be less than 0.5 percent in CH_2 and may be much smaller. If it occurred for 1 out of 200 stopped π^--mesons, and as many as 5 percent of the π^-'s stopped in the target, this would contribute only 2.5 counts per million M counts to the observed CH_2–C difference. This source of error is therefore negligible.

That nuclear gamma-rays cannot contribute much to the CH_2–C difference is also shown by the fact that the results obtained with a 15-Mev minimum detectable electron energy agree with those for 8-Mev minimum, assuming the π^0-decay gamma-ray spectrum (i.e., high energy gamma-rays). The π^--capture gamma-rays would be expected to be much lower in energy, and the change in detection efficiency much more than the observed factor of two. In addition, there is the angular correlation demonstrated by the two quantum coincidences.

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The Absorption of Slow π^- Mesons by He^4 Nuclei*

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On the assumption of central charge-independent two particle nuclear forces, the wave function $(E+K)^{-2}(bE+K)^{-2}$ with E the binding energy, K the kinetic energy, and $b=5.3$ is found to be a better wave function, in the sense of a variational principle, for the He^4 nucleus than the optimum Gaussian. With this wave function, the ratios of the numbers of protons to deuterons to tritons in final states resulting from the absorption of π^- mesons from K shells of mesic helium atoms are found to be 1:1.3:0.7. For reasonable values of the $PS(PV)$ coupling constant, 20 percent of the absorptions take place directly from the $2p$ level. The number of high energy γ-rays is less than 1 percent of the number of nonradiative absorptions. Charge-exchange absorption is energetically forbidden.

INTRODUCTION

When a negative pion is absorbed by a He^4 nucleus, three types of final states are available for the nuclear system, namely,

$$p+3n,\quad d+2n,\quad t+n,$$

to which we shall refer as the proton, deuteron, and triton final states, respectively. The transition to any of these states may be accompanied by the emission of electromagnetic radiation but not, because of the conservation of energy, by the emission of neutral pions. A calculation of the relative probabilities of transitions to these final states is of interest not only to corroborate the information concerning the π-meson which is gained by a study of its interaction with the proton and the deuteron,[1] but also because, through the dependence of the relative probabilities on the nuclear wave function, it sheds light on the structure of the nucleus. The He^4 nucleus is the nucleus next in complexity to the deuteron that is available in sufficient quantity to make an experiment practicable. It is also the lightest nucleus with structural resemblance to heavier nuclei in that both the binding per energy nucleon and the average kinetic energy per nucleon are close to the corresponding figures for heavy nuclei.

For these reasons it was thought worth while to extend to the He^4 nucleus the calculation on the absorption of slow negative pions by nuclei carried out at this laboratory by Marshak,[2] Tamor,[3] and Messiah.[4] In addition to these calculations, several others of similar scope precede ours, namely, two calculations of Bruno,[5] and one of Clark and Ruddlesden.[6] Besides the use of a less carefully chosen wave function for the He^4 nucleus and differences in the detailed treatment of the final states, the latter calculations differ from ours in several important respects. The first calculation of Bruno employed a meson mass of 100 Mev in accordance with the experimental data at that time. Moreover, transitions to the triton final state alone were calculated. In his next calculation Bruno revised the meson mass, and because the larger mass is expected to lead to a larger yield for the proton final states, he calculated the transition rate to such states (for vector mesons) and found it to be larger than the triton rate. In the calculation of Clark and Ruddlesden the nucleons are taken to be infinitely heavy in calculating the interaction Hamiltonian (not, of course, in the kinematics) and the electromagnetic radiation accompanying the absorption is not considered.

The present calculation resembles in many respects the calculation of Messiah for He^3. Section I describes the method of calculation in general terms and allows the construction in Sec. II of the wave functions both of the initial He^4 nucleus and of the final nuclear

* This paper is based upon a thesis submitted to the University of Rochester in partial fulfillment of its requirements for the degree Doctor of Philosophy.

[1] R. E. Marshak, Revs. Modern Phys. **23**, 137 (1951).

[2] R. E. Marshak and A. S. Wightman, Phys. Rev. **76**, 114 (1949).

[3] S. Tamor, Phys. Rev. **82**, 38 (1951).

[4] A. M. L. Messiah, Phys. Rev. **87**, 639 (1952).

[5] B. Bruno, Arkiv Mat. Astron. Fysik **36A**, No. 8 (1948) and Arkiv Fysik **1**, 19 (1949).

[6] A. C. Clark and S. N. Ruddlesden, Proc. Phys. Soc. (London) **64**, 1064 (1951).

fragments. Section III contains the principal part of the calculation, including a calculation of the relative frequency of the three final states for absorption from s and p states of the π-mesic helium atom, an estimate of the ratio of optical transitions from the p state to direct nuclear absorption from it, and an upper limit to the number of gamma-rays accompanying nuclear absorption from s states. Finally, Sec. IV presents an evaluation of the results and a discussion of the validity of some of the approximations employed in their calculation.

I. GENERAL METHOD OF CALCULATION

Throughout this calculation we shall assume that the pseudoscalar nature of the meson is established.[1] Furthermore only pseudovector coupling is considered in order to allow a phenomenological treatment of the nuclear force.[7]

The wave function of the initial He^4 nucleus is calculated as a wave function of four Schrödinger particles bound by two-particle nuclear forces which are taken to be central and charge independent. The effect of the tensor component of the nuclear forces on the wave function is taken into account in the calculation of the absorption of mesons from the mesic p state, where it is most important. The range of the nuclear forces is taken from the two-particle scattering data,[8] while the strength is adjusted to give the proper binding energy for the alpha-particle.

Subsequent to the calculation of the wave function the nucleus is treated as an assemblage of Dirac particles whose momentum distribution is given by the Schrödinger wave function.[4] This and the next approximation are both valid in the limit $v/c \ll 1$. The maximum v/c of interest occurs for the free neutron in the triton final states and is $(v/c)^2 = 0.19$.

