Cloud-Chamber Observations of the Heavy Charged Unstable Particles in Cosmic Rays*

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In a series of pictures taken with a multiplate cloud chamber, we have observed a number of charged unstable particles considerably heavier than π-mesons, of which some decay at rest (S particles) and some decay in flight ($V^{\pm}$ particles). In each case the charged decay product is a particle of mesonic mass. From our results, which are not yet completely conclusive, it appears that most of the observed events can be explained by the two-body decay of a single kind of particle. The evidence concerning the mass of this particle, its mean life, and the nature of the neutral decay product is discussed. Our observations are compared with those of other experimenters.

I. INTRODUCTION

In previous publications[1–3] we have reported the observation of charged particles heavier than π-mesons which appeared to undergo spontaneous decay after coming to rest in a multiplate cloud chamber. In each case the single visible decay product was a particle of mesonic mass with a kinetic energy of the order of 100 Mev. In our laboratory we have been referring to these particles as "anomalous stopped particles," or "S-particles" for short.

In this paper we wish to describe some further experimental evidence concerning S particles, as well as some observations on the so-called "charged V particles." These are particles heavier than π-mesons that are observed to disintegrate in the gas of a cloud chamber. We wish to point out that by using two different names we do not imply that S particles are necessarily different from charged V particles; we merely want to leave this possibility open for discussion.

2. EXPERIMENTAL ARRANGEMENT

We made our observations with the same cloud chamber used in a number of previous experiments. The chamber, however, had a new plate assembly consisting of 11 lead plates, 48 cm long, 22 cm wide, and 0.63 cm thick. Each lead plate carried, on either side, an 0.8-mm thick glass mirror to improve the illumination. The plates were tilted so as to be viewed edgewise by the cameras. At the center of the illuminated region their separation was 2.4 cm. The total thickness of lead and glass was equivalent to 7.7 g cm^{-2} of lead with regard to ionization losses, to 7.1 g cm^{-2} of lead with regard to radiation losses or scattering, and to approximately 7.8 g cm^{-2} Pb in terms of probability for nuclear collisions. The illuminated region was approximately 18 cm deep. Two photographs were taken at 5° to the chamber axis and two more at 30° to the axis.

The cloud chamber was triggered by a penetrating shower detector placed directly above it, as shown in Fig. 1. In this figure, Pb represents a block of lead; A, B, C, D, and E are groups of Geiger-Mueller counters. The triggering event was the simultaneous discharge of at least one counter in each group. Such fivefold coincidences occurred at a rate of approximately 18 per hour. The cloud-chamber recycling time was 4 minutes so that the chamber was expanded on the average about 8 times per hour. Practically all of the triggering events were caused by nuclear interactions which occurred in the lead block. In about 50 percent of the cases penetrating particles from the nuclear interactions appeared in the cloud-chamber pictures.

The equipment was operated for a period of approxi-

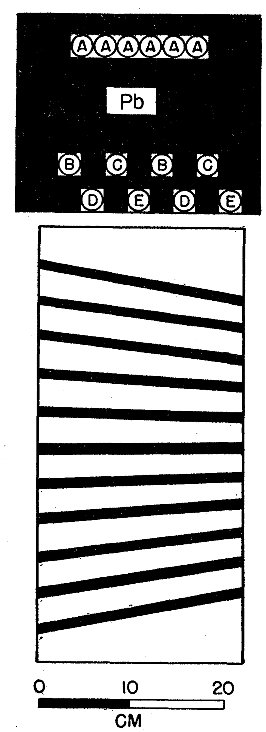

FIG. 1. Experimental arrangement.

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[1] H. S. Bridge and M. Annis, Phys. Rev. **82**, 445 (1951).

[2] Rossi, Bridge, and Annis, Rend. accad. nazl. Lincei **11**, 73 (1951).

[3] Annis, Bridge, Courant, Olbert, and Rossi, Nuovo cimento **9**, 624 (1952).

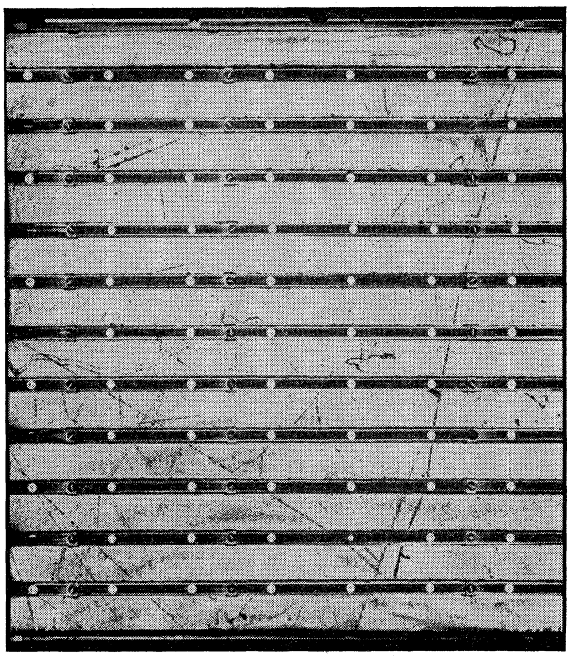

FIG. 2. S particle (event $S7$).

mately 5 months at an altitude of 3260 m above sea level. During this period approximately 22 000 pictures were taken.

3. EXPERIMENTAL RESULTS CONCERNING STOPPED PARTICLES

In the 22 000 pictures we found approximately 1200 particles that stopped in the chamber after traversing at least five plates. Nearly all of these particles originated in the lead block above the chamber. From specific ionization, residual range, and scattering,[4] we estimated that approximately 260 of the stopped particles mentioned above were π-mesons, and the rest mostly protons and perhaps deuterons.

We also observed 33 cases where a particle stops in a lead plate after traversing one or more plates, and a particle which appears to be the decay product of the stopped particle, emerges from the same plate. These 33 cases fall into the following groups, according to the behavior of the decay particle:

Group A (17 cases): the decay particle stops in the lead plate next to the one where it originates, or is scattered back and forth between the two plates.

Group B (6 cases): the decay particle enters the next plate near the edge of the illuminated region and may either stop in this plate or emerge from it outside of the illuminated region.

Group C (2 cases): the decay particle emerges from the plate of production alone, but undergoes multiplication on traversing the next plate.

Group D (1 case): after emerging from the plate of production, the decay particle traverses 1 additional plate and comes to rest in the following plate.

Group E (1 case): the decay particle traverses one plate and then goes out of the illuminated region.

Group S (6 cases): the decay particle traverses two or more plates.

The 6 cases included in the last group are discussed individually below.

Event $S3$

The primary particle enters the chamber from above and stops in the 5th plate from the top. The secondary particle is emitted downward and appears to stop in the 11th plate. However, in this particular picture the chamber was underexpanded; it is possible, but not likely, that the particle has emerged undetected from the 11th plate. Before entering the 11th plate, the particle had traversed 64.4 g cm^{-2} of lead equivalent. Here as in the following cases the range was determined from a reconstruction of the trajectory in space.

Event $S4$

The primary particle appears to originate in the 2nd lead plate with no other particles coming from the same origin. It stops near the top of the 4th plate. The secondary particle is emitted downwards and stops in the 8th plate. Before entering this plate, it had traversed 65.6 g cm^{-2} Pb.

Event $S5$

The primary particle originates from a nuclear interaction in the 6th plate; it stops in the 8th plate. The secondary particle is emitted downward and leaves the cloud chamber after traversing 3 plates. Its range is greater than 36 g cm^{-2} Pb.

Event $S6$

The primary particle enters the chamber from above. It stops in the 9th plate. The secondary particle is emitted upward and leaves the illuminated region after traversing 2 plates. Its range is greater than 16.6 g cm^{-2} Pb.

Event $S7$

The primary particle enters the chamber from above. It stops in the 11th plate. The secondary particle is emitted upward and leaves the illuminated region after traversing 4 plates. Its range is greater than 60.0 g cm^{-2} Pb.

Event $S8$

The primary particle enters the chamber from above and stops in the 3rd plate. The secondary particle leaves the illuminated region after traversing 4 plates. Its range is greater than 43.4 g cm^{-2} Pb.

Events $S7$ and $S8$ are shown in Figs. 2 and 3. Table I summarizes the measurements made on the primary and

[4] Annis, Bridge, and Olbert, Phys. Rev. 89, 1216 (1953).

TABLE I. Summary of measurements on the primary and secondary tracks of events $S1$ to $S8$ as explained in the text.

