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Charge-Exchange Scattering of 34-Mev π-Mesons in Hydrogen and Deuterium*

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We have measured the total cross section for the charge-exchange scattering of π-mesons in hydrogen and deuterium at 34 Mev. Both single photons and two-photon coincidences were detected. The results are relatively independent of the assumed angular distribution of the π-mesons. A value of 5.0 ± 1.5 millibarns was obtained for the total charge-exchange cross section of π^--mesons in hydrogen. The expected null result was obtained for π^+-mesons in hydrogen. In deuterium, values of about one-third the hydrogen cross section were obtained for both signs of meson. The photons observed from carbon and oxygen do not appear to be mainly from π^0-decay, and the cross section for π^0 production in C and O is less than 5 mb for both signs of meson.

The two-photon coincidences yield a rough value of 0.8 mb/sterad for the differential charge-exchange cross section for π^- on hydrogen at 90°. The accuracy is sufficient to rule out the possibility of a very deep minimum at 90°, e.g., a $\cos^2\theta$ angular distribution.

INTRODUCTION

THE charge-exchange scattering of π^--mesons in hydrogen [i.e., the reaction $p(\pi^-,\pi^0)n$], has been observed in the energy range 120 to 135 Mev by Anderson, Fermi, Nagle, and Yodh.[1] They measured the angular distribution of photons from a liquid hydrogen target. Previous attempts to detect charge-exchange scattering at lower energies have not succeeded.[2]

We have measured the yield of photons at 90° from hydrogen and deuterium bombarded by mesons of both signs in the energy range 26 to 41 Mev.[3] The experiment was performed by taking C–CH_2 and H_2O–D_2O differences. It seems plausible that most of the photons are decay products of π^0-mesons.

We can obtain a fairly good value of the charge-exchange cross section from measurements at one angle because of the peculiarities of π^0-decay. At the relatively low meson energy used, the two decay gamma-rays are emitted with a rather large angular separation, and are only weakly correlated in direction with the parent π^0-meson in the laboratory system. Consequently, the photon angular distribution is relatively insensitive to the π^0-angular distribution.

We have also detected photons produced in carbon and oxygen but interpret the majority of them as being of nuclear origin.

The production of coincident pairs of photons has been observed, but the resulting numbers of events are small and have large statistical uncertainty. Nevertheless, the data strongly support the hypothesis that the single photons originate predominantly from π^0-decay. In addition, we obtain a very approximate value for the 90° differential charge-exchange scattering cross section in hydrogen.

EXPERIMENTAL ARRANGEMENT

Mesons were produced by bombarding an internal aluminum target with 240-Mev protons in the Rochester 130-inch synchrocyclotron. They emerged through a thin Al window and passed through the 4×5 in. aperture of a "Z-focusing" magnet. The beam was then deflected through about 45° in the horizontal plane by a horizontal focusing magnet. Thus, a nearly parallel beam of mesons was obtained whose energy was closely defined (50 ± 1 Mev). Either sign of meson was available, depending on the orientation of the magnetic fields of the cyclotron and of focusing magnets.

The counter assembly, consisting of eleven liquid scintillation counters, was imbedded in a lead and copper shielding block, in order to minimize single counting rates. Further shielding was strategically disposed in the vicinity. The general arrangement of the apparatus is shown in Fig. 1.

The arrangement and dimensions of the counters are given in Fig. 2. The assembly consisted of the meson telescope (counters 1–3), the main photon telescope T_1 (counters 6–9), and a subsidiary photon telescope T_2 (counters 10–11). Counter 4, beyond the meson target, was in anticoincidence with the meson telescope, to eliminate the detection of events in which the meson emerged undeviated from the target. Counter 5, immediately above the target, was in anticoincidence with T_1, to desensitize it to charged particles originating in the target.

Events of interest were thus caused by a meson traversing counters 1–3, but not counter 4, and simultaneously producing a coincidence in either the main photon telescope T_1 or in both T_1 and T_2.

* This work was supported by the U. S. Atomic Energy Commission.

[1] Anderson, Fermi, Nagle, and Yodh, Phys. Rev. **86**, 793 (1952); also Proceedings of the Third Rochester High Energy Conference, 1952 (Interscience Publishers, Inc., New York, 1953).

[2] R. Wilson and J. Perry, Phys. Rev. **84**, 163 (1951); Bernardini, Booth, Lederman, and Tinlot, Phys. Rev. **80**, 924 (1950).

[3] A preliminary report of some of these results has been given by Roberts, Spry, and Tinlot, Bull. Am. Phys. Soc. **28**, No. 1, 14 (1953).

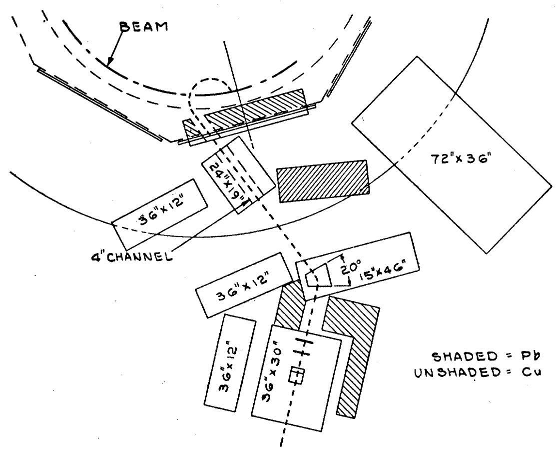

FIG. 1. General arrangement of apparatus, showing shielding, Z focus, and deflecting magnets. In the vertical direction, normal to the plane shown, the shielding is about three feet high, approximately the separation of the cyclotron magnet coils.