The interaction between the meson and the nucleons is taken to be the nonrelativistic limit of the pseudovector interaction, namely,

$$O_i = (2\pi/\mu)^{\frac{1}{2}}(f/\mu)[\boldsymbol{\sigma}_i \cdot \nabla_i \phi - i\mu \boldsymbol{\sigma}_i \cdot \mathbf{P}_i \phi]\boldsymbol{\tau}_i^-, \quad (1)$$

where ϕ is the meson wave function, f, the coupling constant, μ, the meson mass, and $\boldsymbol{\sigma}_i$, $\boldsymbol{\tau}_i$, $\mathbf{P}_i$ the spin, isotopic spin, and momentum of the ith nucleon, respectively. The units used throughout are $\hbar = c = M = 1$. Occasionally, the second term in Eq. (1) has been neglected.[6,9] Not only does this term arise quite naturally in taking the nonrelativistic limit of the $PS(PV)$ theory but it is necessary to preserve the Galilean invariance and the two-particle nature of the meson-nucleon interaction. Galilean invariance requires that the interaction depend only on a relative velocity, and the two-particle nature of the interaction requires that this be the relative velocity of the meson and one nucleon, rather than some other relative velocity, such as that of the meson and the entire nucleus, for instance. An estimate of the relative importance of the two terms may be obtained by assuming that the meson is in the Coulomb field of the nucleus and that this field is cut off at some small radius. The ratio of the first term in Eq. (1) to the second is of the order 1/60 for absorption from s states. In case the meson is absorbed from a p state the first approximation to the first term in the interaction is a constant rather than zero. The relative importance of the two terms is nevertheless maintained in He^4 because of a selection rule associated with operators independent of position. The higher approximations to the first term are then roughly of the same order as for the s state.

In view of these remarks, it is a good approximation to take the meson wave function to be a constant within the nucleus for s states and to have a constant gradient for p states. Wherever it is necessary to assign a value to this constant it is chosen to agree with the wave function corresponding to a pure Coulomb field.

In calculating the emission of radiation resulting from the nuclear absorption of a slow π^- meson, three terms in the interaction Hamiltonian must be taken into account: the coupling between the radiation field and the free meson and nucleons; the triple term in the meson-nucleon-radiation interaction introduced to maintain gauge invariance; and, finally, the coupling between the radiation field and the mesons responsible for the nuclear forces. The first part of the interaction is easily discarded since it is of higher order in v/c than the second. Several reasons may be put forward for neglecting the third part of the interaction also: first, it is of higher order in the coupling constant and would therefore not appear in a consistent weak coupling calculation; second, in the impulse approximation (which is, however, quite hard to justify for a structure as tightly bound as the He^4 nucleus) terms of this nature are neglected; and, third, it is possible to compare the importance of these terms with their importance in other processes. In the photodisintegration of the deuteron this type of term becomes important in the vicinity of 80 Mev because of the rapid fall-off of the deuteron wave function at high energies.[10] In our case such high momenta are not involved (in addition they are abundant in the He^4 nuclear wave function) and the third interaction is being compared with the second rather than the first as is the case in the photodisintegration. On the other hand, there are of course more mesons in transit at any one time in the He^4 nucleus than in the deuteron. It is difficult to evaluate the merits of these arguments,

[7] K. Brueckner, Phys. Rev. **82**, 598 (1951). Pseudoscalar coupling may, however, give similar results. See, for instance, S. D. Drell and E. M. Henley, Stanford Microwave Laboratory Report 165 (unpublished).

[8] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949), J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

[9] Chew, Goldberger, Steinberger, and Yang, Phys. Rev. **84**, 581 (1951).

[10] B. Bruno and S. Depken, Phys. Rev. **86**, 1054 (1952).

but it does seem reasonable to assume that the third interaction should be somewhat less than the second for the 100-Mev photons that are of interest for our case. Therefore, only the second part of the interaction will be considered. The transition is then a first-order process taking place through the operator,

$$O_{\gamma i}=f\phi(2\pi/\mu)(e^2/\mu k)^{\frac{1}{2}}\boldsymbol{\tau}_i^-\boldsymbol{\sigma}_i\cdot\boldsymbol{\varepsilon}\Delta_i(\mathbf{k}), \tag{2}$$

where k is the photon momentum, $\boldsymbol{\varepsilon}$ its polarization, and $\Delta_i(\mathbf{k})$ the transfer of momentum $\mathbf{k}$ to the ith nucleon.

The final states are calculated neglecting the forces between unbound particles. A discussion of this approximation is deferred until Sec. IV.

Finally, the matrix element for a transition is given by

$$\left\langle\psi_F\left|\sum_i\begin{Bmatrix}O_i\\O_{\gamma i}\end{Bmatrix}\right|\psi_\alpha\right\rangle, \tag{3}$$

where ψ_α and ψ_F are the initial and final wave functions, respectively.

II. THE WAVE FUNCTIONS

In this section the wave functions of the initial and final states will be discussed. In order to facilitate this discussion we begin by discussing the symmetry group on four letters. Much of this discussion has been carried out by Gamba,[11] whose notation we have adopted, except for trivial changes. The operators T given by Gamba form part of the normal representation[12] of the group. These operators obey the multiplication table,

$$o_{ij}{}^r o_{km}{}^t=o_{im}{}^t\delta_{jk}\delta_{rt},$$

and their Hermitian conjugates are given by

$$o_{ij}{}^{r\dagger}=o_{ji}{}^r.$$

Finally, if φ is a normalized wave function symmetric in the exchange of m particles and orthogonal to itself upon any other exchange, then a wave function transforming according to the representation s and also normalized is

$$(\theta^s/m!)^{\frac{1}{2}}o_{ij}{}^s\varphi \tag{4}$$

with θ^s the ratio of the number of elements in the permutation group to the dimension of s.