Primary particle

Plate No.	$S1$ φ deg	$S1$ R g cm^{-2}	$S2$ φ deg	$S2$ R g cm^{-2}	$S3$ φ deg	$S3$ R g cm^{-2}	$S4$ φ deg	$S4$ R g cm^{-2}	$S5$ φ deg	$S5$ R g cm^{-2}	$S6$ φ deg	$S6$ R g cm^{-2}	$S7$ φ deg	$S7$ R g cm^{-2}	$S8$ φ deg	$S8$ R g cm^{-2}
10													0.9	82.4		
9	1.0	66.4											3.1	74.4		
8	4.0	59.1									1.3	61.9	1.2	66.4		
7	5.0	51.8									5.4	54.2	1.7	58.4		
6	4.0	44.5									1.2	46.4	0.5	50.4		
5	1.5	37.2									1.5	38.7	1.0	42.4		
4	2.0	29.9	1.5	30.6	1.5	40.8					2.0	31.0	1.2	34.4		
3	1.5	22.6	1.0	23.5	3.0	32.6					0.4	23.2	4.9	26.4		
2	3.5	15.3	4.5	16.2	9.0	24.4					0.5	15.5	17.3	18.4	4.0	18.6
1	22.0	8.0	16.0	8.9	1.5	16.2	18.0	9.2	4.5	18.5	1.2	7.8	8.0	10.4	9.0	10.8
0		0.7		1.6		8.0		0.1		7.5		0.1		2.4		3.1

Secondary particle

Plate No.	$S1$ φ deg	$S1$ x g cm^{-2}	$S2$ φ deg	$S2$ x g cm^{-2}	$S3$ φ deg	$S3$ x g cm^{-2}	$S4$ φ deg	$S4$ x g cm^{-2}	$S5$ φ deg	$S5$ x g cm^{-2}	$S6$ φ deg	$S6$ x g cm^{-2}	$S7$ φ deg	$S7$ x g cm^{-2}	$S8$ φ deg	$S8$ x g cm^{-2}
0		6.6		5.7		0.1		24.2		0.2		0.2		4.2		5.7
1	9.0	13.9	2.0	13.0	3.0	17.0	8.5	39.6	8.2	12.2	9.0	8.4	9.2	18.1	1.5	15.1
2	1.5	21.2	2.5	20.3	12.0	32.5	32.0	57.1	2.0	24.2	10.0	16.6	1.5	32.0	5.5	24.6
3	1.0	28.5	2.0	27.6	10.0	44.1	12.0	65.6	7.0	36.2			0.8	46.0	3.5	34.0
4	5.0	35.8			5.0	54.5	STOPS						1.6	60.0	10.0	43.4
5					1.0	64.4										
6					STOPS											

on the secondary tracks of the six events discussed above, as well as similar measurements made on two events reported previously ($S1$ and $S2$).[3] In Table I, the number 0 denotes the plate where the primary particle stops and decays. The other plates are numbered in order of increasing distance from plate No. 0.

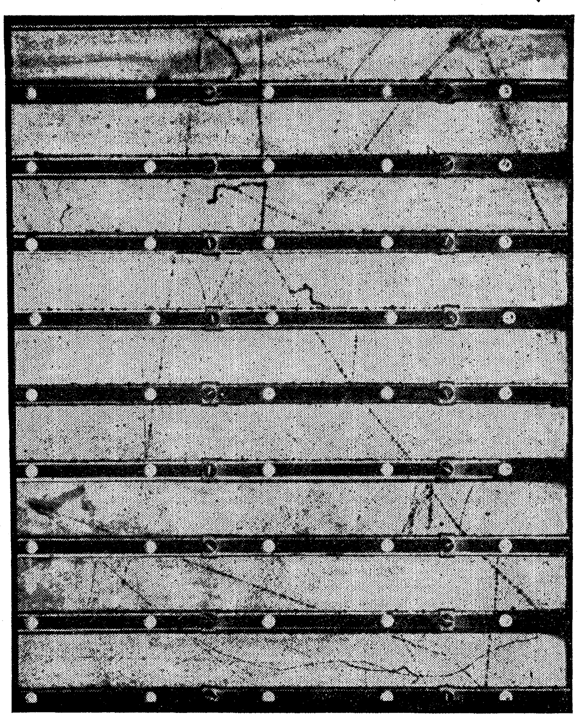

FIG. 3. S particle (event $S8$).

φ represents the projected angle of scattering in the plate. R is the residual range of the incident particle as it enters the plate. x is the total thickness of lead equivalent traversed by the secondary particle from the point of decay to the point where it leaves the plate.

4. INTERPRETATION

The events listed in groups A, C, and D can be readily interpreted in terms of the $\pi\rightarrow\mu\rightarrow e$ decay process. Most of the events in group B are probably of the same nature. According to this interpretation, the incident particle is a π-meson from a nuclear interaction. The μ-meson produced by the decay of the π-meson at rest has an energy of only 4 Mev, and thus stops in the plate where it is produced. The outgoing particle is then the decay electron of the μ-meson. It may be recalled here that the decay electrons of μ-mesons are distributed in energy from 0 to 54 Mev, and that the maximum of the distribution occurs at about 35 Mev.[5]

Our interpretation of the pictures in groups A, B, C, and D is consistent with the fact that most of the decay particles emerging from the plate of production fail to penetrate the next lead plate. It is also consistent with the observation that most of the particles which do penetrate this plate undergo large angle scattering and, occasionally, a small amount of multiplication.

Following the method outlined by Annis, Bridge, and Olbert[4] (see also Appendix) one can use the observed

[5] Leighton, Anderson and Seriff, Phys. Rev. **75**, 1432 (1949); A. Lagarrigue and C. Peyrou, J. phys. et radium **12**, 848 (1951); Sagane, Gardner, and Hubbard, Phys. Rev. **82**, 557 (1951); R. Levi-Setti and G. Tomasini, Nuovo cimento **8**, 994 (1951); Bramson, Seifert, and Havens, Phys. Rev. **88**, 304 (1952).

scattering in the lead plates to estimate the mass of the primary particles in the pictures of groups A, B, C, and D (assumed to be all of the same kind). One obtains a value of

$$\left(230{+40 \atop -30}\right)m_e,$$

which is consistent with either a π- or μ-meson (here and in what follows m_e represents the electron mass).[6]

In the single case in group E, the primary particle appears to be heavier than a π-meson from specific ionization and scattering; but, since the secondary particle leaves the illuminated region after penetrating only one plate, nothing definite can be said about the nature of the event.

The 6 cases in group S cannot be interpreted as $\pi\rightarrow\mu\rightarrow e$ decay processes. Ionization loss alone sets an upper limit of 45 g cm^{-2} to the range of 54-Mev electrons in lead. In cases $S3$, $S4$, and $S7$ the range of the decay particle is greater than this value. Moreover in cases $S3$ and $S4$ the secondary particle stops in the chamber. Specific ionization, scattering, and lack of shower production concur to identify the particle as one of mesonic mass.

In cases $S5$ and $S8$, as well as in case $S1$ reported previously, the range of the secondary particle is greater than 35.8 g cm^{-2} of lead. If one considers only ionization losses, one can place a lower limit of 42 Mev to the energy of a particle whose range is greater than 35.8 g cm^{-2} Pb. Actually, of course, for electrons of the energies under consideration here, radiation losses in lead greatly exceed collision losses. It is thus virtually impossible to identify these particles as electrons of less than 54-Mev energy. Small scattering and lack of multiplication confirm this conclusion.

In case $S6$, and in case $S1$ previously reported, the secondary particles leave the illuminated region after traversing 16.6 and 27.6 g cm^{-2} of lead, respectively. From range alone, one cannot completely rule out the possibility that the secondary particle is the decay electron of a μ-meson. However the lack of multiplication and of large-angle scattering makes this possibility appear as a very remote one. Indeed the root mean square projected angle of scattering of an electron in the lead plates of the chamber is approximately 17° at 54 Mev and increases in inverse proportion to the decreasing energy as the electron traverses matter. The observed root mean square projected angle of scattering is about 5° in case $S5$ and about 2° in case $S2$.