Targets

All meson targets had identical cross-sectional dimensions $1\frac{3}{8}\times3$ in. In order to make a direct difference experiment possible, the C and CH_2 targets used were chosen to have the same number of C nuclei (2.5 g/cm^2). Likewise, the H_2O and D_2O targets had the same number of O atoms (2.25 g/cm^2). The liquids were contained in Lucite boxes with a total wall thickness of 0.10 in.

After traversing the threefold meson telescope, the mesons entered the target with an energy of 41 Mev. In the case of the CH_2 target, they emerged with an energy of 26 Mev. The mean meson energy varied slightly (less than 3 Mev) among the several targets because of their different stopping powers; this effect was not important because of the considerable energy spread that resulted from target thickness.

The Photon Telescopes

The photon telescopes were intended to detect photons by virtue of the electron showers produced in lead converters. In the main telescope T_1, the converter (0.25 in.$\times$2 in.$\times$3 in.) was placed immediately above counter 5 (see Fig. 2). "Wings" of graphite, 0.75 in. $\times$0.5 in.$\times$3 in., were placed on either side of the lead. The graphite had approximately the same stopping power for protons as the lead and served principally to shield the telescope from the target volume.

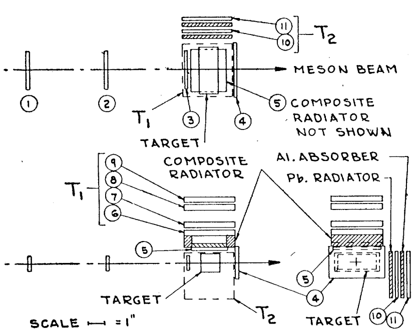

FIG. 2. Counter arrangement, to scale. Only the active volume of each scintillation counter is shown.

The composite converter could be replaced by one of solid graphite 0.75 in. thick. The lead and graphite differed by a factor of about ten in conversion efficiency for 100-Mev photons; it was therefore simple to differentiate photons from other neutral particles. A $\frac{1}{4}$-in. Pb converter was left permanently in front of the secondary telescope T_2. The stopping powers of both telescopes were made equal by the insertion of $\frac{1}{4}$ in. of Al between counters 10 and 11.

In some of the early runs, an absorber was used between counters 7 and 8 in order to limit the background of fourfold coincidences 6789. The corresponding minimum detectable energy of conversion electrons was 15 Mev. With improved shielding, the rate became low enough to permit the removal of the absorber. The minimum detectable energy was thus reduced to 8 Mev, and the photon telescope efficiency was doubled. As it turned out, the calculation of the telescope efficiency was considerably simpler and subject to less uncertainty in the latter case.

Electronic Circuits

A block diagram of the electronic circuits, somewhat simplified, is shown in Fig. 3. The fast coincidences were formed by using diode bridge circuits or Garwin circuits.[4] Resolving times were 10–30 mμsec. The threefold meson telescope rate of 123 was used as a monitor; this rate, hereafter called M, was corrected for accidentals and beam contamination to give the meson flux. Counter 4 in anticoincidence with M gave the anticoincidence rate $123\bar{4}$, hereafter called A. The fourfold coincidence 6789 in anticoincidence with 5 gave the event $\bar{5}6789$, hereafter called B, and the coincidences AB were recorded. Twofold coincidences 10–11 were placed in coincidence with A to form events C, and ABC coincidences were recorded. Since both AB and ABC rates were quite low (one to 40 per hour), it was thought desirable to guard against counting either cosmic-ray events or adventitious pulses from external electrical interference. This was accomplished by gating the recording circuits to count events only during 400-μsec intervals which included the time the beam was on target. Ungated events were also counted. In no case was there a difference between gated and ungated events.

The accidental coincidence rate in AB was monitored throughout the experiment by means of a slow coincidence circuit measuring the coincidence rate with one channel delayed. The number of accidentals in the

[4] R. L. Garwin, Rev. Sci. Instr. **21**, 569 (1950); Phys. Rev. **90**, 274 (1953).

undelayed fast circuit was then obtained from the known ratio of resolving times. This correction was quite small in the AB rate, and the chance background in the ABC rate was so small as to be completely negligible.

PROCEDURE

Calibration and Adjustment of Counters

Since the amplification at various points of the circuit was in most cases fixed, the most convenient method of adjusting pulse heights was to vary the photomultiplier voltages. In adjusting the meson telescope, three checks were available. The M rate was quite sensitive to the adjustment of the focusing magnet currents; it exhibited a good plateau as a function of each photomultiplier voltage; and the nature of the beam could be checked by taking range curves. Ex-

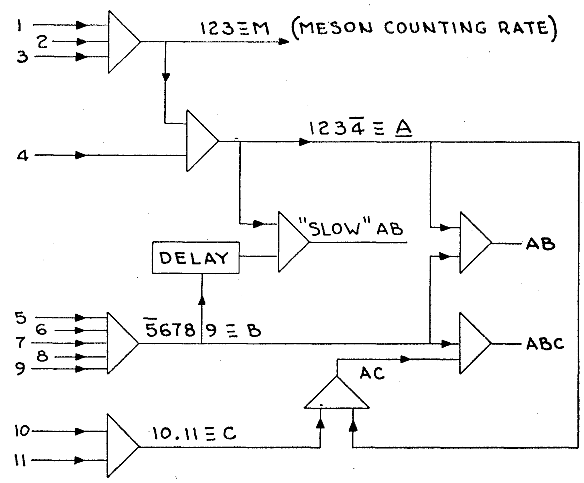

FIG. 3. Simplified block diagram of electronics. The arrangements for recording both gated and ungated coincidences are omitted.

amples of range curves for π^+- and π^--mesons are given in Fig. 4.