Operating with o^s and $o_{j1}{}^+$ on the coordinate $\mathbf{R}_4$ and momentum $\mathbf{P}_4$ of the fourth particle generates our coordinate system,

$$\begin{aligned}
\mathbf{Q}&=\tfrac{1}{2}(\mathbf{R}_1+\mathbf{R}_2+\mathbf{R}_3+\mathbf{R}_4),\\
\mathbf{Q}_1&=\tfrac{1}{2}3^{-\frac{1}{2}}(3\mathbf{R}_4-\mathbf{R}_1-\mathbf{R}_2-\mathbf{R}_3),\\
\mathbf{Q}_2&=2^{-\frac{1}{2}}(\mathbf{R}_2-\mathbf{R}_3),\\
\mathbf{Q}_3&=6^{-\frac{1}{2}}(2\mathbf{R}_1-\mathbf{R}_2-\mathbf{R}_3),\\
\boldsymbol{\Pi}&=\tfrac{1}{2}(\mathbf{P}_1+\mathbf{P}_2+\mathbf{P}_3+\mathbf{P}_4),\\
\boldsymbol{\Pi}_1&=\tfrac{1}{2}3^{-\frac{1}{2}}(3\mathbf{P}_4-\mathbf{P}_1-\mathbf{P}_2-\mathbf{P}_3),\\
\boldsymbol{\Pi}_2&=2^{-\frac{1}{2}}(\mathbf{P}_2-\mathbf{P}_3),\\
\boldsymbol{\Pi}_3&=6^{-\frac{1}{2}}(2\mathbf{P}_1-\mathbf{P}_2-\mathbf{P}_3),
\end{aligned}$$

where the normalization has been chosen so that $\mathbf{Q}_j$ is conjugate to $\boldsymbol{\Pi}_j$ and so that the kinetic energy in the center-of-mass system is $K=\frac{1}{2}\sum\Pi_j{}^2$.

It can be shown[13] that in a reasonable zeroth approximation the space wave function of the ground state of the He^4 nucleus has spin and isotopic spin 0 and is a completely symmetric S state in space. The assumption that this is a sufficient approximation for our calculations will now be justified. Irving[14] has calculated, using the forces of Pease and Feshbach, the admixture of D state to the wave function to the He^4 nucleus and finds it to be only 3 percent. The principal effect of the tensor forces is to reduce the binding energy through this coupling to the D state. That the spin-exchange character of the nuclear forces cannot be neglected in all cases is best illustrated in the calculation below of the wave function for final states involving deuterons. On the other hand, a minimal principle is relied upon to determine the excellence of the space dependence of the wave function, and may equally well be relied upon to determine whether the assumption of space symmetry is satisfactory or not.

If the space part of the wave function is to be symmetric the spin $\times$ isotopic spin part must be completely antisymmetric. From reference 11 it is deduced that the spin and isotopic spin must both transform in R^2, that they therefore correspond to spin 0 and isotopic spin 0, and that a separate assumption about the eigenvalues of these operators is not necessary. It can be seen from the matrices corresponding to the elements of the permutation group given in reference 11 that the determinant of the matrix corresponding to any odd permutation is -1 and that therefore the antisymmetric combination of the spin and isotopic spin wave functions is the determinant

$$\xi^a=\frac{1}{\sqrt{2}}\begin{vmatrix}\chi' & \zeta'\\ \chi'' & \zeta''\end{vmatrix}.$$

[11] A. Gamba, Nuovo cimento **9**, 605 (1951).
[12] D. E. Littlewood, *The Theory of Group Characters* (Clarendon Press, Oxford, 1940). In order to obtain detailed agreement with Gamba, the natural order of the integers has been taken to be 2314 and some of the operators have been rearranged.

[13] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (J. Wiley and Sons, Inc., New York, 1952) p. 202.
[14] J. Irving, Phys. Rev. **87**, 519 (1952).

It is evident that a unitary transformation may be carried out on χ', χ'' to give one wave function that is singlet in 1, 2 and one that is triplet. For these wave functions the form of the antisymmetric wave function is unchanged,

$$\xi^a = \frac{1}{\sqrt{2}} \begin{vmatrix} \chi^S & \zeta^S \\ \chi^T & \zeta^T \end{vmatrix}.$$

With this form of the wave function it is simple to calculate the matrix element of a potential of arbitrary exchange character, yielding

$$\langle \xi^a | a + b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + c\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + d\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 | \xi^a \rangle = a - b - c - 3d.$$

This is the same as the corresponding result of Messiah,[4] so that any difference in the strength of the forces required for H^3 and He^4 nuclei cannot be explained in this approximation by adjusting the exchange character of the forces.

The spin and isotopic spin part of the wave function having been separated out and the exchange character of the potential eliminated, the problem of finding a wave function for the He^4 nucleus reduces to finding a satisfactory space dependence of the wave function. In order that the overlap integrals which will appear later in our calculations be reasonably simple, a wave function with a simple dependence on the momenta is desired. The method chosen was a simple variational method in momentum space such as the zeroth approximation of Svartholm[15] (an iteration of the zeroth approximation is clearly at variance with the requirement of simplicity) using wave functions similar to those of Messiah. This method has the sizable disadvantage that there is no way of predicting the analytic form of the wave function that is necessary in order to obtain a satisfactory minimum and the compensating advantage that the form of the wave function is imposed at the beginning of the calculation and may therefore be taken as simple as desired.

The procedure consists of inserting the chosen wave function into the expression

$$\lambda = \langle \varphi | K + E | \varphi \rangle / \langle \varphi | V | \varphi \rangle,$$

where K is the kinetic energy, E the binding energy, and V the space dependence of the potential energy which we take to be a Yukawa well,[8]

$$V = e^{-\kappa r}/\kappa r, \quad 1/\kappa = 5.54.$$

The parameter λ gives the strength of the potential and is minimized with respect to whatever adjustable parameters may be contained in the wave function. The integrals that appear in the ratio above are quite tedious but in contrast to those arising in the three-body problem may all be evaluated in terms of elementary functions for our wave function. We have calculated the minimum for two different wave functions and record the results below:

λ of Svartholm with Gaussian wave function is 1.73;

λ for wave function $(E+K)^{-1}(bE+K)^{-2}$ at $b=0.4$ is 2.00;

λ for wave function $(E+K)^{-2}(bE+K)^{-2}$ at $b=5.3$ is 1.66.