One may wonder about the possibility of explaining the events discussed above as chance coincidences between unrelated tracks. To discuss this possibility, consider that in all of the events the apparent point of decay is in the well-illuminated region. The incident particle traverses at least one plate and fails to emerge from the plate where the decay seems to occur. It is quite heavily ionizing in the last visible section of its trajectory. The particle that we interpret as the decay product certainly originates in the plate, because if it had originated elsewhere its trajectory would have been seen entering the plate in all cases. The trajectories of the primary and of the secondary particles appear copunctual in all available views. A conservative estimate of the possible experimental uncertainty shows that in each case the secondary particle must have originated within an area of 0.2 cm^2 about the end point of the primary particle. We have explored about 5×10^5 cm^2 of plate surface taken at random in the illuminated region and found one track of a particle which originated in a plate and traversed at least 2 additional plates. On the other hand, in our series of pictures there were about 4000 tracks of heavily ionizing particles (mostly protons) that came to rest in the chamber after traversing at least one plate. The probability that in the entire set of pictures we would have observed one chance coincidence between unrelated particles simulating a decay process is thus of the order of $4000\times0.2/5\times10^5\approx2\times10^{-3}$. We thus conclude that the observed events cannot be explained as chance coincidences. Since we have already rejected the possibility of $\pi\rightarrow\mu\rightarrow e$ decays, we shall regard the events of Group S as examples of S particle decays.

The scattering method mentioned above enables us to estimate the mass of the primary S particles (assumed to be all identical). From the measurements on the eight particles listed in Table I one obtains for this mass a value of

$$m_s=\left(1460{+410 \atop -360}\right)m_e.$$

Comparing this result with that obtained for particles in groups A, B, C, and D it is evident that the S-particles are certainly heavier than π-mesons. They could, however, have protonic mass.

One can obtain some additional information on the mass of S particles from the specific ionizations observed at various distances from the ends of the tracks. One finds that in all cases the mass lies between 1000 m_e and the proton mass.[7]

We now turn our attention to the secondary particles, and we consider first the two cases ($S3$ and $S4$) in which the secondary particle stops in one of the plates. In case $S4$ the specific ionization in the last section of the secondary track is considerably greater than minimum. Thus if the particle is a meson, it must have stopped near the point of entrance into the plate. Taking this fact into account, one obtains for the range a value between 65.6 and 67.7 g cm^{-2} Pb. In case

[6] It should be noted that no special effort was made to detect all events where the decay electron came out of the plate of production. These events are not very conspicuous and many of them may have been missed.

[7] Under the conditions in which our pictures were taken, only a crude estimate of the specific ionization is possible. In general we believe that we can estimate the ionization within a factor of 2, up to about 10 times minimum.

$S3$ the chamber was underexpanded and therefore no confidence can be placed on estimates of specific ionization. Thus the particle might have stopped anywhere in the plate, and the limiting values for its range are 64.4 and 74.3 g cm^{-2} Pb.

A scattering analysis applied to the combined data of events $S3$ and $S4$ yields a value of

$$\left(160{+130 \atop -\ 80}\right)m_e$$

for the mass of the secondary particle. Thus the secondary particle appears to have mesonic mass, as already pointed out. Confirming evidence to the effect that this particle is heavier than an electron and lighter than a proton comes from lack of multiplication and from crude estimates of the specific ionization.

In cases $S1$, $S2$, $S5$, $S6$, $S7$, and $S8$ the secondary particle leaves the illuminated region before coming to rest. Thus, from range alone, one can only place a lower limit to its momentum. In addition, however, one can obtain an estimate of the momentum from the observed scattering (see Appendix). Figure 4 summarizes the information on the momentum of the secondary particles derived from range and from scattering measurements, under the assumptions that the secondary particles are π-mesons [Fig. 4(a)] or μ-mesons [Fig. 4(b)]. Vertical bars indicate "absolute" momentum limits determined by range. Arrowheads indicate "statistical" limits determined by scattering (see Appendix). Dots indicate "most probable" values derived from scattering.

In events $S3$ and $S4$, where the limits of the experimental uncertainty in momentum are quite narrow, the agreement between the values of the momentum is particularly striking. The assumption of a unique momentum for the secondary particle is perfectly consistent with the results obtained for events $S1$, $S5$, $S6$, and $S8$. In events $S2$ and $S7$ the most likely value of the momentum determined by scattering is considerably higher than the secondary momenta observed in events $S3$ and $S4$. However the significance of this result is doubtful on account of the large statistical errors of the scattering measurements.

In conclusion, our data do not seem to contradict the assumption that all secondary particles have the same momentum and thus that the decay of S particles is a two-body process. If this assumption is correct, event $S4$ yields for the momentum of the secondary particle a value from 212 to 215 Mev/c if this particle is a π-meson, and a momentum from 183 to 186 Mev/c if this particle is a μ-meson.

As a further check of internal consistency, we have computed the mass of the secondary particle by assuming that all secondary particles have a range of 66 g cm^{-2} (in agreement with event $S4$) and by taking into

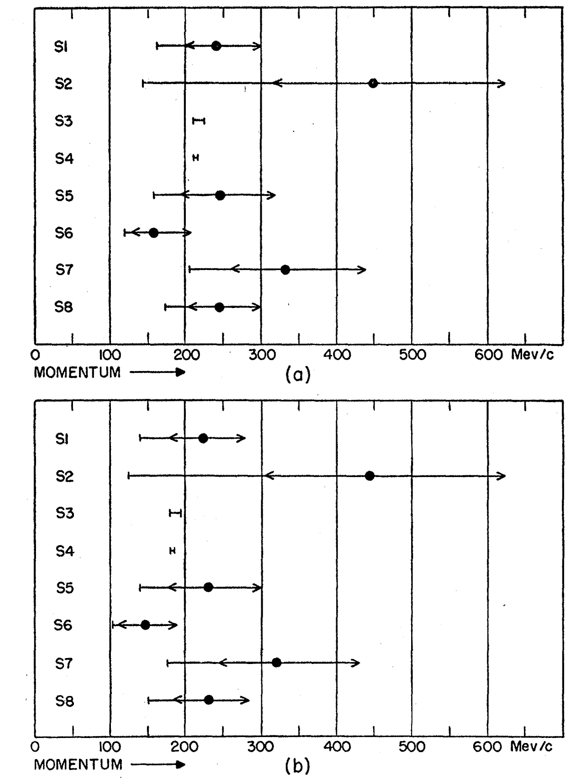

FIG. 4. Momenta of the secondary particles of S particles computed under the assumption that the secondary particles are (a) π-mesons, (b) μ-mesons. Bars indicate "absolute" limits from range; arrowheads indicate "statistical" limits, from scattering. Dots indicate "most probable" values, from scattering.

account all observed angles of scattering. We found

$$m=\left(275{+110 \atop -\ 70}\right)m_e,$$

a value consistent with either the π- or the μ-meson mass.

Although the secondary particles have been observed to traverse 350 g cm^{-2} Pb, no nuclear interactions have been seen. Since it is known that π-mesons, unlike μ-mesons, interact strongly with nuclei, this observation favors the identification of the secondary particles as μ-mesons. However it does not completely rule out the possibility that the secondary particles are π-mesons, especially because there may be some bias against recording S particles whose secondaries undergo nuclear interactions. Indeed, if the nuclear interaction occurs in the first or in the second plate next to the one where the secondary particle originates, and if neither the interacting particle nor any of the products of the interaction emerges from the plate, the event would be classified as a $\pi\rightarrow\mu\rightarrow e$ decay. Neglecting the first and the second plate, the total thickness of matter traversed by the secondary particles is 122 g cm^{-2} Pb.

If one accepts the assumption of a two-body decay,

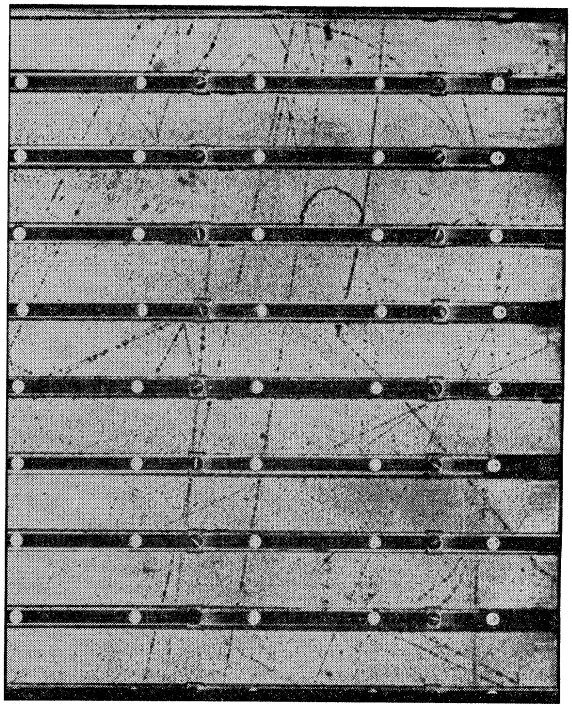

FIG. 5. Charged V particle (event $V2$).

one must conclude that, in each case, a neutral particle of momentum about 200 Mev/c is emitted in the direction opposite to that of the charged decay product. From stereoscopic analysis of the pictures one finds that altogether the neutral decay products of the observed eight S particles have travelled a distance of 58 cm in the gas and of 16 cm in the plates ($\sim$170 g cm^{-2} Pb) before leaving the illuminated region of the cloud chamber. No spontaneous decay and no interaction in matter has been observed.