The range curve for the positive beam agrees well with that expected for 50-Mev π-mesons. There is about 5 percent contamination of μ-mesons and positrons with ranges greater than the π-mesons. The curve for negative mesons shows much more contamination. A careful set of measurements by Camac, McGuire, Platt, and Schulte[5] at 40 Mev shows a contamination of about 22 percent μ-mesons and electrons. The remainder of the tail probably arises from protons produced in the absorber by stopped π^--mesons.

The voltage on counter 4 was set by finding a rough plateau in the rate A; the ratio A/M, which measures the anticoincidence efficiency of 4, was about 0.07 for π^+, 0.14 for π^--mesons, with the meson targets in place. The lower efficiency with negatives is perhaps due to

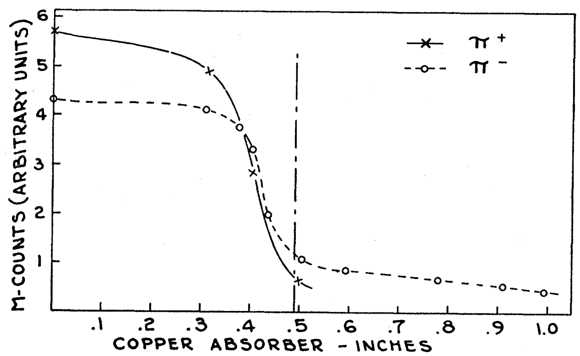

FIG. 4. Range curves for π-meson beams. The vertical dashed line indicates the expected range of the beam, taking into account the energy loss in the meson telescope.

the greater electron contamination and the larger number of accidentals with the lower intensity negative beam.

The photon telescope T_1 was adjusted by counting cosmic-ray μ-mesons with the cyclotron turned off. A reasonable plateau was obtained for 6789 and 56 789 rates as functions of photomultiplier voltages. Since counters 5 to 9 were normally in position to count vertically incident μ-mesons, periodic checks were possible without disturbing the apparatus. Examples of calibration runs are given in Fig. 5. The 56 789 rate agrees very well with that expected[6] for μ-mesons of sufficient energy to penetrate the 15 in. of copper shielding above the telescope. Therefore, we have assumed 100 percent efficiency for detection of single minimum ionization particles in T_1.

Counters 10 and 11 were adjusted by placing them above T_1, and counting cosmic-ray μ-mesons in sixfold coincidence $6789\widehat{10}\widehat{11}$. This measurement could not be repeated without disturbing the apparatus during a run. The constancy of the C rate was therefore used to indicate that no changes in efficiency had occurred.

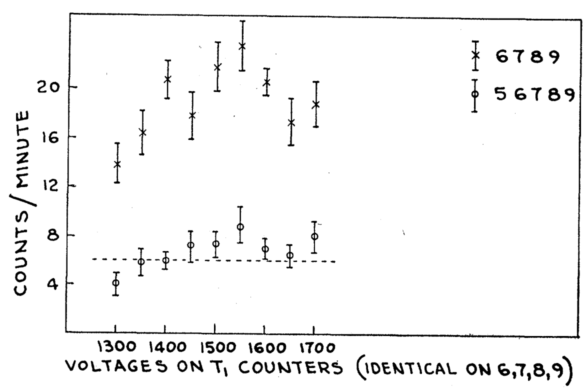

FIG. 5. Plateau of photomultiplier voltages *vs* incident μ-meson cosmic-ray rate. The dotted line indicates the expected coincidence rate assuming 100 percent efficiency in all counters.

[5] Camac, McGuire, Platt, and Schulte (private communication).

[6] B. Rossi, Revs. Modern Phys. 20, 537 (1948).

TABLE I. AB coincidences for C and CH_2. Rates are per million M counts, corrected only for accidental AB coincidences. Errors are standard deviations (including error in accidentals).

Meson beam	Target	Converter Pb	C	CH_2–C difference Pb converter
π^-	C	21.6±2.1	7.7±2.5	
π^-	CH_2	51.0±2.5	12.0±2.5	29.4±3.3
π^+	C	13.4±4.5		
π^+	CH_2	13.9±4.9		0.5±6.6

TABLE II. AB coincidences for H_2O and D_2O. Rates are per million M counts, corrected only for accidental AB coincidences. Errors are standard deviations (including error in accidentals).

Meson beam	Target	Converter Pb	C	D_2O–H_2O difference Pb converter
π^-	H_2O	46.1±2.8		
π^-	D_2O	33.0±3.0		−13.1±4.2
π^+	H_2O	14.0±2.0	10.65±4.4	
π^+	D_2O	21.6±2.6	8.7 ±4.4	7.6±3.1

Adjustment of Delays and Resolving Times

All of the coincidence circuits were checked with artificial pulses of varying amplitude to insure that no threshold jitter could change the coincidence efficiency. Before each run, the delays were checked with the actual circuit voltages and connections used during the experiment. This was done by moving first T_1 and later T_2, so that the incident meson beam passed through it, as well as the meson telescope. The exact delays to be inserted in each line, making all lines the same electrical length, could then be determined with the true coincidences thus produced. In view of the short resolving times (10 mμsec), and long cables (350 ft) used, this was an important precaution.