We have therefore chosen as the wave function of the He^4 nucleus

$$\psi_\alpha = 2^{-\frac{1}{2}}(\chi^S\zeta^T - \chi^T\zeta^S)(E+K)^{-2}(bE+K)^{-2}, \quad b=5.3, \tag{5}$$

which is somewhat better than the optimum Gaussian wave function.

For the final state wave functions we proceed as follows: According to Sec. I the operator responsible for the transitions in which we are interested is (1) or (2), which clearly transforms as a vector in both spin and isotopic spin space and must therefore lead from the initial state to a state in which both of these operators have the eigenvalue one. Furthermore, the space part of the operator transforms under permutations in R^s and R_+^3. Then because of the transformation character of the spin and isotopic spin wave functions, and because R^s is excluded for the space part of the wave function by the over-all antisymmetry requirement, all three wave functions transform in R_+^3. The determinants of the matrices of odd permutations in this representation are all -1 just as they are in R^2, and therefore the three wave functions,

$$6^{-\frac{1}{2}} \begin{vmatrix} \varphi_j^1 & \chi^1 & \zeta^1 \\ \varphi_j^2 & \chi^2 & \zeta^2 \\ \varphi_j^3 & \chi^3 & \zeta^3 \end{vmatrix}, \quad j=1,2,3, \tag{6}$$

are completely antisymmetric. Since $R_+^3 \times R_+^3 \times R_+^3$ contains R^a only once, these three wave functions also exhaust the possibilities. We have chosen to neglect internuclear forces except in bound systems, and therefore the wave function φ on which the operators o operate to generate the φ_j^i is simply the product of Dirac δ functions. Here and also in the deuteron and triton wave functions we have used $\varphi_j^i = 8^{\frac{1}{2}} o_{ij}^+ \varphi$, which is not properly normalized because of Eq. (4). The improper normalization is corrected in the numerical factors in the wave functions.

In the case that the final state includes a bound deuteron it is necessary to take into account spin-dependent forces; otherwise, singlet and triplet "deuterons" would appear with equal frequency. Since the spin wave functions are not eigenfunctions of all the operators $\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$ that then appear in the Hamiltonian of the system, a wave function may no longer be written arbitrarily as a product of space, spin, and isotopic spin wave functions. The wave function φ is now

$$\begin{aligned}\varphi &= \delta(\mathbf{P}_1 - \mathbf{P}_{10})\delta(\mathbf{P}_2 + \mathbf{P}_3 - \mathbf{P}_{d0})\delta(\mathbf{P}_4 - \mathbf{P}_{40})\varphi_d(\mathbf{P}_2 - \mathbf{P}_3) \\ &= \delta(\boldsymbol{\Pi}_1 - \boldsymbol{\Pi}_{10})\delta(\boldsymbol{\Pi}_3 - \boldsymbol{\Pi}_{30})\varphi_d(\boldsymbol{\Pi}_2),\end{aligned} \tag{7a}$$

[15] N. Svartholm, Thesis, Lund, 1945.

which is symmetric in 2, 3 and has been chosen so that o_{i2}^+ vanishes when applied to it. It is easily shown that the most general antisymmetric wave function is

$$6^{-\frac{1}{2}}[A\varphi_1{}^i\xi_-{}^i+B\varphi_3{}^i\xi_-{}^i+C\varphi_-{}^i\xi_+{}^i+(3/2)^{\frac{1}{2}}D(\varphi'\xi''-\varphi''\xi')], \quad (7b)$$

with

$$\xi_-{}^1=2^{-\frac{1}{2}}(\chi^2\zeta^3-\chi^3\zeta^2),$$
$$\xi_-{}^2=2^{-\frac{1}{2}}(\chi^3\zeta^1-\zeta^3\chi^1),$$
$$\xi_-{}^3=2^{-\frac{1}{2}}(\chi^1\zeta^2-\chi^2\zeta^1),$$
$$\xi_+{}^1=6^{-\frac{1}{2}}[-2\chi^1\zeta^1+\chi^2\zeta^2+\chi^3\zeta^3],$$
$$\xi_+{}^2=6^{-\frac{1}{2}}[\chi^1\zeta^2+\chi^2\zeta^1+\sqrt{2}(\chi^3\zeta^2+\chi^2\zeta^3)],$$
$$\xi_+{}^3=6^{-\frac{1}{2}}[\chi^1\zeta^3+\chi^3\zeta^1+\sqrt{2}(\chi^2\zeta^2-\chi^3\zeta^3)],$$
$$\xi'=6^{-\frac{1}{2}}[\sqrt{2}(\chi^1\zeta^2+\chi^2\zeta^1)-(\chi^3\zeta^2+\chi^2\zeta^3)],$$
$$\xi''=6^{-\frac{1}{2}}[\sqrt{2}(\chi^1\zeta^3+\chi^3\zeta^1)+\chi^3\zeta^3-\chi^2\zeta^2].$$

Then if it is possible to choose the constants so that φ which occurs in some of the $\varphi_s{}^i$ is multiplied only by triplet spin wave functions (i.e., χ^1 or χ^3) and singlet isotopic spin wave functions (i.e., ζ^2), a proper wave function for the deuteron final state is achieved. The antisymmetry ensures that any permutation $P\varphi$ of φ is multiplied by proper spin and isotopic spin wave functions. There are two wave functions that satisfy the requirements, to wit,

$$\begin{array}{llll} A=0 & B=2^{-\frac{1}{2}}, & C=6^{-\frac{1}{2}}, & D=-3^{-\frac{1}{2}}; \\ A=-2^{-\frac{1}{2}}, & B=0, & C=3^{-\frac{1}{2}}, & D=6^{-\frac{1}{2}}. \end{array} \quad (7c)$$

In the case of tritons in the final state the situation is much simpler. The space part of the triton wave function transforms in $R^s+R_+{}^3$ and only the latter part of the wave function ($R_+{}^3$) can enter into an antisymmetric wave function with the spin and isotopic spin parts, and only in the following manner:

$$\frac{1}{6}\begin{vmatrix} \varphi^1 & \chi^1 & \zeta^1 \\ \varphi^2 & \chi^2 & \zeta^2 \\ \varphi^3 & \chi^3 & \zeta^3 \end{vmatrix}, \quad (8)$$

which gives the final state wave function for a triton.