5. DOUBTFUL EVENTS

Not included in the decays of stopped particles discussed in the preceding sections are two events whose interpretation is doubtful.

In the first event a particle appears to enter the cloud chamber from above and to stop in the second lead plate, giving rise to a secondary penetrating particle which leaves the illuminated region after traversing 4 lead plates (51 g cm^{-2}). At first sight, this event looks like an S particle decay. It has, however, several peculiar features.

(a) The primary particle has not more than twice minimum ionization above the first plate and becomes heavily ionizing on traversing this plate. This behavior is inconsistent with the large mass of S particles, unless one assumes that the particle loses energy in a nuclear collision.

(b) Halfway between the first and the second plate the trajectory of the primary particle undergoes a deflection of 20°. There is a blob of ionization at the point where the deflection occurs, suggesting that the deflection may be due to a collision with an argon nucleus.

(c) The secondary particle scatters less than one would expect for a π- or μ-meson of about 200 Mev/c momentum even though the difference may not be statistically significant (the observed angles of scattering indicate a momentum of

$$\left(720\begin{matrix}+270\\-230\end{matrix}\right)\ \text{Mev}/c).$$

The following is a possible, even though unlikely, alternate interpretation for the event described. A neutron causes a nuclear interaction in the second lead plate, and produces a penetrating particle in a downward direction and a slow π-meson (or S particle) in an upward direction. This particle stops and decays in the first lead plate. The decay product leaves the plate following a trajectory which is the almost exact continuation of the trajectory of the parent particle.

In the second event a particle appears to enter the chamber from above, to traverse three lead plates and to stop in the fourth plate. Seemingly a secondary particle arises from the point where the primary particle stops; it traverses seven lead plates (69 g cm^{-2}), and then leaves the chamber. The peculiar features of this event are:

(a) the fact that the "primary particle" is heavily ionizing only in the last section of its trajectory, which is inconsistent with the identification of this particle as an S-particle, unless an energy loss by nuclear collision occurs;

(b) the high momentum of the secondary particle (the visible range sets a lower limit of 217 Mev/c to this momentum if the particle is a π-meson and a limit of 188 Mev/c if it is a μ-meson; the scattering is extremely small and indicates a momentum greater than 1000 Mev/c).

Here again an alternate interpretation is possible. One can assume that a neutron causes a nuclear interaction in the fourth lead plate; a penetrating particle is produced in a downward direction, and an S particle of very low energy in an upward direction. The S particle stops in the third plate and its decay product is projected upward.

Because of their doubtful interpretation, the two events described above will be disregarded in the following discussion. They are mentioned here merely as an indication for the possible existence of S particles which give rise to secondaries of momentum considerably greater than 200 Mev/c.

6. CHARGED V PARTICLES

In the same set of pictures that yielded the six examples of S particles ($S3$ through $S8$) described in Secs. 3 and 4, we found five clear examples of particles different from π- or μ-mesons which disintegrate in the gas. We classify these events phenomenologically as charged V particle (or $V^{\pm}$ particle) decays. Two of them are reproduced in Figs. 5 and 6. In what follows we describe in some detail the above five events ($V2$ through $V6$) as well as one similar event reported previously ($V1$).[1,2]

In event $V2$ (Fig. 5) the primary particle enters the chamber from above. It traverses 4 plates and has a very small velocity or perhaps is at rest when it disintegrates. From range and specific ionization it appears to have a mass greater than about 600 m_e. The small amount of scattering indicates an even larger mass. The secondary particle is emitted at an angle of 56° and traverses three lead plates before leaving the illuminated region. If this particle is a π-meson, its visible range sets a lower limit of 155 Mev/c to the momentum, whereas scattering indicates a momentum of

$$\left(266\begin{matrix}+110\\-\ 70\end{matrix}\right) \text{Mev}/c.$$

If the secondary particle is a μ-meson, the corresponding values are 132 Mev/c and

$$\left(254\begin{matrix}+110\\-\ 60\end{matrix}\right) \text{Mev}/c.$$

In event $V3$ (Fig. 6) the primary particle enters the chamber from above. It ionizes more than minimum all along its visible trajectory and has an estimated specific ionization of about twice minimum where it decays, which indicates a velocity of the order of 0.6 times the velocity of light. The observed scattering and specific ionization indicate a mass greater than about 1000 m_e. The secondary particle is emitted at an angle of 32.5° and traverses four lead plates before leaving the chamber. This places a lower bound of 166 Mev/c to its momentum if the particle is a π-meson, and a lower bound of 142 Mev/c if it is a μ-meson. From scattering, the momentum appears to be

$$\left(490\begin{matrix}+165\\-140\end{matrix}\right) \text{Mev}/c$$

irrespective of whether the particle is a π- or a μ-meson.

By applying the Lorentz transformation, one finds the following results for the momentum of the secondary particle in the frame of reference where the primary particle is at rest: (1) If the secondary particle is a π-meson: $p_{\min}=90$ Mev/c (from range); 204 Mev/$c<p<430$ Mev/c (from scattering). (2) If the particle is a μ-meson: $p_{\min}=76$ Mev/c (from range); 202 Mev/$c<p<424$ Mev/c (from scattering). The limits indicated include the uncertainty in the estimate of the primary velocity.

In event $V4$ the primary particle originates in a nuclear interaction in the chamber and it decays after traversing one plate. Its ionization at the point of decay is close to minimum. The secondary particle is emitted at an angle of 57.5°. It traverses 2 plates, becomes heavily ionizing and stops in the following plate. If the particle is a π-meson, its momentum is between 141 and 159 Mev/c. If it is a μ-meson, the momentum limits are 120 and 137 Mev/c. One cannot compute the corresponding momenta in the frame of reference of the primary particle because the velocity of this particle is not known. One can say, however, that the momentum in the frame of reference of the primary particle cannot be smaller than the transverse component of the momentum in the laboratory system. This is between 119 and 134 Mev/c if the secondary particle is a π-meson, and between 101 and 115 Mev/c if the secondary particle is a μ-meson.

In event $V5$ the primary particle enters the chamber from above and traverses four plates before decaying. The ionization is everywhere more than twice minimum, which places a lower limit of about 1000 m_e to the mass. The secondary particle is emitted at an angle of 135° and leaves the illuminated region before traversing any lead plate. The event cannot represent a $\pi\rightarrow\mu$ or a $\mu\rightarrow e$ decay because the mass of the primary particle is definitely greater than that of a μ- or a π-meson. Moreover, μ-mesons do not usually originate from nuclear interactions. Furthermore, if a π-meson disintegrating in flight ejects the secondary μ-meson at an angle greater than 90° in the laboratory system, both

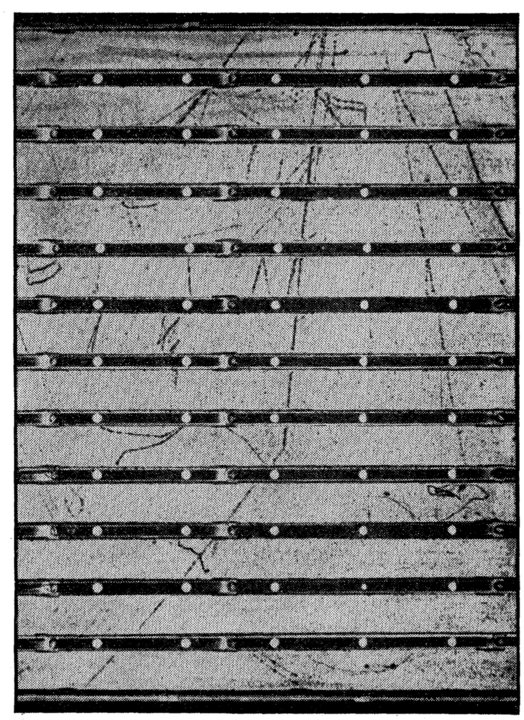

FIG. 6. Charged V particle (event $V3$).

the parent and the daughter particles must have velocities smaller than the velocity of a μ-meson originating from the decay of a π-meson at rest ($\beta=0.38$). Such small velocities are incompatible with the observed ionizations.