Operating Procedure

Relatively short runs, with different targets or radiators, were alternated to reduce the effect of long-term drifts. Check runs were occasionally made at half- or quarter-intensity beam levels, so as to prove that none of the coincidence circuits was being jammed by too high a singles rate anywhere.

Since no effects of jamming were detected, most of the data were taken at maximum beam level, corresponding to π^--rates of ~10 000/min and π^+-rates of ~25 000/min.

RESULTS

Single-Photon Detection

A summary of the data on AB coincidences appears in Tables I and II. The numbers listed are the numbers of coincidences per million M counts, corrected only for accidental coincidences in the AB rate. The uncertainty in the correction is included in the quoted errors.

Charge-Exchange Scattering in Hydrogen

There are three obvious features of the data for π^--mesons in Table I. (1) The Pb–C converter differences are certainly real and large. (2) The CH_2–C difference with the Pb converter is large; and (3) the CH_2–C difference with the C converter is small, perhaps even zero. Ionizing particles produced in the target and incompletely rejected by counter 5 would show no Pb–C difference. Furthermore, no known reaction in hydrogen produces ionizing particles with enough energy to traverse the telescope T_1.[7]

The above observations thus strongly support the view that the CH_2–C difference is the result of the production of photons in hydrogen. The absence of such photons in the case of π^+-mesons on hydrogen in Table I (no CH_2–C difference) is consistent with this interpretation.

The considerable Pb–C differences observed with π^- on C and π^+ on D_2O (Table II) are perhaps the most direct indications that photons are produced in other nuclei as well. The results for π^- on H_2O and D_2O show immediately that the yield from D is less than from H.

Two-Photon Emission

The apparatus for counting events ABC was placed in operation only in the latter part of the experiment, and there was not sufficient running time available to obtain good statistics. In particular, there was no time to take long runs with carbon converter for T_2 in order to prove the electromagnetic character of the detected radiation or to try angles at which no coincidences would be expected. However, we find some significance in the data, which are given in Table III and discussed below.

INTERPRETATION OF DATA

A. Single-Photon Detection

In order to deduce the cross section for the specific process $p+\pi^-\rightarrow n+\pi^0\rightarrow n+2\gamma$, we must know several important factors. These are: (1) the dependence of

TABLE III. Two-photon detection. The number of M counts is given, and the number of corresponding ABC events observed.

Target	Mesons T_1 converter	Pb	C	Pb	C
C	$M\times10^{-6}$	1.5	—	0.7	—
	ABC	0	—	0	—
CH_2	$M\times10^{-6}$	1.0	—	2.15	0.5
	ABC	0	—	2	0
H_2O	$M\times10^{-6}$	4.5	1.0	7.2	0.7
	ABC	0	0	10	0
D_2O	$M\times10^{-6}$	4.5	1.0	5.9	1.1
	ABC	2	0	4	0

[7] Excepting the high energy electron pairs arising from the infrequent internal conversion of a photon from a neutral meson decay; counter 5 discriminates against such events. Steinberger, Sachs, and Lindenfeld, Bull. Am. Phys. Soc. 28, No. 1, 14 (Jan. 22, 1953).

the photon spectrum and angular distribution upon the angular distribution of the π^0-mesons, (2) the intrinsic efficiency $\epsilon(E)$ of the telescope T_1 as a function of photon energy E, (3) the net efficiency η of the telescope for detecting π^0-mesons, (4) the contamination of the meson beam and the fraction of mesons lost to scattering and absorption in the target, and (5) the cross section and probability of detecting the competing process of radiative capture. We shall now discuss these factors in detail.

Kinematics of Charge Exchange in Hydrogen

Consider the decay of the neutral meson in the c.m. system. Let the π^0 rest mass be μ, its velocity β, and its total energy $\gamma\mu$. The decay photons are distributed in energy between limits $E_{\max}$ and $E_{\min}$ given by

$$2E_{\min}/\mu=\mu/2E_{\max}=[(1-\beta)/(1+\beta)]^{\frac{1}{2}}. \quad (1)$$

If the π^0 mesons are emitted in the c.m. system with an angular distribution given in terms of spherical harmonics by

$$H(\theta)d\Omega=\sum_l a_l P_l(\cos\theta)d\Omega, \quad (2)$$

then the energy spectrum of photons emitted at the angle θ can be shown to be

$$J(\theta,E)dEd\Omega = (2\pi\beta\gamma\mu)^{-1}dEd\Omega\sum_l a_l P_l(\cos\theta)P_l\left(\frac{1}{\beta}-\frac{\mu}{2\beta\gamma E}\right), \quad (3)$$

where $J(\theta, E)$ is normalized to one π^0-meson (two gamma-rays), and the angle θ in Eqs. (2) and (3) is measured relative to the direction of the incident π^--meson beam.

For a mean incident π^--energy of 34 Mev, the emitted meson has a c.m. energy of 28.9 Mev. The velocity of the c.m. system is $\beta_{\text{c.m.}}=0.092$, and the extremal photon energies are $E_{\max}=129$ Mev, $E_{\min}=36$ Mev.