For the deuteron and triton wave functions, we have used the Hulthén

$$\varphi_d=(\gamma^2+K)^{-1}-(\alpha^2+K)^{-1},\ \alpha=0.0485,\ \gamma=0.326, \quad (7d)$$

and the Messiah

$$\varphi_t=(E+K)^{-1}(bE+K)^{-2},\quad b=8, \quad (8a)$$

wave functions, respectively. Here, K is internal kinetic energy of the deuteron or triton.

In order to determine the effect of the wave function on the results of the calculations, some of them have been repeated using the wave functions of Bruno, and of Clark and Ruddlesden which read, in momentum space,

$$\varphi_\alpha=\exp(-K/8a), \quad (5a)$$
$$\varphi_d=\exp(-K/4a), \quad (7e)$$
$$\varphi_t=\exp(-K/6a), \quad (8b)$$
$$a^{-\frac{1}{2}}=18.8.$$

The average kinetic energy for this He^4 nuclear wave function is 48 Mev, which is much less than the value 130 Mev obtained for our wave function.

An attempt is made elsewhere to assess the effect upon the absorption of mesons of the forces between unbound systems which have been neglected in calculating the wave functions above, but the following remark may be in order. Since no forces are taken into account in calculating the wave functions of the proton final states, forces between only two particles are taken into account in calculating the wave functions of the deuteron final states, while the forces between three particles are taken into account in calculating the wave functions of the triton final states, the three kinds of wave functions are not eigenfunctions of the same Hamiltonian and are therefore not automatically orthogonal. That they are in fact orthogonal results from the stability of the bound systems which ensures that

$$(\Pi_1{}^2)_{\text{tritons}}>(\Pi_1{}^2)_{\text{deuterons}}$$

and

$$[\Pi_1{}^2+\Pi_3{}^2]_{\text{deuterons}}>[\Pi_1{}^2+\Pi_3{}^2]_{\text{protons}},$$

which, in turn, is sufficient to ensure the orthogonality of the wave functions that are here considered.

III. THE TRANSITION PROBABILITIES

The evaluation of the various transition probabilities now proceeds in a perfectly straightforward manner. Except for the differences in the normalization of the space wave functions indicated in Eq. (4) and the differences in the amplitude of that part of the final wave function that transforms in $R_+{}^3$, both of which have been incorporated into the numerical factors appearing in Eqs. (6)–(8), the calculation of the matrix elements for the direct absorption of mesons is independent of the final state.

In particular, the square of matrix element for absorption from an s state of the pi-mesic helium atom leading to the proton final state is given by

$$\frac{1}{12}\left\langle\begin{vmatrix} \varphi_j{}^1 & \chi^1 & \zeta^1 \\ \varphi_j{}^2 & \chi^2 & \zeta^2 \\ \varphi_j{}^3 & \chi^3 & \zeta^3 \end{vmatrix}\,|c_0\textstyle\sum\boldsymbol{\sigma}_i\cdot\mathbf{P}_i\tau_i{}^-|\,\varphi_\alpha\begin{vmatrix} \chi^S & \zeta^S \\ \chi^T & \zeta^T \end{vmatrix}\right\rangle^2.$$

The scalar product appearing here may be written

$$\boldsymbol{\sigma}\cdot\mathbf{P}=2\sigma^-P^++\sigma_zP_z+2\sigma^+P^-,\quad P^\pm=\tfrac{1}{2}(P_x\pm iP_y).$$

Written in this way it is evident that each of the terms leads to a distinct final state (with spin magnetic quantum number 1, 0, −1, respectively), and because of the zero spin of the initial system all of the matrix elements may be calculated from symmetry when one of them is known. It is therefore only necessary to calculate

$$\frac{c_0^2}{3}\left\langle \begin{vmatrix} \varphi_j^1 & \chi^1 & \zeta^1 \\ \varphi_j^2 & \chi^2 & \zeta^2 \\ \varphi_j^3 & \chi^3 & \zeta^3 \end{vmatrix} \middle| \sum \sigma_i^- P_i^+ \tau_i^- \middle| \varphi_\alpha \begin{vmatrix} \chi^S & \zeta^S \\ \chi^T & \zeta^T \end{vmatrix} \right\rangle^2,$$

or, upon evaluation of the spin and isotopic spin matrix elements,

$$(4c_0^2/9)\{\sum_l \langle \varphi_j{}^l | \Pi_l{}^+ | \varphi_\alpha \rangle\}^2 = 32 c_0^2 \langle \varphi | \Pi_j{}^+ | \varphi_\alpha \rangle^2. \quad (9)$$

Finally, summation over j and over the spin magnetic quantum number of the final state, and an evaluation of the space part of the matrix element give

$$(\mathrm{ME})_p{}^2 = 32 K_0 c_0^2 \varphi_\alpha{}^2(K_0), \quad (10\mathrm{p})$$

where K_0 is the kinetic energy of the final states.