In event $V6$, the primary particle emerges from a nuclear interaction in the fourth plate and decays in the gas at a distance of 2 cm from its point of origin. The secondary particle is emitted at an angle of 78°, traverses one lead plate with a scattering angle of 10° and stops in the next plate. The event cannot be a $\pi\rightarrow\mu$ decay. It could be a $\mu\rightarrow e$ decay, but this appears to be only a remote possibility. In the first place μ-mesons are very rarely emitted from nuclear interactions. In the second place a μ-meson has a long mean life and is thus very unlikely to decay in flight in a cloud chamber. Thirdly, few among the decay electrons of μ-mesons traverse one of the plates in our chamber without multiplication (see Sec. 3 above). Thus the event under discussion represents most probably a $V^{\pm}$ particle decay.

If this is the case and if we assume the secondary particle to be a π-meson, its momentum lies between 138 and 150 Mev/c. If the particle is a μ-meson, the corresponding limits are 116 and 127 Mev/c. The primary particle ionizes between 2 and 4 times minimum at the point where it disintegrates; the corresponding velocity is between 0.6 and 0.4 times the velocity of light. By means of the Lorentz transformation, one finds the following limits for the momentum of the secondary particle in the frame of reference where the primary particle is at rest: if the particle is a π-meson: 146 to 187 Mev/c; if the particle is a μ-meson: 122 to 153 Mev/c.

Event $V1$, which was described in previous publications,[1,2] has been reanalyzed with the following results. The primary particle has an ionization between 2 and 5 times minimum. The secondary particle is emitted at an angle of 95°, undergoes nuclear scattering in an aluminum plate, and comes to rest in the chamber after traversing two lead plates, each 0.63 cm thick, and two aluminum plates, each 0.8 cm thick. From scattering and specific ionization it appears to have mass near to that of the π- or the μ-meson. Since it undergoes nuclear scattering, it is presumably a π-meson. Its residual range at the point of production corresponds to a momentum between 142 and 145 Mev/c, *if one assumes that the particle does not lose any energy in the process of nuclear scattering*. Transformation to the frame of reference where the primary particle is at rest gives a value between 172 and 225 Mev/c for the momentum of the decay product in this frame of reference.

From the similarity of these results and those obtained for events $S3$ and $S4$, it would appear that, in many cases at least, charged V particles are S particles disintegrating in flight. The phenomenological identity between S particles and charged V particles is also forcefully suggested by event $V2$, which could equally well be classified as an S particle or as a charged V particle.

In event $V6$ the momentum of the secondary particle in the frame of reference of the parent particle appears to be somewhat lower than the momentum of the secondary particle from $S3$ or $S4$. However, one cannot rule out the possibility that the secondary particle might have undergone an anomalous energy loss by nuclear collision. Conversely, if event $V1$ has to be interpreted as the decay of a particle similar to $S3$ or $S4$, one must assume that the nuclear scattering of the secondary particle is not accompanied by a large loss of energy.

7. COMPARISON WITH OTHER EXPERIMENTAL RESULTS ON V PARTICLES

Observations of $V^{\pm}$ particles have been reported by various authors.[8–14] Recently Armenteros *et al.*[12] and Barker *et al.*[14] have published the results of measurements on 27 charged V's photographed in a magnetic cloud chamber; however, only in 18 cases were they able to determine the momentum of the secondary particle. They found that the distribution of the transverse momenta did not seem to correspond to the distribution to be expected from a two-body decay and therefore concluded that if all charged V particles were the same, the decay scheme was probably a three body process.[12,15] On account of the small number of events, a conclusion based on a statistical argument is perhaps open to some doubt. It is therefore worthwhile to examine the individual observations in detail.

In one case an estimate of the specific ionization of the primary particle is available. From the corresponding velocity estimate, from the angle of emission of the secondary particle and from its measured momentum one obtains the following values for the momentum, p, of the secondary particle in the frame of reference where the primary particle is at rest: if the secondary particle is a π-meson: $p=(225\pm20)$ Mev/c; if the secondary particle is a μ-meson: $p=(217\pm20)$ Mev/c.

In six additional cases simultaneous measurements of the momenta of the primary and of the secondary particles are available. In these cases one can compute the momenta of the secondary particles in the frame of reference of the parent $V^{\pm}$ particles if one assumes specific values for the masses of the primary and of the secondary particles. Figure 7 shows the result of such a computation, made under the assumptions that the pri-

[8] G. D. Rochester and C. C. Butler, Nature **160**, 855 (1947).
[9] Seriff, Leighton, Hsiao, Cowan, and Anderson, Phys. Rev. **78**, 290 (1950).
[10] Thompson, Cohn, and Flum, Phys. Rev. **83**, 175 (1951).
[11] W. B. Fretter, Phys. Rev. **83**, 1053 (1951).
[12] Armenteros, Barker, Butler, Cachon, and York, Phil. Mag. **43**, 597 (1952).
[13] Astbury, Chippendale, Millar, Neuth, Page, Rytz, and Sahiar, Phil. Mag. **43**, 1283 (1952).
[14] Barker, Butler, Sowerby, and York, Phil. Mag. **43**, 1201 (1952).
[15] C. M. York, Phil. Mag. **43**, 985 (1952).

mary mass is 1500 m_e and that the secondary particles are π-mesons. One sees that the computed momenta are consistent with a single value. Moreover this value does not differ appreciably from our best estimate of the momentum of hypothetical π-mesons originating from the decay of S particles.[16] We have also computed the secondary momenta under the assumptions that the secondary particles are μ-mesons and that the primary mass is 1200 m_e. Again we found data consistent with the view that the secondary charged decay products of charged V particles and of S particles form a mono-energetic group.

In the remaining eleven cases neither the momentum nor the specific ionization of the primary particle is known. From these observations one can only obtain a lower limit for the momentum of the secondary particle in the frame of reference of the primary particle, this being the observed transverse momentum. Occasionally, this limit appears to be higher than the upper limit for the secondary momentum found in some $V^\pm$ or S decays. However the magnitude of the experimental errors makes it difficult to base any firm conclusion on such observations.

For comparison with the results of Armenteros *et al.*, Fig. 7 also shows the secondary momenta obtained from our measurements on $V^\pm$ particles, under the assumption that the secondary particles are π-mesons.

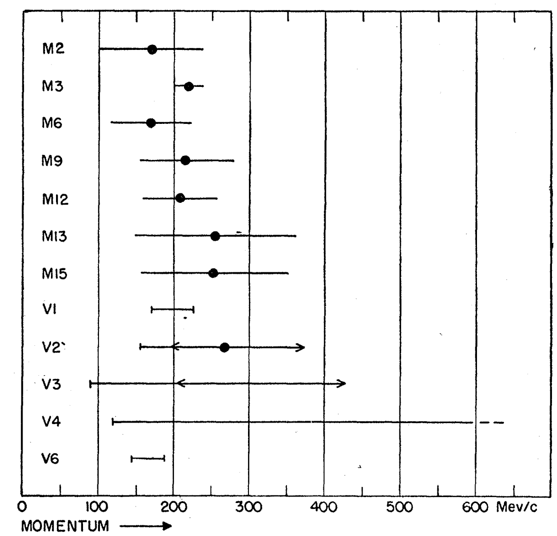

FIG. 7. Momenta of the secondary particles of charged V's, in the frame of reference of the parent particle, computed under the assumption that the secondary particles are π-mesons. Data obtained from the measurements of the Manchester group are indicated with "M." Data obtained in the present experiment are indicated with "V."

8. DISCUSSION

Our results and those of other cloud-chamber experiments suggest that many of the so-called S particles and charged V particles are in fact particles of one kind, disintegrating either at rest or in flight. From the present evidence, which is not yet conclusive, the decay appears to be a two-body process. In the frame of reference of the primary particle, the momentum of the charged secondary particle is slightly greater than 200 Mev/c if this particle is a π-meson, or slightly smaller than 200 Mev/c if it is a μ-meson. However in many cases only a crude estimate of the secondary momentum is available, and it is perfectly possible that some of the S events as well as some of the $V^\pm$ events correspond to a different type of decay. One should also notice that an S particle whose secondary charged product is not energetic enough to traverse at least two lead plates could have been classified as a π-meson in our analysis.