The Intrinsic Efficiency $\epsilon(E)$

The conventional cosmic-ray shower theory is not suitable for most calculations involving photon energies much below 500 Mev. We have therefore made use of the theory of Wilson,[8] which is based on calculations made by the Monte Carlo method, and takes into account multiple scattering. Wilson divides the shower electrons into two groups: those which have energies above a critical energy E' and are supposed to have negligible Coulomb scattering, and those with energy less than E', which are assumed to be scattered so much as to be isotropically distributed. The average number of electrons in each group (a function of photon energy and converter thickness), is denoted by n_{st} and n_d, respectively. The critical energy E' in lead is 8 Mev. Since this is also the minimum energy of electrons capable of traversing the telescopes, the intrinsic telescope efficiency for any photon energy is equal to the probability that n_{st} is different from zero. As shown by Wilson, this probability is obtained by assuming a Poisson distribution. The efficiency *vs* photon energy for a $\frac{1}{4}$ in. lead converter is plotted in Fig. 6. The curve is quite well fitted over the relevant energy range by the empirical formula: $\epsilon(E)=-0.484+0.229\ln E$ (with E in Mev).

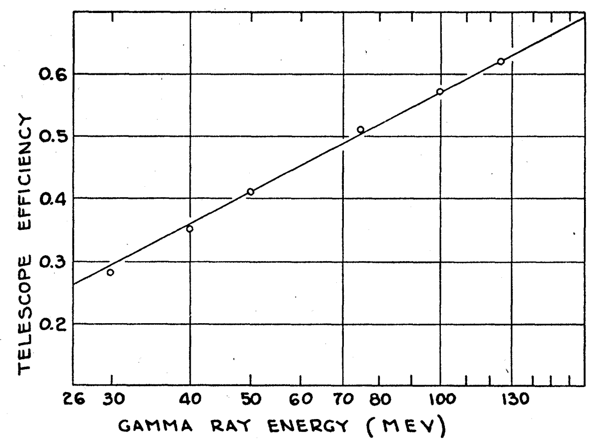

FIG. 6. Calculated intrinsic efficiency of gamma-ray telescope as a function of energy. The radiator is $\frac{1}{4}$ lead, the minimum energy of detected electrons 8 Mev. The straight line represents the empirical equation used (see text).

As mentioned previously, in our earlier work the minimum detectable electron energy was 15 Mev. One can estimate[9] that the efficiency in this case should be about one-half the value given above, over the relevant range of photon energies. The predicted difference in efficiency was observed (with a rather large statistical error). Such agreement constitutes an order of magnitude check on the energy of the detected photon.

An important consequence of the "two-group" analysis is that the photon telescope efficiency does not depend on the area of the lead converter, if the converter subtends a solid angle larger than that subtended by the most distant counter in the telescope. Thus, the composite converter shown in Fig. 2 should be as effective as one having the same area as the counters. This prediction was verified (although the measurements have large statistical errors).

The Effective Telescope Efficiency, η

Knowing the intrinsic efficiency $\epsilon(E)$ of the telescope as a function of energy and the differential intensity $J(\theta, E)$ of the photons produced at a given point in the target, we can now calculate the net probability η' of observing a given neutral meson by detection of one decay photon. This is given by

$$\eta'=\int_{\Omega'}\sin\theta d\theta d\phi\int_E J(\theta,E)\epsilon(E)dE. \quad (4)$$

[8] R. R. Wilson, Phys. Rev. 86, 261 (1952).

[9] See, for example, J. A. Richards and L. W. Nordheim, Phys. Rev. 74, 1106 (1948).

TABLE IV. Net detection efficiency η for π^0-mesons of telescope T_1, assuming different π^0-angular distributions $H(\theta)$.

$H(\theta)$	η
$1/4\pi$	0.0307
$(3/4\pi)\cos^2\theta$	0.0269
$(3/8\pi)\sin^2\theta$	0.0325

Here Ω' is the solid angle subtended by counter 9 (the furthest one), at the point of production of the photons. The limits on the energy integral are the $E_{\max}$ and $E_{\min}$ in the laboratory system, which are functions of θ and of the π^0-energy. The average efficiency η for π^0-detection is then the average of η' over all points in the target, weighted by the probability of π^0-production at each point. We have evaluated η only approximately. First, it is assumed that the production of mesons is uniform over the target; this is justified, since the calculated efficiency depends very little on the assumed beam density distribution. Second, the motion of the c.m. system (proton+meson) is neglected. This assumption introduces an error in the angle θ of 5°, at most. The solid angle aberration may amount to 10 percent for the values of θ furthest from 90°, but the effect almost cancels out when averaged over the angular range. The changes of counter efficiency due to Doppler shift in going from the c.m. to the laboratory are likewise of no practical importance.

The values of η so obtained are given in Table IV, for several different assumptions as to the angular distributions of the π^0-mesons. We neglect the possibility of higher powers of $\cos\theta$ than the second. The small dependence on the assumed angular distribution of the π^0's is evident.

Beam Contamination and Particle Loss in the Target

It is not easy to assess accurately the contamination of the negative beam from range curves like Fig. 4. For example, the beam composition may change during a run, or from one run to another, because changes in counter voltages may change the relative sensitivities of the telescope counters to electrons and mesons. Changes in focusing magnet currents may also change the composition. From curves like Fig. 4 and data on pulse-height distributions of the meson telescope counters, we believe the proportion of π^--mesons in the negative beam to be 0.78 ± 0.08. In order to derive a correction factor which reduces M counts to meson counts, one must include the effect of accidental coincidences and the effect of losses in the target due to nuclear absorption, large angle scattering, and incomplete traversal because of deviations from ideal parallelism. The final estimate of the correction factor is, $\alpha_-=0.72\pm0.08$.