The formulas derived on exactly the same basis for the deuteron and triton final states give, after evaluation of the space part of the matrix elements,

$$(\mathrm{ME})_d{}^2 = 8K_d c_0^2 \left\{ \int \varphi_\alpha (K_d + \tfrac{1}{2}\Pi_2{}^2) \varphi_d (\tfrac{1}{2}\Pi_2{}^2) d\mathbf{\Pi}_2 \right\}^2, \quad (10\mathrm{d})$$

$$(\mathrm{ME})_t{}^2 = (16/3) K_t c_0^2 \left\{ \int \varphi_\alpha (K_t + \tfrac{1}{2}\Pi_2{}^2 + \tfrac{1}{2}\Pi_3{}^2) \times \varphi_t (\tfrac{1}{2}\Pi_2{}^2 + \tfrac{1}{2}\Pi_3{}^2) d\mathbf{\Pi}_2 d\mathbf{\Pi}_3 \right\}^2, \quad (10\mathrm{t})$$

where K_d, K_t are the kinetic energies available in the final states, and the wave functions have been assumed to depend on the kinetic energies alone, as ours do.

In this method of calculation the sum over the final spin states has already been performed, and therefore the density of states is calculated without taking into account the possible final spin orientations. The final formulas for the transition rates are

$$(\mathrm{T.R.})_p = (2048/315)\pi^5 \sqrt{2} c_0^2 K_0^{9/2} \varphi_\alpha{}^2(K_0) = 4.0 c_0^2, \quad (11\mathrm{p})$$

$$(\mathrm{T.R.})_d = 32\pi^4 c_0^2 K_d{}^3 \left\{ \int \varphi_\alpha (K_d + \tfrac{1}{2}\Pi_2{}^2) \times \varphi_d (\tfrac{1}{2}\Pi_2{}^2) d\mathbf{\Pi}_2 \right\}^2 = 5.2 c_0^2, \quad (11\mathrm{d})$$

$$(\mathrm{T.R.})_t = (128/3)\pi^2 \sqrt{2} c_0^2 K_t{}^{\frac{5}{2}} \left\{ \int \varphi_\alpha (K_t + \tfrac{1}{2}\Pi_2{}^2 + \tfrac{1}{2}\Pi_3{}^2) \times \varphi_t (\tfrac{1}{2}\Pi_2{}^2 + \tfrac{1}{2}\Pi_3{}^2) d\mathbf{\Pi}_2 d\mathbf{\Pi}_3 \right\}^2 = 2.8 c_0^2. \quad (11\mathrm{t})$$

The total transition rate is 2.4×10^{18} sec^{-1} at $f^2 = \frac{1}{4}$.

In considering the absorption of mesons from the p state of the π-mesic helium atom, somewhat more care is necessary. In the first place, since the spin of the He^4 nucleus is zero, the magnetic quantum number of the mesic p state is evidently irrelevant, and will be taken zero in what follows. The meson wave function may now be written to good approximation, as we have already indicated, as $\phi = z\,\mathrm{grad}\phi$, $\mathrm{grad}\phi$ being taken constant, and the interaction becomes

$$O_i = (2\pi/\mu)^{\frac{1}{2}}(f/\mu)\,\mathrm{grad}\phi[\sigma_{zi} - i\mu\sigma_i\cdot\mathbf{P}_i z_i]\tau_i^-.$$

The first part of the above operator gives zero if the initial wave function is completely symmetric as we have assumed but there may be an appreciable contribution from the D state of the He^4 nucleus. The estimate of this contribution is rendered uncertain by the fact that the momentum dependence of the S and D parts of the wave function are not identical, as we have been forced to assume below. The D state is a quintet and therefore the spin wave function is completely symmetric. If it has zero isotopic spin, that is, if the tensor force is charge independent, then it must have space part in R^2. Since the operator being considered does not affect the space coordinates, the final state also has space part in R^2 and therefore no tritons are possible. Moreover, since a given initial symmetry must be preserved, there is only one possible final space symmetry. This materially reduces the number of possible final states. Departing for a moment from the assumption that the initial meson is in an $m=0$ state, consider the part of the He^4 nuclear D state with $m_s = -m_L = 2$. Evidently only σ^-, not σ_z or σ^+, lead to acceptable final states; therefore $\frac{1}{3}$ of the π^- mesons are absorbed by this final state or, alternately, only $\frac{1}{3}$ of the D state is effective in the absorption of a π^- meson. The amount of D state initially present is 3 percent (see above) so that finally the ratio of the squares of the matrix elements for transitions from the D and S states of the He^4 nucleus is

$$0.01\psi^2 : \mu^2 P_i^2 P_{iz}^2 (d\psi/dK)^2 = 1:1.$$

Therefore, because of the difference in the number of available final states, the absorption from the p state through the small part of the operator dominates by a factor of 2–3.

We have therefore performed the calculation only for the small part of the interaction operator and the S state of the initial He^4 nucleus. Inasmuch as the absorption from the p state is not a large effect, certain additional approximations have been made. In discussing them we shall refer to the momenta within bound systems in the final states as internal momenta, and to the momenta between bound systems as external momenta. In the calculation for the absorption from s states only external momenta appeared, while for p states the internal momenta appear as well. Upon integration over the internal momenta, because of the spherical symmetry of the wave functions, the internal momenta remain in only $\frac{1}{3}$ of the matrix elements (they tend to reduce the matrix elements). Furthermore, since the wave functions decrease quite rapidly with increasing energy, the average square of the internal momenta is less than the square of the external momenta. Therefore, the terms involving the internal momenta have been neglected. Finally, again because of the rapid variation of the wave function, the factor $\varphi_\alpha^{-1}d\varphi_\alpha/dK$ in the integrals may be treated as a constant. This again overestimates slightly the matrix elements. The above approximations together should not introduce an error above 25 percent in the relative numbers of protons, deuterons, and tritons arising in the absorption from the p state. Even a larger error in these ratios makes, however, very little difference in the observed ratios because of the small amount of absorption taking place from the p state.