Charged particles with masses from 1000 to 1500 electron masses, giving rise to single charged particles of mesonic mass by their decay, have also been detected in photographic emulsions.[17–20] The Bristol observers have tentatively assumed that these particles are of two distinct kinds: (1) a χ-meson, which decays into a π-meson of momentum about 210 Mev/c and a single neutral particle; (2) a κ-meson, which decays into a μ-meson and at least two neutral particles.[20] χ-mesons and κ-mesons appear to have a mean life greater than several times 10^{-10} sec. Thus they certainly live long enough to be observed as $V^\pm$ particles, and possibly long enough to be observed as S particles. Indeed the results presented above make it appear likely that most S particles and $V^\pm$ particles are identical to χ-mesons.

A difficult problem arises when one inquires about the nature of the neutral particles which must necessarily originate from the decay of S and $V^\pm$ particles. We have considered various possible decay schemes involving a single neutral particle and either a π-meson or a μ-meson of range 66.6 g cm^{-2} Pb. The corresponding masses of the primary particle are listed below.

(a)	$S\rightarrow\pi+\nu$,	$m_s=$ 920 m_e,
(a′)	$S\rightarrow\mu+\nu$,	$m_s=$ 780 m_e,
(b)	$S\rightarrow\pi+\gamma$,	$m_s=$ 920 m_e,
(b′)	$S\rightarrow\mu+\gamma$,	$m_s=$ 780 m_e,
(c)	$S\rightarrow\pi+\pi^0$,	$m_s=$ 995 m_e,
(c′)	$S\rightarrow\mu+\pi^0$,	$m_s=$ 870 m_e,
(d)	$S\rightarrow\pi+V_2^0$,	$m_s=$1520 m_e,
(d′)	$S\rightarrow\mu+V_2^0$,	$m_s=$1410 m_e,
(e)	$S\rightarrow\pi+n$,	$m_s=$2385 m_e,
(e′)	$S\rightarrow\mu+n$,	$m_s=$2310 m_e.

[16] It is perhaps interesting to point out that a similar computation based on an assumed mass of 1000 m_e for the parent particle yields a much greater spread for the secondary momenta.
[17] C. O'Ceallaigh, Phil. Mag. **42**, 1032 (1951).
[18] R. Levi-Setti and G. Tomasini, Nuovo cimento **9**, 1244 (1952).
[19] Crussard, Mubboux, Morellet, Tremblay, and Orkin-Lecourtois, Compt. rend. **234**, 84 (1952).
[20] C. F. Powell, Copenhagen Conference, June, 1952.

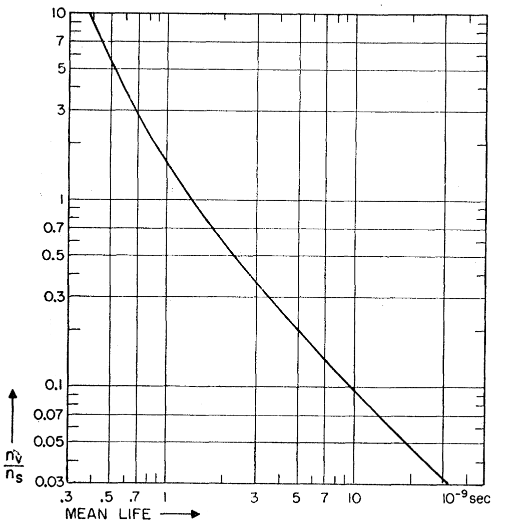

FIG. 8. Ratio, n_v/n_s, of the number of particles disintegrating in flight to the number of particles disintegrating at rest, as a function of mean life, computed under the assumptions mentioned in the text.

Here ν represents a neutrino, γ a photon, π^0 a neutral meson, n a neutron, and V_2^0 the hypothetical neutral V particle, disintegrating into two π-mesons, postulated by Armenteros *et al.*[21] We have assumed the mass of this particle to be 920 m_e, corresponding to a Q value of 190 Mev.

Assumptions (e) and (e′) yield values of m_s appreciably greater than that indicated by scattering. Conversely assumptions (a), (a′), (b), (b′), (c), and (c′) yield values of m_s that appear somewhat too low. Moreover, the fact that we have not observed an electronic component associated with S particle or $V^\pm$ particle decays is an argument against the existence of photons or neutral mesons among the decay products.‡

Assumptions (d) and (d′) yield values of m_s perfectly consistent with the observations, as has already been pointed out by Powell.[20] Moreover, Armenteros *et al.*,[12] have published a cloud-chamber picture in which the decay of a neutral V particle appears to be associated with the decay of a charged V particle. However, our observations on S particles are difficult to reconcile with either assumption (d) or (d′). As already pointed out, if the decay of S-particles is a two-body process, the neutral decay products of our eight S particles have traversed 58 cm of gas and 16 cm of solid material without undergoing a visible decay process. A decay process giving rise to charged particles is certainly detected if it occurs in the gas and is probably detected if it occurs in a plate. If we identify the neutral secondary particles with V_2^0 particles, and if we assume that all V_2^0 particles decay into charged π-mesons, we conclude that their mean free path before decay could not be much shorter than 70 cm. Therefore, considering the mass ($\sim$900 m_e) and the momentum ($\sim$200 Mev/c) of the V_2^0 particles, their mean life could not be much shorter than 5×10^{-9} sec. If half of the V_2^0 particles decayed into pairs of π^0 mesons, the time available for the observation of the decay would be reduced to 2.5×10^{-9} sec. Even so, assumptions (d) and (d′) seem to be inconsistent with existing data on the mean life of V_2^0 particles.[22]

If we assume that $V^\pm$ particles are S particles disintegrating in flight, we can use our data on the relative abundance of S particles and $V^\pm$ particles to obtain some information on their mean life. Indeed, the ratio n_v/n_s between the numbers of $V^\pm$ and S particles observed with a given experimental arrangement is a function of the mean life, τ.

We have computed this function with the following assumptions: (1) the particles entering the chamber are produced at the center of the lead block placed above the chamber; (2) they are distributed uniformly in range and in angle, so that the number of particles stopping in a given plate is proportional to the solid angle subtended by this plate at the center of the lead block; (3) a particle is classified as an S particle if it stops and decays in a plate, as a $V^\pm$ particle if it decays in the gas, and is rejected as an apparent nuclear scattering if it decays in flight while traversing a plate. For the evaluation of the ratio n_v/n_s, a $V^\pm$ particle was only counted as such if it could have appeared in the chamber as an S particle; i.e., its velocity had to be low enough so that, had the particle not decayed in flight, it would have stopped in the chamber and decayed at rest. The results of this computation are shown in Fig. 8.

In the group of pictures considered in this paper we found six S particles, of which four entered the chamber from above; thus $n_s=4$. In the same group of pictures we found three $V^\pm$ particles entering the chamber from above. Of these one ($V2$) would have certainly stopped in a plate if it had not decayed in the gas. The other two had a velocity appreciably smaller than the velocity of light, as shown by their specific ionization. The estimate of the velocity, however, was not sufficiently reliable to decide whether or not the particles would have stopped had they lived long enough. Thus, the number of $V^\pm$ particles which might have come to rest in the chamber lies between 1 and 3. To obtain n_v this number should be divided by two because negative particles may decay in flight, whereas they presumably undergo nuclear absorption after coming to rest in lead. In addition, of course, one must consider the large statistical errors of a determination based on such an extremely small number of events. With these con-

[21] Armenteros, Barker, Butler, and Cachon, Phil. Mag. 42, 1113 (1951).

‡ *Note added in proof.*—More recent observations indicate that, at least occasionally, γ-rays are associated with the decay of S particles.

[22] Fretter, May, and Nakada [Phys. Rev. 89, 168 (1953)] found for this mean life a value of $(4\pm3)\times10^{-10}$ sec; Bridge, Peyrou, Rossi, and Safford (to be published in Phys. Rev.) derived from their observations a mean life of the order of 10^{-10} sec.

siderations in mind, we feel that probably the ratio n_v/n_s lies between 0.05 and 1.0. From Fig. 8 we then conclude that the mean life is probably between 10^{-9} and 2×10^{-8} sec. It may be recalled that Bridge and Annis already gave a lower limit of the order of 10^{-9} sec for the mean life of S particles.[1] For the charged V particles, Barker and his collaborators placed a lower limit of 10^{-10} sec and an upper limit of 10^{-8} sec to the mean life,[14] while Astbury and his collaborators estimated a mean life somewhat greater than 10^{-9} sec.[13]

Our results give some information on the relative abundance of S particles and π-mesons. In the present experiment we found 4 (presumably positive) S particles entering the chamber from above and stopping in one of the plates. We also found 3 particles presumably identical to S particles, which might have stopped in the chamber if they had not decayed in flight; some of these particles, however, were probably negatively charged. In the same set of pictures we found about 130 positive π-mesons coming from above and stopping in the lower half of the chamber. Taking into account the geometry of the experiment, we have crudely estimated that there were about 400 positive π-mesons coming from above and stopping anywhere in the chamber. We then conclude that in the interval of ranges explored in the present experiment the ratio of S particles to π-mesons was approximately 1/70.