For the π^+-beam, the contamination is much smaller. A 5 percent μ-meson and positron contamination and a similar correction for target losses and accidentals gives a correction factor, $\alpha_+=0.89\pm0.03$.

Radiative Capture in Hydrogen

The reaction $p(\pi^-,\gamma)n$ will give rise to high energy single photons having the unique energy of 153 Mev in the c.m. system. The T_1 telescope has an intrinsic efficiency 0.67 and a net efficiency $\eta_\gamma=0.0212$ at this energy. We consider the experimental data of Steinberger and Bishop[10] on the cross section for the reaction $p(\gamma,\pi^+)n$ to estimate the contribution of this reaction to the observed counting rate. It is reasonable to assume the cross section to be about the same as that for the reaction $n(\gamma,\pi^-)p$. The probability of radiative capture can then be found by applying the principle of detailed balancing. We find the cross section for the radiative capture of 34-Mev π^--mesons in hydrogen to be $\sigma_\gamma\approx0.26$ millibarn. For the CH_2 target used this corresponds to 1.0 counts per 10^6 M counts.

π^--H Charge-Exchange Cross Section

We consider the CH_2–C difference with Pb converter (see Table I) to be a measure of the photon yield from hydrogen. The actual number of hydrogen events per M count (n/M) is then directly related to the charge exchange and radiative capture cross sections, σ_x and σ_γ:

$$n/M=(NH/\alpha_-)(\sigma_x\eta+\sigma_\gamma\eta_\gamma).$$

In this expression, H is the surface density of hydrogen in the target (0.42g cm^{-2}), and N is Avogadro's number. η, the telescope efficiency for π^0-decay photons, is to be chosen (see Table IV) according to the assumed π^0 angular distribution. (It should be noted that the radiative capture process accounts only for a few percent of the observed effect.) For an isotropic π^0-distribution, one finds for the charge-exchange cross section,

$$(\sigma_x)_{\text{iso}}=5.0\pm0.75 \text{ mb}.$$

A $\cos^2\theta$-angular distribution would increase this value by about 14 percent, while a $\sin^2\theta$-distribution would decrease it by 7 percent. Although the angular distribution is not known, it is reasonable to assume that higher powers of $\cos\theta$ will not contribute significantly. Thus, the lack of knowledge about the angular distribution means only that uncertainty in the cross section is somewhat larger than that given above. The most likely source of error in the computation lies in the application of the shower theory of Wilson, which is admittedly approximate and has been only partially checked by experiment. As a consequence of these factors, we conclude that the charge-exchange cross section in hydrogen has been ascertained within a provisional uncertainty of 30 percent.

Charge-Exchange Scattering in Deuterium

The relevant data for deuterium are given in Table II. For π^+-mesons, the D_2O–H_2O difference yields the

[10] J. Steinberger and A. S. Bishop, Phys. Rev. 86, 171 (1952).

deuterium effect directly. The π^--result can be interpreted by using the hydrogen results of the previous section. Assuming that the results are reduced to equal members of hydrogen nuclei,

$$D_2 \text{ effect} = (D_2O - H_2O) + H_2 \text{ effect}.$$

Because of the large H effect, this procedure is much less accurate than the direct subtraction possible with π^+-mesons.

The net deuterium effect, expressed as events per 10^6 M counts is given below, together with the hydrogen effect reduced to the same number of atoms:

$$\pi^+ + D: \quad 8.0 \pm 3.3$$

$$\pi^- + D: \quad 9.0 \pm 6.5$$

$$\pi^+ + H: \quad 0.4 \pm 5.2$$

$$\pi^- + H: \quad 23.2 \pm 2.6.$$

A direct comparison of cross sections between H and D is inexact for several reasons. The π^0-mesons are not monochromatic; the angular distributions may be quite different; and it is difficult to estimate the relative importance of radiative capture.

If one neglects these difficulties and assumes the charge-exchange processes to be identical in H and D, then one concludes that the deuterium cross section is about one-third as large as the hydrogen cross section.

The smaller cross section of deuterium compared with hydrogen can be understood in terms of the exclusion principle, parity conservation, and the momentum distribution in the deuteron. As has been pointed out by Marshak[11] and Cheston,[12] s-wave scattering on deuterium cannot lead to a ^1S state of the product nucleons. ^3P states of the nucleons are possible for both s- and p-wave incident mesons, but the transition probabilities are expected to be small, because of the predominance of low momentum components in the deuteron wave function.

Equal cross sections for charge exchange with both π^+- and π^--mesons are expected on the hypothesis of charge symmetry.

B. Two-Photon Detection

Detection Efficiency

As shown in Fig. 2, the axes of T_1 and T_2 define a plane perpendicular to the incident meson beam. The relative directions of the axes in this plane were chosen to favor the detection of photons whose angular separation was near 112°, the most probable angle between the decay photons of a 28.7 Mev π^0-meson. Since the acceptable range of angles varies considerably over the target volume, it is difficult to derive an exact expression for the two-photon detection efficiency η_2. We can, however, develop an approximate formula, using the π^0-decay kinematical relations. It is known[13] that the most probable correlation angle ϕ between the two photons is the minimum angle ϕ_0 and that the corresponding photon energy is $E_0 = \gamma\mu/2$.