The result of the calculation of the absorption from the p state gives for the transition rates

$$(\text{T.R.})_p = \frac{1024}{315}\frac{8.72}{9}\mu^4 e^{10} f^2 \pi^5 \sqrt{2} K_0^{11/2}\left(\frac{d\varphi_\alpha}{dK}\right)^2_{K_0} = 7.3\times 10^{11} f^2 \text{ sec}^{-1}, \quad (12\text{p})$$

$$(\text{T.R.})_d \cong 16\frac{13}{9}\mu^4 e^{10} f^2 \pi^4 K_d^4\left(\frac{1}{\varphi_\alpha}\frac{d\varphi_\alpha}{dK}\right)^2_{K_d} \times\left\{\int \varphi_\alpha(K_d+\tfrac{1}{2}\Pi_2^2)\varphi_d(\tfrac{1}{2}\Pi_2^2)d\mathbf{\Pi}_2\right\}^2 = 13.4\times 10^{11} f^2 \text{ sec}^{-1}, \quad (12\text{d})$$

$$(\text{T.R.})_t \cong \frac{128}{3}\frac{16}{9}\mu^4 e^{10} f^2 \pi^2 \sqrt{2} K_t^{5/2}\left(\frac{1}{\varphi_\alpha}\frac{d\varphi_\alpha}{dK}\right)^2_{K_t} \times\left\{\int \varphi_\alpha(K_t+\tfrac{1}{2}\Pi_2^2+\tfrac{1}{2}\Pi_3^2) \times\varphi_t(\tfrac{1}{2}\Pi_2^2+\tfrac{1}{2}\Pi_3^2)d\mathbf{\Pi}_2 d\mathbf{\Pi}_3\right\} = 8.5\times 10^{11} f^2 \text{ sec}^{-1}. \quad (12\text{t})$$

The optical transition rate from the p to the s state of the mesic atom is[16]

$$28\times 10^{11} \text{ sec}^{-1},$$

so that if f^2 is $\frac{1}{4}$, 20 percent of the absorptions of π^- mesons take place from the p state of the mesic helium atom.

In calculating the electromagnetic radiation accompanying the absorption from the s state, we have used the closure approximation. In this approximation the square of the matrix element (3) is summed over all final nuclear states regardless of whether or not they conserve energy, and the sum and the density of states by which it is multiplied are both evaluated at their maximum with respect to the photon momentum. This approximation certainly gives an upper limit to the number of photons to be expected. In the case of deuterium and He^3, where the final states include no more than two similar particles so that the matrix elements leading to low excitations of the nuclear system are large and where, because of the large amount of energy taken away by the photon, the nuclear forces are strong and tend to bind the final particles and thus increase the average photon energy further, the closure approximation is very good. In He^4 on the other hand, the final states contain three neutrons and the effect of the Pauli principle is to give nuclear wave functions of low energy small values near the origin, thus depressing the nuclear matrix elements for small excitations of the nucleons. Therefore, for large photon energies, matrix elements leading to nuclear states of high excitation which conserve energy badly are prominent and also in a correct calculation there would be no peak for high energy photons. Both these effects mean that the closure approximation is no longer a good approximation to the actual transition rate but rather an upper limit.

The actual calculation is quite simple. The square of the matrix element

$$\langle\psi|\textstyle\sum_i O_{\gamma i}|\psi_\alpha\rangle^2 = \langle\psi_\alpha|\sum_i O_{\gamma i}^+ \sum_i O_{\gamma i}|\psi_\alpha\rangle$$

is rewritten, by taking advantage of the antisymmetry of the He^4 nuclear wave function, in the form

$$4\langle\psi_\alpha|O_{\gamma 2}^+[O_{\gamma 2}+3O_{\gamma 3}]|\psi_\alpha\rangle.$$

The polarization sums and spin and isotopic spin matrix elements are calculated to give

$$2c_0^2(2\pi e^2/\mu^2 k)\langle\varphi_\alpha|1-\Delta_2(-\mathbf{k})\Delta_3(\mathbf{k})|\varphi_\alpha\rangle = 4\pi(e^2c_0^2/\mu^2 k)\left\{1-\int\varphi_\alpha(\mathbf{\Pi}_1, \mathbf{\Pi}_2+\sqrt{2}\mathbf{k}, \mathbf{\Pi}_3) \times\varphi_\alpha(\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{\Pi}_3)d\mathbf{\Pi}_1 d\mathbf{\Pi}_2 d\mathbf{\Pi}_3\right\}. \quad (14)$$

Equation (14) yields for the transition rate

$$4c_0^2 e^2\mu^{-2}k(1+k/4)^{-1}\left\{1-\int\varphi_\alpha(\mathbf{\Pi}_1, \mathbf{\Pi}_2+\sqrt{2}\mathbf{k}, \mathbf{\Pi}_3) \times\varphi_\alpha(\mathbf{\Pi}_1, \mathbf{\Pi}_2, \mathbf{\Pi}_3)d\mathbf{\Pi}_1 d\mathbf{\Pi}_2 d\mathbf{\Pi}_3\right\} = 0.113c_0^2, \quad (15)$$

which is 1 percent of the total nonradiative transition rate.

Some of the calculations here reported have been repeated using the wave functions of Bruno, Ruddles-

[16] A. M. L. Messiah and R. E. Marshak, Phys. Rev. 88, 678 (1952).

den, and Clark. The results of these calculations, in which only the integrations are different from ours, are,

for absorption from the S state,

protons	$0.76c_0^2$	(11′p)
deuterons	$0.30c_0^2$	(11′d)
tritons	$0.019c_0^2$	(11′t)

and for absorption from the p state,

protons	$6.2\times10^{11}f^2$ sec^{-1}	(12′p)
deuterons	$3.6\times10^{11}f^2$ sec^{-1}	(12′d)
tritons	$0.066\times10^{11}f^2$ sec^{-1}.	(12′t)

The reasons for the difference between (11) (12) and (11′) (12′) are quite evident. First, the low average kinetic energy of the Gaussian wave function means that this wave function is quite small at the final kinetic energy of the fragments, so that the rate of absorption from the s state is reduced. The rapid decrease of the wave function for higher kinetic energies means that the matrix elements that depend on the value of the wave function there are depressed even relative to the matrix elements for pure absorption. Since the triton is more tightly bound than the deuteron, this affects the triton transition rate more strongly than the deuteron rate.