The experiments discussed in this paper were performed at the Inter-University High-Altitude Laboratory at Echo Lake, Colorado. The authors gratefully acknowledge the assistance of Mr. Robert Hewitt in the operation of the cloud chamber. They are indebted to Messrs. S. Olbert, H. Courant, and M. S. Rifai for their help in the analysis of the data.

APPENDIX

1. Estimate of the Momentum

The paper by Annis, Bridge, and Olbert[4] (referred to as "II" in what follows) describes a method for obtaining the maximum likelihood estimate of the momentum of a particle in the case that the momentum loss in the plates is negligible. Here we shall discuss the same problem in the case that the momentum loss in the plates is not negligible.

Consider a particle crossing n plates without reaching the end of its range. We use the following notations:

θ, the angle of the trajectory with the perpendicular to the plates (for simplicity, we assume that θ is the same for all plates, thus disregarding small changes due to scattering and to the different tilt of the plates);

t_0, the thickness of the plates;

$t=t_0/\cos\theta$, the thickness of the plates in the direction of the trajectory;

φ_i, the projected angle of scattering in the ith plate;

Π_i, the value of the product $p\beta c$ at the center of the ith plate;

R_i, the corresponding residual range.

Under the normal approximation, the probability that the projected scattering angle in the ith plate has a value between φ_i and $\varphi_i+d\varphi_i$ is given by

$$\frac{1}{\sqrt{\pi}}\frac{1}{\sigma_i}\exp(-\varphi_i^2/2\sigma_i^2), \tag{A1}$$

where

$$\sigma_i=K_2t^{\frac{1}{2}}m_ec^2/\Pi_i \tag{A2}$$

represents the root-mean-square angle of scattering for a particle of $p\beta c=\Pi_i$ (see Eq. (II-17)). If we put

$$K_2t^{\frac{1}{2}}m_ec^2=h, \tag{A2$'$}$$

Eq. (A2) becomes

$$\Pi_i\sigma_i=h. \tag{A3}$$

For our plate thickness the constant h has the value $h=900/(\cos\theta)^{\frac{1}{2}}$ Mev degrees.

The probability of observing a consecutive series of scattering angles between φ_i and $\varphi_i+d\varphi_i$ in the n plates is the product of the expressions (A1) relative to the various plates. Apart from a constant, the logarithm of this probability is given by the equation

$$\Phi=\sum(\ln\Pi_i-\Pi_i^2\varphi_i^2/2h^2). \tag{A4}$$

Let p be the initial momentum, i.e., the momentum of the particle before it enters the uppermost plate. For given observed values of the scattering angles, the quantity $(\exp\Phi)dp$ is proportional to the probability that the initial momentum lie between p and $p+dp$, *provided all values of p are* a priori *equally probable.* Thus the maximum likelihood estimate of p is given by the equation

$$\partial\Phi/\partial p=0. \tag{A5}$$

It is convenient to regard Φ as a function of p through the intermediary of the quantity R_0, which represents the residual range at the midpoint of the trajectory. This point lies at the center of one plate or midway between two plates depending on whether n is odd or even. We shall number the plates from this point; more specifically, we shall ascribe to i the values $\frac{1}{2}(n-1)$, $\cdots 1$, 0, -1, $\cdots -\frac{1}{2}(n-1)$ if n is odd, and $\frac{1}{2}(n-1)$, $\cdots\frac{1}{2}$, $-\frac{1}{2}$, $\cdots -\frac{1}{2}(n-1)$ if n is even. We have, then,

$$R_i=R_0+it. \tag{A6}$$

Equation (A5) yields

$$\partial\Phi/\partial R_0=0,$$

or, remembering Eq. (A4) and considering that $d\Pi_i/dR_0=d\Pi_i/dR_i$,

$$\sum\frac{1}{\Pi_i}\frac{d\Pi_i}{dR_i}-\frac{1}{h^2}\sum\Pi_i\frac{d\Pi_i}{dR_i}\varphi_i^2=0. \tag{A7}$$

Let

$$\alpha=\frac{d(\ln\Pi)}{d(\ln R)}=\frac{R}{\Pi}\frac{d\Pi}{dR} \tag{A8}$$

be the slope of the curve representing $\ln\Pi$ *vs* $\ln R$. Equation (A7) can then be written as follows:

$$\sum \frac{\alpha_i}{R_i} - \frac{1}{h^2} \sum \frac{\Pi_i^2}{R_i} \alpha_i \varphi_i^2 = 0.^{23} \qquad \text{(A9)}$$

Consider now that α is a very slowly varying function of Π, whose value in the region of interest for our experiments is close to $\frac{1}{2}$, so that R is nearly proportional to Π^2. Consider further that

$$\sum \frac{1}{R_i} = \frac{1}{R_0} \sum_{-(n-1)/2}^{(n-1)/2} \frac{1}{1+it/R_0} = \frac{2}{R_0} \sum_{0,\frac{1}{2}}^{(n-1)/2} \frac{1}{1-(it/R_0)^2} \approx \frac{n}{R_0}.$$

One sees that Eq. (A9) is approximately equivalent to

$$\frac{n}{R_0} - \frac{1}{h^2} \frac{\Pi_0^2}{R_0} \sum \varphi_i^2 = 0, \qquad \text{(A10)}$$

or

$$\sigma_0^2 = (1/n) \sum \varphi_i^2, \qquad \text{(A11)}$$

where Π_0 is the value of Π at the center of the trajectory and $\sigma_0 = h/\Pi_0$ is the corresponding value of the rms angle of scattering.

We conclude from Eq. (A11) that the most probable momentum of the particle, computed without consideration of the ionization loss, represents, with good approximation, the most probable momentum of the particle at the center of the trajectory, p_{0l}, when ionization loss is taken into account. If we call R_{0l} the corresponding range, the most probable value of the *initial* momentum, p_l corresponds to the range $R_{0l} + \frac{1}{2}nt$, and is thus obtained from the equation

$$p_l = p(R_{0l} + \tfrac{1}{2}nt), \qquad \text{(A12)}$$

where $p(R)$ represents the functional relation between momentum and range.

Starting from this approximate solution of Eq. (A9), it is easy to obtain an exact solution either by successive approximations, or by graphical construction. It turns out, however, that the approximate solution is usually adequate to all practical purposes.

The uncertainty in the estimate of the momentum shall be characterized by the quantities Δp^+ and Δp^-, such that the probability function $\exp\Phi$ at $p^+ = p_l + \Delta p^+$ and at $p^- = p_l - \Delta p^-$ equals $e^{-\frac{1}{2}}$ times the same function at p_l.

[23] It may be appropriate to point out that the solution of Eq. (A9) gives the maximum likelihood estimate not only for p, but for any other quantity (such as the initial residual range, R, the residual range at the center of the trajectory, R_0, the momentum p_0 at the center of the trajectory, etc.) which is a unique function of p and whose value, therefore, determines the values Π_i uniquely. The above statement means that if x is one of these quantities, Eq. (A9) defines the most probable value of x, *under the assumption that all values of x are* a priori *equally probable.*

We begin by computing p^+ and p^- without considering ionization loss, as explained in II. We then interpret the quantities thus computed as approximate values for the upper and lower limits of the momentum *at the center of the trajectory*, p_0^+ and p_0^-. Starting from these approximate values, it is comparatively easy to obtain exact solutions of the equations

$$\Phi(p_0^+) = \Phi(p_0^-) = \Phi(p_{0l}) - \tfrac{1}{2}, \qquad \text{(A13)}$$

by a method of graphical interpolation. One finds that the exact value of p_0^+ never differs appreciably from the approximate value, whereas the exact value of p_0^- differs appreciably from the approximate value only if the particle is near the end of its range when it leaves the last plate. Once p_0^+ and p_0^- have been determined, one computes the corresponding ranges R_0^+ and R_0^- and one obtains p^+ and p^- with the equations:

$$p^+ = p(R_0^+ + \tfrac{1}{2}nt); \quad p^- = p(R_0^- + \tfrac{1}{2}nt). \qquad \text{(A14)}$$

For the effect of noise level scattering on momentum determinations, see II, Appendix II.