Now suppose a π^0-meson decays at some point in the target into photons having a particular correlation angle ϕ. The meson will be detected, provided (a) it is emitted within a particular solid angle Ω_π, (b) the photons lie in a plane whose azimuthal angle about the direction of π^0-motion is within a particular range $\Delta\chi$, and (c) both photons convert into electrons capable of detection.

We have graphically evaluated Ω_π and $\Delta\chi$ for various points in the target and for different angles ϕ. The average efficiency η_2 is approximately

$$\eta_2 = \left(\frac{\langle\Omega_\pi\rangle_{\text{Av}}}{4\pi}\right)\left(\frac{\langle\Delta\chi\rangle_{\text{Av}}}{2\pi}\right)\langle\epsilon_1(E_1)\epsilon_2(\gamma\mu - E_1) + \epsilon_2(E_1)\epsilon_1(\gamma\mu - E_1)\rangle_{\text{Av}}, \quad (6)$$

where $\langle\Omega_\pi\rangle_{\text{Av}}$ and $\langle\Delta\chi\rangle_{\text{Av}}$ are averages over the target, weighted by the angular correlation function for the photon pairs. For simplicity, the product of the conversion efficiencies can be approximated by $2[\epsilon(E_0)]^2$. This is justified in view of the high probability of detection for $\phi = \phi_0$. At the mean π^0-energy of 29 Mev,

$$\langle\Omega_\pi\rangle_{\text{Av}}/4\pi = 0.0145, \quad \langle\Delta\chi\rangle_{\text{Av}} = \pi/4, \quad \text{and} \quad \epsilon(E_0) = 0.53.$$

The resulting efficiency is $\eta_2 = 1.0 \times 10^{-3}$. This is four times as large as the value which would be obtained if there were no angular correlation between the two photons.

Two-Photon Data

Let us now consider the results with the H_2O target (Table III). It seems reasonable to attribute the 10 ABC events obtained with Pb converter with π^--mesons to reactions in hydrogen; if they were produced in oxygen, it would be difficult to understand the absence of a similar yield with π^+-mesons incident. Under this assumption we find for the differential cross section,

$$d\sigma_H(90°)/d\Omega = ABC[4\pi M\eta_2\alpha_- NH]^{-1} = 0.8 \pm 0.25 \text{ mb/sterad}.$$

The indicated error here is only the statistical error. This result and that obtained from single-photon detection are easily compatible with an isotropic or $\sin^2\theta$-distribution. For a $\cos^2\theta$-distribution we would have expected 0.7 detected ABC events rather than the observed ten.

The events obtained with the D_2O target are too few in number to be significant. They do, however, lend

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[12] W. B. Cheston, Phys. Rev. **83**, 1118 (1951).

[13] Panofsky, Steinberger, and Steller, Phys. Rev. **86**, 180 (1952).

support to the supposition that the photons produced in D are, as in the case of H, principally caused by charge-exchange.

C. Photons from Carbon and Oxygen

The Pb–C converter difference for π^- on C (Table I) is about half as large as the H effect. This would give a cross section equal to that for hydrogen, if the yield of photons from C were all the result of isotropically emitted π^0-mesons. To assume this seems unjustified, however. The Pb–C difference for π^+ on H_2O is small, in fact compatible with zero. There are no data for the Pb–C difference for π^+ on C, but one may reasonably expect that the photon yield from C is about as small as that from H_2O. One may take this as evidence that π^0-production from carbon causes only a small part of the total effect, since the principle of charge symmetry (neglecting Coulomb effects) would require equal production of π^0-mesons by π^+ and π^- in nuclei with isotopic spin zero. Such a conclusion would be borne out by the results of Kessler *et al.*[14] who find an upper limit of 15 mb for charge-exchange of 150 Mev π^- in C.

The small cross section for charge-exchange scattering in nuclei with isotopic spin zero can be understood in terms of beta-ray theory, as Petschek[15] has pointed out. The nuclear transition involved is always unfavored, since a change of the partition is required. One might therefore expect larger charge-exchange cross sections when the nuclear transition is favored. This would occur for π^+-bombardment of the $T_\zeta=\frac{1}{2}$ nuclei Li^7, Be^9, etc.

The data with two-photon detection are insufficient to permit any conclusions concerning π^0-production in C or O.

The single photons observed from C and O might be tentatively ascribed to radiative capture processes; but these, too, might be expected to be equal for π^+ and π^-. It is perhaps possible that the effect is in part due to stopped π^--mesons. This and other possible effects of stopped mesons are discussed briefly in an appendix.

CONCLUSIONS

The analysis presented in this article leads to several conclusions.

(1) The cross section for charge-exchange scattering of 34-Mev π^--mesons in hydrogen is 5.0 ± 1.5 millibarns, assuming any reasonable angular distribution.

(2) The differential cross section for charge-exchange scattering of 34-Mev π^--mesons in hydrogen at 90° is about 0.8 mb/sterad. This is compatible with an isotropic or $\sin^2\theta$-distribution of mesons but not with a predominantly $\cos^2\theta$-distribution.

(3) The total cross section for charge-exchange scattering in deuterium is about one-third the value in hydrogen, and is the same for π^+- and π^--mesons, within wide limits of error. The latter result is in accordance with the notion of charge symmetry in nucleon-meson interactions.

(4) The cross section for charge-exchange scattering in carbon and oxygen is not larger than the hydrogen cross section and may well be much smaller.

