/C}$, where $G$ is the conductance, we obtain:

$$G=\frac{\hbar\omega S}{2}[1-\exp-(\hbar\omega/kT)]=\pi\omega[1-\exp-(\hbar\omega/kT)]\times\int_0^\infty \rho(E_r+\hbar\omega)\langle E_r|Q|E_r+\hbar\omega\rangle^2\rho(E_r)f(E_r)dE_r, \quad (20)$$

in agreement with the result of Callen and Welton. In terms of (20), (19) becomes

$$U=\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right][1-e^{-Gt/C}]+U_0e^{-Gt/C}. \quad (19a)$$

## EQUIVALENCE OF RESISTANCE AND A NOISE GENERATOR

In this section we prove the following theorem: The Johnson[4] noise plus "spontaneous" emission is entirely

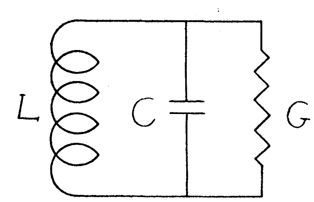


FIG. 2. Damped electrical oscillator.

† The procedure of this section is based on the discussion of E. C. Kemble, *The Fundamental Principles of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), Chapter 12.

[4] J. B. Johnson, Phys. Rev. **32**, 97 (1928).

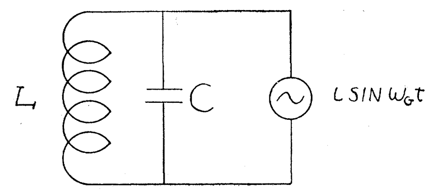


FIG. 3. Electrical oscillator and current generator.

equivalent to all damping effects which the resistance has upon the oscillating circuit. The noise is seen to play a role in the damping process. To show this let us imagine that the resistance is removed, and that it is replaced by a "current generator" which has an infinite internal impedance.

It may readily be shown by comparison with the classical differential equations that the Hamiltonian of the system of Fig. 3 is

$$H=\tfrac{1}{2}(p^2+\omega^2q^2)-(qi/\sqrt{C})\sin(\omega_0 t). \tag{21}$$

It is well known that an interaction term of the form of the last term of (21) will not give a transition probability proportional to time. To obtain transitions equivalent to those of the resistance we need a current generator with a continuous spectrum in the vicinity of $\omega$. Under these conditions the transition probability is

$$W_G=\frac{\pi[i^2(\omega)]_{\text{Av}}}{C\hbar^2}[\langle E_F|q|E_F+\hbar\omega\rangle^2 + \langle E_F|q|E_F-\hbar\omega\rangle^2], \tag{22}$$

where $C$ is the capacitance, and the mean square value of the current over a range $d\omega$ is $[i^2(\omega)]_{\text{Av}}d\omega$. Inserting the harmonic oscillator matrix elements, (22) becomes

$$W_G=\frac{\pi[i^2(\omega)]_{\text{Av}}}{2C\hbar\omega}[(n+1)+n], \tag{23}$$

where $n$ is the quantum number of the oscillator.

In order to compare the transition probability induced by the current generator with that induced by the resistance we insert the appropriate matrix elements into (16). Making use of (15) and (20) and rearranging terms, (16) becomes

$$W_r=\frac{G}{C}\left[\frac{(n+1)+n}{\exp(\hbar\omega/kT)-1}+n\right]. \tag{24}$$

Comparison of (23) and (24) shows that the transition probability will be the same, in so far as the first term of (24) is concerned, if

$$[i^2(\omega)]_{\text{Av}}=\frac{2G\hbar\omega}{\pi[\exp(\hbar\omega/kT)-1]}. \tag{25}$$

Equation (25) is the Nyquist[5] formula in the equivalent current representation, modified for quantum effects.

There is still the last term of (24). It is apparent that the last term in (24) is just the transition probability at $T=0$, that is, the transition probability if the resistance is in its lowest state and the quantum number of the circuit is $n$. This is closely analogous to the spontaneous emission which atoms undergo even if the radiation fields are in their lowest states. We therefore conclude that the transitions required by (24) will be produced by a noise current generator described by (25) plus spontaneous emission, that is, plus the effect of the absorber in its lowest state.