2. Estimate of the Mass

The problem of estimating the mass, m, of a particle, or of a group of identical particles stopping in the chamber has been discussed in II.

For each measured projected angle of scattering, φ_i, we compute the quantity

$$\eta_i = \varphi_i R_i^{\alpha} (\cos\theta_i)^{\frac{1}{2}}, \qquad \text{(A15)}$$

where R_i is the residual range, measured from the center of the plate, θ_i is the angle of the trajectory with the perpendicular to the plate and α is the exponent in the approximate equation,

$$A_z \Pi / mc^2 = (R/mc^2)^{\alpha}, \qquad \text{(A16)}$$

connecting the range with the value of $\Pi = p\beta c$ (see Eq. (II–36); for the correction due to the momentum loss in each individual plate, see II, Appendix I). We then form the quantity

$$S_2^2 = \frac{1}{n} \sum_1^n \eta_i^2. \qquad \text{(A17)}$$

Under the normal approximation, the probability distribution of S_2 is given by

$$\Psi_n(S_2, \rho) = \text{const}\, \rho^{-n} S_2^{n-1} \exp(-nS_2^2/2\rho^2), \qquad \text{(A18)}$$

where ρ is the quantity defined by Eq. (II–38), which may be rewritten as follows:

$$\rho = a t_0^{\frac{1}{2}} (m_e/m)^{1-\alpha}. \qquad \text{(A19)}$$

The equation

$$\partial \Psi_n / \partial \rho = 0 \qquad \text{(A20)}$$

defines the maximum likelihood estimate of ρ, ρ_l. Substitution of this value in Eq. (A19) gives the most probable value of the mass, m_l, computed under the assumption that all values of m are *a priori* equally probable. Equation (A20) is equivalent to

$$\frac{\partial}{\partial\chi}[\chi^n \exp(-\chi^2)]=0, \tag{A21}$$

where

$$\chi^2=nS_2^2/2\rho^2. \tag{A22}$$

Equation (A21) has the solution

$$\chi_l=(n/2)^{\frac{1}{2}}, \tag{A23}$$

from which it follows that

$$\rho_l=S_2. \tag{A24}$$

To estimate the uncertainty in the mass determination, we compute the quantities $\Delta\chi^+$ and $\Delta\chi^-$ such that the function $\chi^n \exp(-\chi^2)$ at $\chi_l+\Delta\chi^+$ and at $\chi_l-\Delta\chi^-$ equals $e^{-\frac{1}{2}}$ times the value of the same function at χ_l. Equations (A19) and (A22) give then the corresponding values of Δm^+ and Δm^-. Notice that $\Delta\chi^+/\chi_l$ and $\Delta\chi^-/\chi_l$ are identical to the quantities $\Delta\Pi^+/\Pi_l$ and $\Delta\Pi^-/\Pi_l$ listed in Table I of reference 4.

The value of α has an uncertainty of the order of a few percent, which introduces a similar uncertainty in the value of m. In our computations we have used the value $\alpha=0.55$. The value of $at_0^{\frac{1}{2}}$ in Eq. (A19) corresponding to this value of α and to the thickness of our plates is 660.

The influence of noise level scattering on mass determinations is discussed in II, Appendix II.

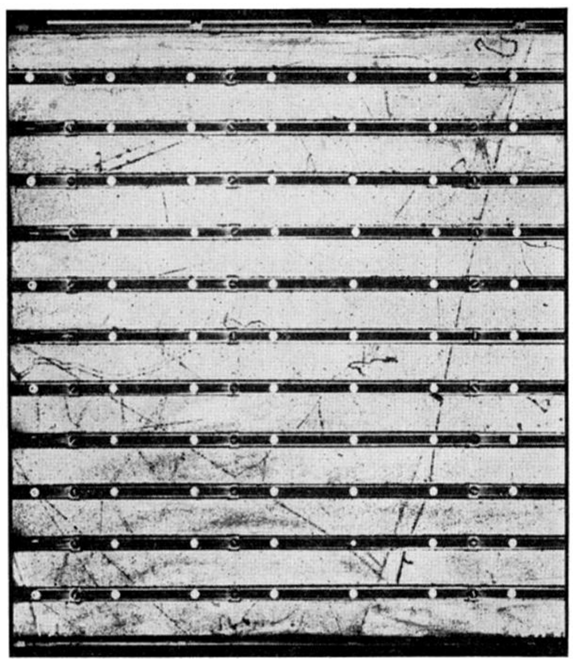

FIG. 2. S particle (event $S7$).

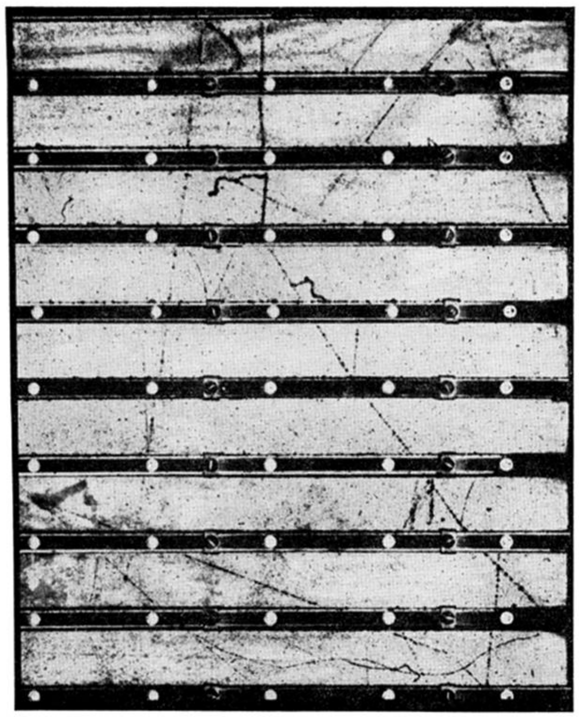

FIG. 3. *S* particle (event *S*8).

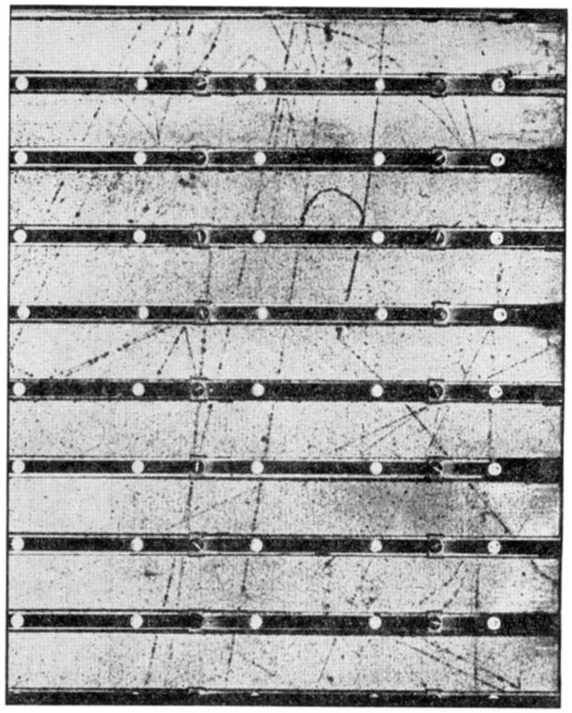

FIG. 5. Charged *V* particle (event *V*2).

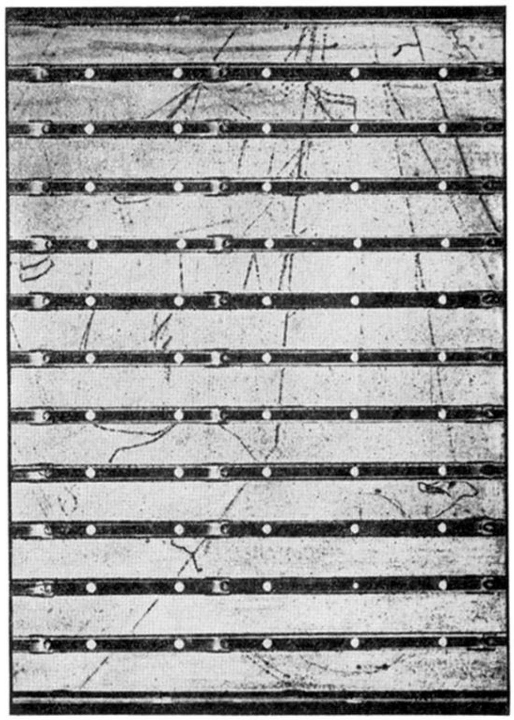

FIG. 6. Charged *V* particle (event *V*3).