Comparison with Other Results

Anderson, Fermi *et al.* found that at higher meson energies (115 Mev and up) the charge-exchange scattering of π^--mesons in hydrogen accounted for nearly all of the total scattering cross section. The total cross section for 37-Mev π^--mesons in hydrogen has been measured by Barnes *et al.*[16] in this laboratory. Their preliminary results indicate that the total cross section is at least 17 millibarns. Thus, one would conclude that charge-exchange is no longer dominant at low energy. However, Shutt *et al.*[17] found for the elastic scattering cross section at 57 Mev the value 3 ± 2 mb. It is possible that the cross sections are sharply energy dependent at low energies.

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APPENDIX: EFFECT OF STOPPED π^--MESONS

If a few π^--mesons have somehow lost large amounts of energy and stop in the target, some gamma-rays may well be emitted in the ensuing nuclear capture. The Pb–C difference with the C target is about ten counts per million M counts greater with π^- than with π^+-mesons. This difference could be accounted for if one π^--meson per thousand yields a capture gamma-ray of sufficient energy to be counted with reasonable efficiency.

This effect would of course tend to cancel in the CH_2–C difference, except in so far as the difference in stopping power of the targets may change the number of stopped mesons, or nuclear capture in hydrogen may occur. The latter effect[18] is known to

[14] Kessler, Byfield, Lederman, and Rogers, Bull. Am. Phys. Soc. **28**, No. 1, 14 (1953).

[15] A. G. Petschek, thesis, University of Rochester, 1952 (unpublished).

[16] Angell, Perry, and Barnes, Rochester Third High Energy Conference, 1952 (Interscience Publishers, Inc., New York).

[17] Shutt, Fowler, Miller, Thorndike, and Fowler, Phys. Rev. **84**, 1247 (1951).

[18] Panofsky, Aamodt, and Hadley, Phys. Rev. **81**, 565 (1951).

be less than 0.5 percent in CH_2 and may be much smaller. If it occurred for 1 out of 200 stopped π^--mesons, and as many as 5 percent of the π^-'s stopped in the target, this would contribute only 2.5 counts per million M counts to the observed CH_2–C difference. This source of error is therefore negligible.

That nuclear gamma-rays cannot contribute much to the CH_2–C difference is also shown by the fact that the results obtained with a 15-Mev minimum detectable electron energy agree with those for 8-Mev minimum, assuming the π^0-decay gamma-ray spectrum (i.e., high energy gamma-rays). The π^--capture gamma-rays would be expected to be much lower in energy, and the change in detection efficiency much more than the observed factor of two. In addition, there is the angular correlation demonstrated by the two quantum coincidences.

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The Absorption of Slow π^- Mesons by He^4 Nuclei*

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On the assumption of central charge-independent two particle nuclear forces, the wave function $(E+K)^{-2}(bE+K)^{-2}$ with E the binding energy, K the kinetic energy, and $b=5.3$ is found to be a better wave function, in the sense of a variational principle, for the He^4 nucleus than the optimum Gaussian. With this wave function, the ratios of the numbers of protons to deuterons to tritons in final states resulting from the absorption of π^- mesons from K shells of mesic helium atoms are found to be 1:1.3:0.7. For reasonable values of the $PS(PV)$ coupling constant, 20 percent of the absorptions take place directly from the $2p$ level. The number of high energy γ-rays is less than 1 percent of the number of nonradiative absorptions. Charge-exchange absorption is energetically forbidden.

INTRODUCTION

When a negative pion is absorbed by a He^4 nucleus, three types of final states are available for the nuclear system, namely,

$$p+3n, \quad d+2n, \quad t+n,$$

to which we shall refer as the proton, deuteron, and triton final states, respectively. The transition to any of these states may be accompanied by the emission of electromagnetic radiation but not, because of the conservation of energy, by the emission of neutral pions. A calculation of the relative probabilities of transitions to these final states is of interest not only to corroborate the information concerning the π-meson which is gained by a study of its interaction with the proton and the deuteron,[1] but also because, through the dependence of the relative probabilities on the nuclear wave function, it sheds light on the structure of the nucleus. The He^4 nucleus is the nucleus next in complexity to the deuteron that is available in sufficient quantity to make an experiment practicable. It is also the lightest nucleus with structural resemblance to heavier nuclei in that both the binding per energy nucleon and the average kinetic energy per nucleon are close to the corresponding figures for heavy nuclei.

For these reasons it was thought worth while to extend to the He^4 nucleus the calculation on the absorption of slow negative pions by nuclei carried out at this laboratory by Marshak,[2] Tamor,[3] and Messiah.[4] In addition to these calculations, several others of similar scope precede ours, namely, two calculations of Bruno,[5] and one of Clark and Ruddlesden.[6] Besides the use of a less carefully chosen wave function for the He^4 nucleus and differences in the detailed treatment of the final states, the latter calculations differ from ours in several important respects. The first calculation of Bruno employed a meson mass of 100 Mev in accordance with the experimental data at that time. Moreover, transitions to the triton final state alone were calculated. In his next calculation Bruno revised the meson mass, and because the larger mass is expected to lead to a larger yield for the proton final states, he calculated the transition rate to such states (for vector mesons) and found it to be larger than the triton rate. In the calculation of Clark and Ruddlesden the nucleons are taken to be infinitely heavy in calculating the interaction Hamiltonian (not, of course, in the kinematics) and the electromagnetic radiation accompanying the absorption is not considered.

The present calculation resembles in many respects the calculation of Messiah for He^3. Section I describes the method of calculation in general terms and allows the construction in Sec. II of the wave functions both of the initial He^4 nucleus and of the final nuclear

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