We can get a more formal analogy with the spontaneous emission induced by the radiation modes in atoms if we imagine the second term of (24) to be equivalent to that of a current generator which can only induce downward transitions. Comparing the second terms of (23) and (24) we see that the equivalent current for such a generator is

$$[i^2(\omega)]_{\text{Av}}=2G\hbar\omega/\pi. \tag{26}$$

This result is formally analogous to that obtained in the treatment of spontaneous emission of radiation by Park and Epstein.[6]

## MEAN SQUARED NOISE VOLTAGE AND AVAILABLE POWER

We can calculate the result of precise measurements of the mean squared noise voltage of the damped oscillator by averaging the equilibrium value of the quantity $[\int E\cdot dl]^2$ over the ensemble. From (11) we obtain

$$\overline{V^2}=\left\langle\left[\int E\cdot dl\right]^2\right\rangle =\frac{\sum_n\langle E_{FN}|p^2/C|E_{FN}\rangle\exp[-(n+\tfrac{1}{2})\hbar\omega/kT]}{\sum_n\exp[-(n+\tfrac{1}{2})\hbar\omega/kT]}. \tag{27}$$

Carrying out the summations indicated in (27) we obtain

$$\overline{V^2}=\frac{1}{C}\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right] =\frac{\Delta\omega}{G}\left[\tfrac{1}{2}\hbar\omega+\frac{\hbar\omega}{\exp(\hbar\omega/kT)-1}\right], \tag{28}$$

where $\Delta\omega$ is the classical half-breadth. The first term of (28) is $\hbar\omega/2C$ and represents noise which would be observable even if the oscillating circuit were in its lowest state. It represents the fluctuations of the vacuum surrounding the circuit. It will now be shown that this term cannot be removed by making formal changes in the Hamiltonian which remove the zero-point energy.[7] The proof follows the discussion of the corre-

[5] H. Nyquist, Phys. Rev. 35, 110 (1928).

[6] D. Park and H. T. Epstein, Am. J. Physics 17, 301 (1949).

[7] The author is indebted to the referee for suggesting investigation of this point.

sponding problem[8] in the quantum theory of the fields in vacuum. We introduce the auxiliary variables $\beta$ and $\beta^*$, defined by

$$q=\beta+\beta^*,\quad p=-i\omega(\beta-\beta^*).$$

The Hamiltonian 8 can be written

$$H=\omega^2[\beta^*\beta+\beta\beta^*]. \tag{29}$$

The correspondence with the classical theory is equally good if (29) is written

$$H=2\omega^2\beta^*\beta=\tfrac{1}{2}(p^2+\omega^2q^2)-\tfrac{1}{2}\hbar\omega. \tag{29A}$$

The eigenvalues of the Hamiltonian (29A) do not have the zero-point energy, but the eigenfunctions of this Hamiltonian are the same as those of the Hamiltonian (8). The quantity

$$\langle E_{FN}|p^2/C|E_{FN}\rangle=\int\psi_{FN}^*(p^2/C)\psi_{FN}d\tau$$

is unchanged, and the summation (27) is also unchanged. The zero-point noise contribution is seen to be independent of the choice of zero-point energy. It is in fact due to the random interaction of the apparatus for measuring the electromotive force, with the circuit, and is related to the uncertainty principle. Equation (28) does not assert that one can observe the zero-point energy; it does assert that one can observe the zero-point fluctuations.

Another quantity which is specified in the classical discussion of resistance noise is the available power. To calculate the power which a resistance would transfer to another resistance within a specified frequency range we consider the experimental arrangement of Fig. 4. The Hamiltonian of such a system is

$$H=\tfrac{1}{2}(p^2+\omega^2q^2)+(p/\sqrt{C})[Q_1+Q_2]. \tag{30}$$

We can deduce the expression for the rate of change of the field energy in the same manner as (18) is obtained, the result is

$$\frac{dU}{dt}=\frac{(\hbar\omega)^2}{2C}[S_1[(n+1)\exp-(\hbar\omega/kT_1)-n] +S_2[(n+1)\exp-(\hbar\omega/kT_2)-n]], \tag{31}$$

where

$$S_1=\frac{2\pi}{\hbar}\int_0^\infty\rho(E_{R1}+\hbar\omega)\langle E_{R1}|Q_1|E_{R1}+\hbar\omega\rangle^2 \times\rho(E_{R1})f(E_{R1})dE_{R1},$$

$$S_2=\frac{2\pi}{\hbar}\int_0^\infty\rho(E_{R2}+\hbar\omega)\langle E_{R2}|Q_2|E_{R2}+\hbar\omega\rangle^2 \times\rho(E_{R2})f(E_{R2})dE_{R2}.$$