The small values of the Gaussian wave function at the final kinetic energy of the fragments is compensated to some extent, for the absorption from p states, by a large logarithmic derivative so that the transition rates leading to free particles are quite comparable for the two wave functions. For the same reason as above, deuterons and tritons are much less frequent.

IV. DISCUSSION

Qualitatively, the results of the preceding calculations are hardly surprising. As pions are absorbed in larger and larger nuclei with greater and greater binding energies, the effect of the nuclear forces should become more prominent and lead to larger and larger proportions of bound fragments until ultimately in the vicinity of neon the absorption of a pion to produce a completely unbound system becomes energetically impossible. In particular, the kinetic energy available in the final state for absorption by He^4 nuclei is less by 20 Mev than in the case of He^3 because of the larger binding energy of the He^4 nucleus and must be shared by three degrees of freedom, as opposed to two for He^3. The ratio of free to bound products, based on the preceding calculations and the similar calculations of Messiah is 1:2 for He^4 and 3:1 for He^3.

The number of photons that is calculated is also in agreement with the trend from deuterium through He^3 (deuterium, 47 percent; tritium, 4.5 percent; He^3, 10 percent of nonradiative absorption). The reason for this decrease in the amount of radiation is due to the fact that as the nucleus gets heavier the high energy part of the wave function is accentuated and augments the pure absorption whereas the low energy part is unaffected; hence, the radiative absorption is unable to keep pace. This trend is expected to continue for larger nuclei and appears to be in agreement with an estimate in the literature[17] that the radiative absorption is less than 1.3 percent of the total absorption in heavy nuclei.

A word might be said about the absorption of negative pions leading to final states involving neutral pions. According to the calculation of Messiah, upon which reliance will be placed for all the relevant facts, the transition rate is proportional to the momentum of the emitted pion and to the square of a matrix element that does not involve the spin but is proportional to the overlap of the wave functions of the initial and final nuclei, for negligible momentum transfers. Inasmuch as the emission of γ-rays also involves low momentum transfers, the neutral pion emission may profitably be compared with it. Because of the large rest energy of the neutral pion, less than 5 Mev is available for the reaction and only low-lying final nuclear states are important. This is not so for photon emissions as has already been pointed out in connection with the closure calculation.

The rate of neutral pion emission is comparable to the rate of photon emission for He^3. It is evident that the number of neutral pions will be negligible (<0.1) with respect to the number of photons, which is already small, unless the nucleus in which the negative pion is absorbed is the decay product of a negatron beta-decay superallowed with Fermi selection rules. Since the only two such nuclei are the proton and He^3, absorption leading to neutral pions will be completely negligible for any other nucleus. An equivalent formulation of this result without recourse to beta-decay theory may be given as follows: In order for the overlap integral mentioned above to be large, it is necessary that the initial and final nuclei belong to the same supermultiplet and have the same spin. Two nuclei in the same supermultiplet with the same spin must have the same isotopic spin. However, the only stable nuclei that can decrease their charge by 1 without changing their isotopic spin are the proton and He^3.

The effect of the nuclear forces on the nonradiative processes is brought in evidence more clearly by considering only states whose symmetry is such that they admit the bound states. In the case of the He^4 nucleus this state is the one with space symmetry o_{jl}^+ and the ratio of protons to deuterons to tritons is 1:2:2.1, while for He^3 it is one of the states with spin $\frac{1}{2}$ and the ratio of protons to deuterons is 1:1.

It is, however, a disturbing feature of calculations of this nature that, although it is evident from the number of bound systems occurring in the disintegration

[17] Brueckner, Serber, and Watson, Phys. Rev. 84, 258 (1951).

products that nuclear forces play an important part in the reactions, the nuclear forces have been completely neglected except within bound systems. Bruno has argued that the binding forces are equivalent to an increase in the kinetic energy of the disintegration products and that therefore the number of unbound products is overestimated by neglecting the forces. This seems a reasonable idea upon which to base at least an estimate of the order of magnitude of the effect of nuclear forces. Irving[18] gives about 60 Mev for the potential energy of the triton. On the assumption of Serber forces the potential energy in a proton final state is probably somewhat less because of the poorer correlation between the particles. The transition rate to proton states would be about halved if the effective final kinetic energy were increased by 40 Mev. Actually, it is by no means evident that such an estimate has a sound basis.

Another disturbing approximation that has been made is the neglect of higher powers of $(v/c)^2=0.19$. There does not even appear to be any basis for estimating the effect of this approximation, except to assert that it is probably quite good for radiative absorption where the momentum transfers are small.

Finally, serious consideration must be given to possible inadequacies in the He^4 nuclear wave function besides those involving neglect of v/c. Inasmuch as the part of the wave function relevant to the absorption calculation is in the vicinity of the average kinetic energy and the part of the wave function near the average kinetic energy is the part that enters most importantly into the variational calculation of the wave function, the present wave function appears quite well founded for use in this problem. It is far better founded than the deuteron wave function, to mention an extreme case. No account has been taken of possible correlations among the nucleons leading to a wave function that is not a function of K alone. Here again, there is no basis for estimating in what direction the results of the calculation might be changed.

At this time no extensive comparison with experiment is possible. A preliminary experiment (involving the observation of 23 absorptions in a high pressure helium cloud chamber[19]) on the absorption of negative pions in helium has led to the following results: the number of tritons per absorption is 0.1 ± 0.1, as evidenced by one possible triton final state observed. Also, low energy charged nuclear fragments seem to be much more abundant than one would expect from a statistical spectrum; in fact, the observed spectrum of charged fragments has no maximum above 5 Mev. The number of tritons observed appears to be within range of our calculations, but the distortion of the spectrum cannot be explained since our calculations always lead to a statistical spectrum (the sum of the squares of the matrix elements Eq. (10) is independent of the distribution of momenta) with an average energy of about 30 Mev. In order to obtain a momentum distribution different from the statistical distribution, it is necessary to use a wave function which does not depend only on the total kinetic energy.

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[18] J. Irving, Phil. Mag. 42, 338 (1951).

[19] C. P. Leavitt (private communication).