The stationary value of $dU/dt$ is obtained for a value of $n$ which makes (31) zero. In terms of the conductances $G_1$ and $G_2$ this is

$$n=\left[\frac{G_1}{\exp(\hbar\omega/kT_1)-1}+\frac{G_2}{\exp(\hbar\omega/kT_2)-1}\right]\Big/(G_1+G_2). \tag{32}$$

The average rate at which $G_1$ transfers energy to the system is obtained from (18) if we insert the stationary value of $n$ as given by (32). The result is

$$\frac{G_1\hbar\omega}{C}\left[\frac{1}{\exp(\hbar\omega/kT_1)-1} -\left(\frac{G_1}{\exp(\hbar\omega/kT_1)-1}+\frac{G_2}{\exp(\hbar\omega/kT_2)-1}\right)\Big/(G_1+G_2)\right]. \tag{33}$$

Equation (33) gives us the net power transferred to the system by $G_1$. Equation (33) will be a maximum if $T_2\to 0$ and $G_2/G_1\to\infty$.

$$P_{\max}\to\frac{G_1\hbar\omega}{C[\exp(\hbar\omega/kT_1)-1]}=\frac{\hbar\omega(\Delta\omega)_1}{\exp(\hbar\omega/kT_1)-1}, \tag{34}$$

where $(\Delta\omega)_1=G_1/C$. Equation (34) is somewhat different from the classical value because we have chosen to specify the maximum power in a way which is different from the classical one but more precise for our purposes.

## NOISE MEASUREMENT EXPERIMENTS

Callen and Welton have given an integral for the noise of a pure resistance. They did not discuss the spectrum of the noise, and their integral contains an infinite zero point contribution.

We might measure the power spectrum of the noise by employing a filter with a flat response within the pass band and infinite rejection outside of the pass band. For simplicity we choose instead to measure the power spectrum in the vicinity of $\omega$ by connecting an LC circuit of natural frequency $\omega$ to the resistance, as in Fig. 2, and measuring the expectation value of the square of the electromotive force. The result of such an experiment is given by Eq. (28). Although (28) was

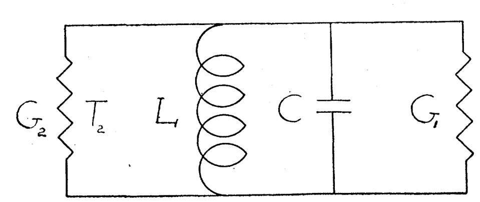


FIG. 4. Electrical oscillator coupled to conductances at different temperatures.

[8] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1944), p. 60.

calculated as an average over the ensemble, the ergodic theorem guarantees that the same result will be obtained if repeated measurements are made with a single resistance. This is because the measurements do not affect the known partial specification of the state of the system. This measurement gives the noise contribution in the vicinity of $\omega$. To obtain the noise over all frequencies we would need an infinite number of circuits of the kind discussed in this paper. The resulting zero-point contribution is therefore infinite. This is believed to be the interpretation of Callen and Welton's result.

All resistances have physical size, and there will always be a certain amount of inductance and distributed capacity. We would always have an arrangement somewhat similar to that of Fig. 2. In making the measurements we can always couple to either a single mode or at most a finite number of modes, and the zero-point noise contribution is finite.

Equation (34) shows that the maximum power which a resistance can transfer to a system tends to zero at low temperatures, while according to (28) the mean squared value of the electromotive force approaches the limit $\hbar\,\omega/2C$. Lawson[9] suggested noise measurements as a method for measuring temperature. If the noise measurements are made by measuring the transitions induced by the resistance (power measurements) there will be no zero-point contribution, according to (34). On the other hand if we measure the mean squared value of the electromotive force there will be a zero-point contribution as given by (28).

[9] A. W. Lawson and E. A. Long, Phys. Rev. **70**, 220 (1946).

## CONCLUSION

In this paper we have examined some of the consequences of the application of field quantization to electrical circuits. The theory gives the familiar classical effects and includes in addition the noise and quantum effects. It shows clearly the role of noise in damping. The zero-point noise contribution which appeared first in the theory of Callen and Welton is shown to represent an observable effect, independent of the choice of zero-point energy. Experiments at low temperatures and high frequencies offer an opportunity to study in detail the quantum effects of a single mode of the electromagnetic field. When *precise* noise measurement techniques are developed it should be possible to observe directly the vacuum fluctuations in a low temperature noise experiment. In a subsequent paper the interaction of circuits with radiation fields and with electrons will be discussed. I wish to acknowledge stimulating discussions with Dr. M. H. Johnson.